Three-dimensional Anderson localization in nearly magnetic fermion systems

M. T. Beal-Monod
Laboratoire de Physique des Solides, Université de Paris-Sud,
Bâtiment 510, 91405 Orsay, France
(Received 19 February 1985; revised manuscript received 8 October 1985)

In disordered three-dimensional itinerant fermion systems close to a magnetic instability, (a) the uniform paramagnons, becoming stronger in the presence of weak disorder, drive the system closer to magnetism and away from the localization transition, in agreement with recent theories in $d=2+\epsilon$. However, (b) since the paramagnons also tend to become momentum independent, they are expected to become localized in the strongly disordered regime; if this situation is achieved before the magnetic instability is reached, then the old results, according to which local paramagnon theory scales towards a vanishing-interaction theory, could possibly be used to understand how the metal-insulator transition can occur in the presence of a strong interaction.

One of the most interesting and puzzling problems, at present, is to understand the metal-insulator transition in the presence of strong interactions among the fermions.[1] The latest developments, so far, are those using either the n-orbital model in two and three dimensions,[2] or, in $2+\epsilon$ dimensions, field-theoretical renormalization-group calculation,[3] generalized diagrammatically.[4] According to these theories, the present status of the problem is described, qualitatively, as follows:

(1) While the metal-insulator transition is well understood in noninteracting systems,[5] the presence of strong interactions tends to develop spin alignment, and strong spin fluctuations are generated which increase the conductivity and thus prevent the localization transition from occurring at least in the absence of spin-orbit coupling for nonmagnetic impurities.

(2) However, these spin fluctuations tend, upon scaling, to become momentum (q) independent so that, actually, "local" spin fluctuations are formed. The authors of Refs. 3 and 4 then conjecture that the onset of local spin fluctuations may allow the localization transition by a possible "suppression of triplet fluctuations and the crossover to the singlet-only regime," as concluded in Ref. 4, or, alternatively, in Ref. 3, the "localized spin-density islands will magnetize the remaining electrons" and thus "the direct exchange of localized moments . . . leads to an antiferromagnetic interaction of moments which will block the ferromagnetic interaction occurring due to the indirect exchange."

Part (1) of these conclusions had emerged already from perturbative theories of disordered interacting electrons;[6] on the other hand, the near-q independence arose in the n-orbital model near a magnetic instability[2] and has also recently been derived from a different approach,[7] within the paramagnon model. Within that last model, I will show that such a result indeed implies that the uniform (to start with) paramagnons tend to switch, for a much stronger disorder, to local paramagnons. Therefore, one can finally make use of somewhat old results obtained for the problem of local paramagnons,[8] which yield a scaling to an effective interaction which is vanishingly small (although the interaction one started with was actually strong). In that case, one recovers the result obtained for noninteracting fermions,[5] i.e., the metal-insulator transition can occur, in agreement with the conjecture of Refs. 3 and 4. Modifications due to the addition of spin-orbit coupling will also be noted.

In the following I will use the paramagnon model as in Ref. 7: a dimensionless, Hubbard-type, instantaneous contact repulsion I among opposite spins, of order 1, so that the system is close to a ferromagnetic instability at $T=0$ (the temperature that I consider in the following). I will also restrict the discussion to the case of a single parabolic band of fermions for simplicity, although in real systems and for quantitative estimates band-structure effects ought to be included. Then the $T=0$ conductivity σ contains two types of corrections to the Boltzmann conductivity σ_0:

$$\sigma=\sigma_0(1+\delta\sigma_L/\sigma_0+\delta\sigma_{I,L}/\sigma_0) \ , \tag{1}$$

where $\delta\sigma_L/\sigma_0$ is the localization correction in the absence of interactions;[5] in perturbation theory[1]

$$\delta\sigma_L/\sigma_0=-(3/4\pi)(\epsilon_F\tau_0)^{-2}, \quad (\epsilon_F\tau_0)^{-2}<1 \ , \tag{2}$$

where $\delta\sigma_{I,L}/\sigma_0$ is the perturbative correction due to the mixed effect of interactions and disorder through the particle-hole diffusion propagators (phDP) and the particle-particle diffusion propagators (ppDP);[1,6] it was computed, in the Hubbard model of Isawa and Fukuyama[9] (IF)

$$\delta\sigma_{I,L}/\sigma_0=-(3\sqrt{3}/16)g(\epsilon_F\tau_0)^{-2}, \quad g(\epsilon_F\tau_0)^{-2}<1 \ . \tag{3}$$

In both formulas (2) and (3), τ_0 is the elastic lifetime and ϵ_F the Fermi energy. g is a coupling constant containing both interactions and disorder. Let us consider the two following cases:

(a) In the absence of spin-orbit coupling, g is

$$g=g_1-2g_3+g_2-2g_4 \ , \tag{4}$$

as given in Ref. 10 to first order in the interaction as well as by IF to higher orders. The phDP contribution enters into g_1-2g_3 and the ppDP one enters into g_2-2g_4 (g_1 here corresponds to g_{1H} of IF in the Hubbard model; as in Ref. 7, I have replaced the notation $F/2$ of IF by $\bar{I}$, $0<\bar{I}\lesssim 1$). Recall that g_3,g_4 and g_1,g_2 are coupling constants entering as, respectively, Hartree and Fock types of self-energy corrections to the fermion Green's functions. One gets

$$g_1=(4/3\bar{I})[-2+(1+\bar{I})^{-1/2}+(1-\bar{I})^{-1/2}] \ , \tag{5a}$$

$$g_3=(4/3\bar{I})[-2-\bar{I}+2(1-\bar{I})^{-1/2}] \ , \tag{5b}$$

$$g_c=g_2-2g_4=-g_4=-g_2\cong-\bar{I}[1+\bar{I}\ln(1.13\epsilon_F/T)]^{-1} \ . \tag{5c}$$

Note that the ppDP contribution to g, g_c is negative whatever $\bar{I}$ is, between 0 and 1. Although "multiply crossed diagrams," in the absence of interactions, contribute to decrease the conductivity σ, when they contain interaction insertions (or paramagnons here) they will contribute to increase σ, although by a very modest amount. Note that from Eqs. (5), there effectively appear three coupling constants (rather than four): two phDP type ones g_1, g_3 and one ppDP type $g_c = g_2 - 2g_4 = -g_4$. This is in qualitative agreement, to lowest order, with the three coupling constants in Refs. 3 and 4. Let us write the phDP contribution differently:

$$g_1 - 2g_3 = g_s + g_t \ , \tag{6a}$$

$$g_s = g_1 - g_3/2 \ , \tag{6b}$$

$$g_t = -3g_3/2 \ , \tag{6c}$$

so that (4) actually reads

$$g = g_s + g_t + g_c \ . \tag{7}$$

The form (6a) explicitly separates the "singlet" (g_s) and the "triplet" (g_t) contributions of the fluctuations as emphasized in Ref. 11 and later on in Refs. 3 and 4 [the factor 3 in (6c) corresponds to the multiplicity in the triplet contribution]. With the $\bar{I}$ dependences of (5) inserted into (6) one gets

$$g_s = (4/3\bar{I})[-1 + \bar{I}/2 + (1+\bar{I})^{-1/2}], \quad 0 \leq \bar{I} \lesssim 1 \ , \tag{8a}$$

$$g_t = (4/\bar{I})[1 + \bar{I}/2 - (1-\bar{I})^{-1/2}] \ . \tag{8b}$$

Therefore, first, it is clear that g_s is positive whatever $\bar{I}$ is; its contribution, were it the only one, would *decrease* σ. However, in contrast, g_t is negative and, moreover, becomes very large when $\bar{I} \to 1$, in which case g_t [$\sim -(1-\bar{I})^{-1/2}$] predominates over $g_s (\sim \text{const})$ in (6a) in order to *increase* σ and prevent Anderson localization from occurring. Such a result was used in Ref. 7 to show that near the magnetic instability ($\bar{I} \to 1$) the combined effect of normal impurity scattering and strong interactions decreases the transport relaxation rate τ_{tr}^{-1} (proportional to σ^{-1}) so that the pair-breaking parameter (τ_{tr}^{-1}) preventing triplet-pairing superconductivity is weakened. The above remarks also agree with the conclusions of Refs. 3 and 4 that "triplet" contribution is responsible for driving the system away from the localization transition while the "singlet-only" contribution would favor it. Let us also note that the neglect of the ppDP contribution in Ref. 4 appears reasonable within the present model, where g_c, although negative like g_t, is negligible compared to g_t when $\bar{I} \to 1$. Finally, while, separately, g_s is positive and g_t negative whatever $\bar{I}$ is, the *combination* $g_s + g_t$ is *negative* for *all* values of $\bar{I}$ between 0 and 1. In other words, in the Hubbard model, i.e., for a contact interaction among opposite spins, the $T = 0$ conductivity is always increased no matter what the strength of the interaction is, while for the usual long-range Coulomb interaction[1,6,9] it can be decreased.[12]

(b) In the presence of spin-orbit coupling, of importance for some experimental cases, g is modified and becomes g_{SO}. It was computed to lowest order in the interaction in Ref. 13. Note that although Ref. 13 is, in principle, devoted to the two-dimensional (2D) case, it actually computes the spin-orbit scatterings in 3D to account for the finite thickness of the film [as is clear from formulas (2) of Ref. 13 and the few lines afterwards]; g_{SO} exhibits[13] a combination of the g_l's different from that of g[9,10] recalled in formula (4); the coefficients of the g_l's, aside from the number of diffusion propagators, depend on specific summations over spin indices which are different in the presence and in the absence of spin-orbit coupling (independent of the space dimensionality[14]). Then, from Ref. 13, one gets

$$g_{SO} = g_1 - (g_2 + g_3 + g_4)/2 \tag{9a}$$

$$= g_s - (g_2 + g_4)/2 \ , \tag{9b}$$

where I have used the formal definition (6b) for g_s. The relation $g_2 - 2g_4 = -g_4$ given by IF and taken at $q = 0$ and $\omega = 0$ [Eq. (2.23) of IF], which yields $g_2 = g_4$, tells us that (9b) reads

$$g_{SO} = g_s + g_c \ . \tag{10}$$

Note that, although τ_0^{-1} switches[13] to $\tau^{-1} = \tau_0^{-1} + \tau_{SO}^{-1}$ in the presence of spin-orbit coupling, the lifetime does not explicitly appear in the $\bar{I}$ dependences of the g_l's in (5); however, it is, in principle, hidden in $F/2$ itself in IF (or $\bar{I}$ here).

But since this is considered as an averaged constant phenomenological parameter, one assumes one can speak of a unique one valid in (a) and (b). Using the relation $g_2 = g_4$ here requires the same approximation.

Now comparing (10) with (7), the important result is that the *triplet* term has *disappeared* and, therefore, since g_s is *positive,* the overall g_{SO} can be positive so that σ may decrease and the localization transition can be reached. This is in agreement with the conclusions of Refs. 11 and 15; while the spin-orbit contribution can be an antilocalization factor in the absence of interactions, when strong interactions are present, the spin-orbit contribution acts to suppress the triplet contribution, and the combined effect of spin-orbit plus "singlet-only" contributions can actually allow the localization to take place. However, from Eq. (10), since g_s is positive and g_c negative for $0 < \bar{I} < 1$, the final result now will depend crucially on the balance between them. Equation (5c), as emphasized by IF, is an underestimate of the actual g_4; a better estimate was provided by IF, but only for two dimensions. I do not elaborate on this point here, where only qualitative features are considered. If the balance between g_s and g_c is such that $g_s + g_c$ happens to be negative, then the situation may be qualitatively similar to that in the absence of spin-orbit coupling: σ may be increased, at least if (3) still wins against (2), τ_{tr}^{-1} would then decrease and triplet superconductivity could occur despite impurities *and* spin-orbit coupling. Instead, if $g_s + g_c$ is positive, the impurities plus the spin-orbit contribution will be very effective in preventing triplet pairing.

To summarize this first part, the above remarks contained in (a) and (b) are in agreement with those obtained in a more sophisticated way in Refs. 3, 4, and 15.

Let us now forget about case (b), since it is not incompatible with the occurrence of the metal-insulator transition, and go back to case (a), which remains, by far, the puzzling one. I then turn to what I think might be a key point in disentangling that case. As shown in Ref. 7, the paramagnon spectrum $\mathrm{Im}\chi(q, \omega)$ considered as a function of ω for fixed q, is narrower and more peaked in the presence of disorder than in the pure system ($\tau_0^{-1} = 0$): The disorder renders the paramagnons stronger and the system is expected to become closer to magnetism (depending on the band structure, if the interaction $\bar{I}_{ren}$, renormalized by these stronger paramagnons,[7] happens to be closer to 1 than the

bare $\bar{I}$ one started with, as can be evaluated through the zero-temperature enhancement of the renormalized spin susceptibility[16]). This agrees with the results of Refs. 3 and 4 that spin alignment is favored by disorder and that strong spin fluctuations are generated. It also agrees with the result of the n-orbital model of Ref. 2: The q dependence of the inverse Stoner factor is weakened by disorder. I found in Ref. 7

$$S^{-1}=1-\bar{I}+\bar{I}\frac{q^2}{12k_F^2}\left[1-\frac{1}{3(4\epsilon_F\tau_0)^2}\right] , \tag{11}$$

to be compared with a similar result of Ref. 2:

$$S^{-1}=1-J_0\rho_F+J_0q^2\left[R_J-R_M\left[1-\frac{1}{2E_0\rho_F}\right]\right] , \tag{12}$$

in which "R_J and R_M denote the range of exchange interaction and disorder scattering, respectively," and $E_0\rho_F=2(E_0^2-\epsilon_F^2)^{1/2}\pi E_0$, with $2E_0$ the width of the semielliptic impurity band in Ref. 2 ($J_0\rho_F$ identifying with $\bar{I}$). It is clear that, for increasing disorder, the coefficient of q in (12) decreases [as in (11)] when E_0 increases [the disorder increases with $(E_0-E_F)^{1/2}$]; therefore, formulas (11) and (12) are very similar. From (11) for instance, it follows that the paramagnon correlation length in the disordered case λ_{dis} is

$$\begin{aligned}&\lambda_{\text{dis}}=p\,\lambda_{\text{pure}} ,\\ &\lambda_{\text{pure}}=(2\sqrt{3})^{-1}\lambda_F\sqrt{\bar{I}/(1-\bar{I})} ,\\ &p=\sqrt{1-(1/3)(4\epsilon_F\tau_0)^{-2}} .\end{aligned} \tag{13}$$

λ_{dis} is shorter compared to λ_{pure}, since $p<1$ for finite τ_0^{-1}, and is closer to the Fermi wavelength λ_F when the disorder increases. Obviously (11), obtained as a perturbation expansion in powers of $(\epsilon_F\tau_0)^{-1}$, holds as such only for weak disorder; an extrapolation to the strong disorder regime, when $\epsilon_F\tau_0\sim 1$, would imply the calculation of higher-order terms. For the same perturbative reason [$g<(\epsilon_F\tau_0)^2$ from (3)] p remains larger than $\sqrt{1-\bar{I}}$, and λ remains larger than λ_F. Therefore, at this stage one can just note that the coefficient of q^2 in (11) is still positive (corresponding to a *nearly ferromagnetic* instability when $\bar{I}\to 1$), but decreases when the disorder increases. In this weakly disordered regime, the paramagnons are just stronger as shown in Ref. 7, the system is closer to becoming magnetic, and all the conclusions recalled above in (a) apply: σ is increased, triplet superconductivity is favored, and the metal-insulator transition prevented. However, in the following, I would like to point out some (possibly crucial) consequences, with a crossover towards a finally different situation, if one supposes that these pessimistic perturbative results persist in the strong-disorder regime.

More precisely, let us examine what could happen if a more elaborate calculation of p in the strong disorder regime $\epsilon_F\tau_0\lesssim 1$ would show that p still decreases when τ_0^{-1} increases, and that it eventually vanishes: either the magnetic instability is reached before the q dependence disappears —then the system would switch to an unconventional magnetically ordered phase (possibly of spin-glass type[17]) before the metal-insulator transition—, or the q dependence disappears before the magnetic instability is reached.

(i) Firstly, the coefficient of q^2 would vanish: The paramagnons would become q independent (the Kramers-Kronig relation would require that if the momentum dependence is lost in the real part of the spin correlation function, then it would be also in the imaginary part). Then the uniform (to start with) paramagnons would switch to *local* paramagnons (as inferred in Refs. 3 and 4). This sounds reasonable: If the electrons tend to localize, the paramagnons which are formed out of these electrons are expected to localize also. These remarks will be further used below, after (ii), to offer a possible explanation of the localization transition in the presence of strong interactions. (Such spin fluctuations, with no q structure, are similar to the Gutzwiller-Brinkmann-Rice-type spin fluctuations.[18])

(ii) Increasing the disorder further, $\epsilon_F\tau_0<1$, if the coefficient of q^2 may eventually become negative, this would correspond to a magnetic instability at finite q, i.e., of an *antiferromagnetic* type. This was indeed remarked in the n-orbital model of Ref. 2. Such a possibility, although very qualitative here, might be important in view of some experimental results. For instance, the compound $Ce(In_xSn_{1-x})_3$ studied in Ref. 19 behaves at very low temperatures, for low x values, as a strongly enhanced paramagnet with all the characteristic features of a nearly ferromagnetic Fermi liquid; however, when the In concentration x increases (and thus the disorder increases) above a certain concentration x_c, the compound orders with a (complicated) antiferromagnetic structure. Less clear-cut examples could be cited: $CeAl_3$ behaves as a nearly ferromagnetic paramagnet down to 0 K, while $CeAl_2$ orders antiferromagnetically; liquid ^{3}He is a nearly ferromagnetic Fermi liquid as well as a nearly solid,[7] but switches, above a certain pressure, to an antiferromagnetic solid. I do not claim that these cases are identical, but it is striking that, very often, nearly ferromagnetic itinerant systems, when they order, do so with some kind of antiferromagnetic structure.

I now ignore (ii) and come back to (i), and to the main point that I wish to raise here. Let us suppose that, in the *strongly disordered* regime, the momentum independence of the paramagnon overtakes the closeness to magnetism: Crudely speaking, suppose that a formula analogous to (13) holds in the strongly disordered regime and that p vanishes before $1-\bar{I}_{\text{ren}}$ can do so, with $\bar{I}_{\text{ren}}$ the renormalized interaction defined earlier. Then, *uniform* paramagnons become *local* ones before the magnetic instability is reached. The problem of local spin fluctuations, i.e., local paramagnons, is not new,[20] but has been studied, so far, as follows: a host of itinerant fermions interacting through the above Hubbard-type contact repulsion $\bar{I}$, with $\bar{I}$ between 0 and 1 was considered; then the effect of the randomly distributed impurities was just supposed to modify the value of $\bar{I}\to\bar{I}'=\bar{I}+\Delta\bar{I}$ locally, with $\Delta\bar{I}>0$ or <0. The problem was treated as a uniform paramagnon one with the Stoner enhancement $(1-\bar{I})^{-1}$ everywhere, plus, on the impurity sites, local paramagnons with the local Stoner enhancement $(1-\Delta\bar{I})^{-1}$, but the fact that the impurities also introduce a mean free path for the host electrons was never taken into account. Had it been, it would have been precisely the problem we are concerned with here. On the other hand, in the literature concerning localization of interacting electrons, a possibly different value $\bar{I}'$ among the electrons when they interact on the impurity sites has not been taken into account.

In conclusion, we have seen in this paper that, from Refs. 3 and 4 as well as 2 and 7, as shown above, the uniform paramagnons are expected to become localized when the

disorder becomes strong; in that case these new local paramagnons add to the local ones that were present to start with (due to the difference $\Delta\bar{I}$ in the interaction values on the impurity sites). Therefore, both types of paramagnons tend to merge into a *unique* one of *local type*. Then it seems reasonable to consider the system, at this stage, as an overall system of *only local* paramagnons. If that is so, then one could use what has been done in the past[8] in the theory of "local paramagnons" on nearly magnetic impurities: When these impurities were far from being magnetic it was shown in Ref. 8 that interactions among local paramagnons ("mode-mode coupling" contributions) render these impurities even farther from becoming magnetic. In other words, the system is expected to scale to a situation for which the *effective* local interaction would tend to *zero,* i.e., a weak coupling theory (a correspondence was proposed in the second of Ref. 8 between such a vanishing interaction and the $J=-\infty$ Kondo impurity case). In that vanishing-interaction limit and if the problem we are concerned with here effectively reduces to a problem of local paramagnons with effective, vanishingly small, local interactions, then, in (1), $\delta\sigma_{I,L}/\sigma_0$ would become first smaller and eventually negligible compared to $\delta\sigma_L/\sigma_0$, i.e., one would recover a situation similar to the noninteracting case of Ref. 5, where σ is decreased compared to σ_0 and the Anderson localization occurs for a certain amount of disorder.

To summarize this last part, the uniform strong spin fluctuations present in a disordered nearly magnetic fermion system tend to become localized under the influence of strong disorder; then, since the local spin-fluctuation theory tends to scale to a free one, one can expect this strongly interacting system to *actually behave as a free one* for which Ref. 5 tells us that the Anderson localization can be reached. Note, however, that in order to become possible, the metal-insulator transition does not require that the effective interaction have reached its zero-value fixed point. The interaction may still be *finite,* but such that $\delta\sigma_{I,L} < |\delta\sigma_L|$, as mentioned above. Such a situation might be illustrated by the Si:P case,[18,21] where one has experimental signatures of the presence of interactions when one reaches the metal-insulator transition from the metallic side.

The importance of spin fluctuations has been stressed elsewhere[18,22] but what is new here is the proposed crossover for increasing disorder from an effective interaction first strengthened by disorder, to one which becomes weaker according to Ref. 8 and thus allows the localization transition to take place.

One more remark is in order: If one reaches the zero-value fixed point of the effective interaction before the metal-insulator transition is achieved, then the transition may follow just because of the localization theory[5] for free electrons and independent impurities; but even if, depending on their concentration, the impurities can no longer be considered as independent from one another, one may still get localization of the electrons. It has indeed been shown long ago[23] (in a very simple way within the first Born approximation) that interference effects among scattered waves of *free electrons on coupled impurities* always decrease the conductivity; a local order among the impurities acts as a localizing factor for the electrons. As suggested by Friedel, this might apply to the case of amorphous quench-condensed Al_xGe_{1-x} mixtures[24] where the metal-insulator transition may be reached by weak annealing of metallic samples initially close to the transition. In other words, one can qualitatively understand the occurrence of a metal insulator transition even in the presence of interactions either among the electrons or between impurities.

Finally, I emphasize that in two dimensions, the nearly magnetic fermion system cannot be understood the same way, since it is still an open problem both in the pure case[25] and the disordered one.[26]

Note added. Since this paper was submitted, a short summary was presented, but without detailed explanations, at the International Conference on Magnetism, San Francisco, 1985.

I am grateful for useful discussions with C. di Castro, G. Forgacs, P. A. Lee, R. Oppermann, and T. M. Rice, and with my colleagues in Orsay, H. Raffy, H. Schulz, and T. Giarmarchi. The Laboratoire de Physique des Solides is a "laboratoire associé au Centre National de la Recherche Scientifique."

---

[1]See the review by P. A. Lee and T. V. Ramakrishnan, Rev. Mod. Phys. **57**, 287 (1985).

[2]R. Oppermann, Solid State Commun. **44**, 1297 (1982); in *Proceedings of the International Conference on Localization, Interaction and Transport Phenomena in Impure Metals, Braunschweig, West Germany, 1984,* edited by B. Kramer, G. Bergman, and Y. Bruynseraede, Springer Tracts in Solid-State Sciences, Vol. 61 (Springer-Verlag, New York, 1985), p. 121.

[3]A. M. Finkel'stein, Zh. Eksp. Teor. Fiz. **84**, 168 (1983) [Sov. Phys. JETP **57**, 97 (1983)]; Z. Phys. B **56**, 189 (1984); Pis'ma Zh. Eksp. Teor. Fiz. **40**, 63 (1984) [JETP Lett. **40**, 796 (1984)].

[4]C. Castellani *et al.*, Phys. Rev. B **30**, 527 (1984); **30**, 1593 (1984); **30**, 1596 (1984).

[5]E. Abrahams, P. W. Anderson, D. C. Licciardello, and T. V. Ramakrishnan, Phys. Rev. Lett. **42**, 673 (1979); L. P. Gor'kov, A. I. Larkin, and D. E. Khmel'nitskii, Pis'ma Zh. Eksp. Teor. Fiz. **30**, 248 (1979) [JETP Lett. **30**, 228 (1979)].

[6]See the reviews by B. L. Altshuler and A. G. Aronov, in *Electron-Electron Interactions in Disordered Systems,* edited by A. L. Efros and M. Pollak (North-Holland, Amsterdam, in press); H. Fukuyama, *ibid.*

[7]M. T. Béal-Monod, Phys. Rev. B **31**, 1647 (1985).

[8]M. T. Béal-Monod, K. Maki, and J. P. Hurault, J. Low Temp. Phys. **17**, 439 (1974); **17**, 467 (1974); in Eq. (2) of the first of these papers the coefficient in front of the right-hand side should read $\frac{1}{2}$ instead of $\frac{1}{4}$.

[9]Y. Isawa and H. Fukuyama, J. Phys. Soc. Jpn. **53**, 1415 (1984); note a slight misdrawing of Fig. 1(b) of this reference: each doubly-dashed line should be drawn as a crossed one in order to be consistent with the definition of Fig. 2(a).

[10]H. Fukuyama, J. Phys. Soc. Jpn. **50**, 3407 (1981).

[11]B. L. Altshuler and A. G. Aronov, Solid State Commun. **46**, 429 (1983).

[12]Note that in Ref. 9, in the long-range Coulomb case, g does not vanish with $F/2$, while it does in the Hubbard case.

[13]H. Fukuyama, J. Phys. Soc. Jpn. **51**, 1105 (1982).

[14]It is, for instance, clear in IF that g is expressed the same way as function of the g_l's in $2d$ and in $3d$. The space dimensionality enters only to give different temperature dependences of the physical properties and different $\bar{I}$ dependences of each g separately, but plays no role in the combination of the g_l's entering in g, or in g_{SO} as well.

[15]C. Castellani, C. Di Castro, G. Forgacs, and S. Sorella, Solid State Commun. **52**, 261 (1984).
[16]M. T. Béal-Monod, Phys. Rev. B **32**, 5966 (1985).
[17]H. Fukuyama (private communication).
[18]P. W. Anderson, in *Proceedings of the International Conference on Localization, Interaction and Transport Phenomena, Braunschweig, West Germany, 1984,* edited by B. Kramer, G. Bergman, and Y. Bruyneseraede, Springer Tracts in Solid-State Sciences, Vol. 61 (Springer-Verlag, New York, 1984), p. 12.
[19]J. M. Lawrence, Phys. Rev. B **20**, 3770 (1979); M. T. Béal-Monod and J. M. Lawrence, Phys. Rev. **21**, 5400 (1980); M. T. Béal-Monod, J. Phys. **41**, 1109 (1980).
[20]See a review by D. L. Mills, M. T. Béal-Monod, and P. Lederer, in *Magnetism,* edited by H. Suhl (Academic, New York, 1973), Vol. V, p. 89.
[21]See, for instance, T. F. Rosenbaum *et al.*, Phys. Rev. Lett. **46**, 568 (1981).
[22]A. E. Ruckenstein, M. A. Paalanen, and G. A. Thomas, in *Proceedings of the Seventeenth International Conference on Semiconductors, San Francisco, 1985,* edited by J. D. Chadi and W. A. Harrison (Springer-Verlag, Berlin, 1985), p. 1083.
[23]M. T. Béal and J. Friedel, Phys. Rev. **135**, A466 (1964).
[24]J. Lesueur, third cycle thesis, Orsay, 1985 (unpublished).
[25]A. Theumann and M. T. Béal-Monod, Phys. Rev. B **29**, 2567 (1984).
[26]M. T. Béal-Monod, Phys. Rev. B **31**, 2764 (1985).