Fermi superfluids at a rough surface

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This paper gives the boundary conditions for the quasiclassical theory of static superfluid Fermi systems in contact with an arbitrary reflecting surface. We demonstrate the application of the final closed circle of equations by displaying a self-consistent numerical solution of ^{3}He-B in contact with a "randomly rough" wall. The results exhibit how great the sensitivity of p-wave pairing is to surface scattering.

INTRODUCTION

This paper is sequent to an earlier one,[1] hereafter referred to as QCB (quasiclassical boundary conditions), in which the boundary conditions for superfluid Fermi systems were discussed within the quasiclassical approach. That paper provides the foundation for a theory of boundary behavior at reflective surfaces and, as one of its central results, prescribes a quasiclassical boundary condition obeyed by superfluid Fermi systems at arbitrary surfaces. Further, QCB proposed two model surfaces to which the theory might be applied. These were the "specular surface" and the "statistically rough" or "randomly-rippled-wall" models.

The intent of this paper is twofold. We first point out that the application of the QCB boundary condition requires further nontrivial prescriptive steps. We emphasize that the QCB condition is fully true but still incomplete without the further specification provided here. Second, we demonstrate the completeness of augmented theory by presenting a self-consistent numerical calculation of the order parameter for superfluid ^{3}He-B in proximity with a rough wall. This study considers only the static limit, leaving the formidable topic of time-dependent behavior to some later effort. By means of this effort we hope to indicate the implementation of the full theory and produce results of interest in their own right.

This paper is divided into three sections. In Sec. I we review the quasiclassical formulation, introduce notation, and sketch the general theory of surfaces. In Sec. II we discuss the quasiclassical boundary condition and its generalization, and in Sec. III we present the numerical results for the specific case of a rough wall.

I. THE QUASICLASSICAL METHOD

In this section we provide a brief summary of the logical and mathematical framework constituting the quasiclassical method. We rely heavily on Ref. 1 for detailed discussions and derivations, as well as for notation.

The observation motivating the quasiclassical method is that the characteristic scales of energy and length in superfluid Fermi phenomena are set by $k_B T_C$ and $\hbar v_F/k_B T_C$, respectively, where T_C and v_F refer to the transition temperature and Fermi velocity. The quasiclassical formulation proceeds by restricting one's attention exclusively to variations along these scales. This is affected by elimination, at the very outset, of variations on the scale of the Fermi wavelength $K_F^{-1} \ll \hbar v_F/k_B T_C$ and the Fermi energy $E_F \gg k_B T_C$. The advantage gained is a very considerable simplification, elegance, and computational ease. The method recognizes explicitly that, since the traditional Bardeen-Cooper-Schrieffer (BCS) approach is essentially an expansion in the small quantity $k_B T_C/E_F$ anyway, one has made progress by fully acknowledging this fact and eliminating as many intermediate steps as possible. Further, the method lends itself to immediate generalization encompassing "strong-coupling" phenomena as well.

The underlying mathematical structure proceeds from formal many-body perturbation theory expressed in terms of thermodynamic (imaginary time) Green's functions. We use the 4×4 matrix notation which contains the "anomalous" Green's functions in the off-diagonal quadrants. The central object of study is the "quasiclassical Green's function $\hat{g}(\hat{\mathbf{k}};\mathbf{R}:\epsilon_n)$. Loosely speaking, $\hat{g}$ has been derived from the full one-particle Green's function by integrating over the magnitude of the relative-spatial variable's Fourier components. As written, $\mathbf{R}$ is the center-of-mass spatial coordinate, $\hat{\mathbf{k}}$ is the remaining direction of the Fourier-transformed relative-spatial coordinate, ϵ_n is the Fourier-transformed relative-time coordinate at the Matsubara frequencies, and the superior caret in $\hat{g}$ denotes the 4×4 matrix notation (unfortunately, a superior caret is also used to denote unit vectors, but this should cause no confusion).

The importance of quasiclassical Green's functions stems from the fact that the expectation values of all interesting observables may be expressed through them as follows:

$$\langle A(\mathbf{R})\rangle = T\sum_{\epsilon_n} N(E_F)\int \frac{d^2\hat{\mathbf{k}}}{4\pi}\tfrac{1}{2}\,\mathrm{Tr}_4[\hat{a}_{\mathrm{qp}}(\hat{\mathbf{k}})\hat{g}(\hat{\mathbf{k}};\mathbf{R}:\epsilon_n)]\ , \tag{1.1}$$

where $\hat{a}_{\mathrm{qp}}(\hat{\mathbf{k}})$, is in the notation of Ref. 2, the quasiparticle operator corresponding to the quantity $A(\mathbf{R})$. For example, the operator corresponding to mass current density is $\hbar k_F \hat{\mathbf{k}}$. The equations determining $\hat{g}$ are derived in turn from the Dyson equation determining the full Green's

function. They take the form of a transportlike equation plus a normalization condition. In the static limit these are

$$i\epsilon_n[\hat{\tau}_3,\hat{g}(\mathbf{k};\mathbf{R}:\epsilon_n)]+iv_F\hat{\mathbf{k}}\cdot\nabla_R\hat{g}(\hat{\mathbf{k}};\mathbf{R}:\epsilon_n)$$
$$-\hat{\tau}_3\hat{\sigma}(\hat{\mathbf{k}};\mathbf{R}:\epsilon_n)\hat{g}(\hat{\mathbf{k}};\mathbf{R}:\epsilon_n)+\hat{g}(\hat{\mathbf{k}};\mathbf{R}:\epsilon_n)\hat{\sigma}(\hat{\mathbf{k}};\mathbf{R}:\epsilon_n)\hat{\tau}_3=0 \tag{1.2a}$$

and

$$\hat{\tau}_3\hat{g}(\hat{\mathbf{k}};\mathbf{R}:\epsilon_n)\hat{\tau}_3\hat{g}(\hat{\mathbf{k}};\mathbf{R}:\epsilon_n)=-\pi^2\hat{\mathbb{1}} \tag{1.2b}$$

where $\hat{\sigma}$ is the quasiparticle self-energy and $\hat{\tau}_i$ refers to the Pauli matrices operating on the quadrants of the 4×4 matrices. Closing the circle of equations requires defining $\hat{\sigma}$ as a functional of $\hat{g}$. This is denoted the "self-energy equation" and will provide the order-parameter "gap equation." As examples we list the results to be used in Sec. III, i.e., those for a p-wave superfluid in no external fields. Our notation consistently exploits the fact that $\hat{g}$ is most simply viewed by quadrants. The upper on- and off-diagonal quadrants contain the conventional and anomalous 2×2 Green's function which we denote g and f. The lower quadrants contain functions, denoted $\tilde{f}$ and $\tilde{g}$, which are related to f and g via simple symmetric operations. Thus the physical information in $\hat{g}$ is actually contained twice. This lavish expenditure of symbols is justified only by the extreme condensation in notation it ultimately affords.

Explicitly written out, we have then

$$\hat{g}=\begin{bmatrix} g & f \\ \tilde{f} & \tilde{g} \end{bmatrix},$$

where

$$\tilde{g}(\hat{\mathbf{k}},\epsilon_n)=-g^T(-\hat{\mathbf{k}},-\epsilon_n),$$
$$\tilde{f}(\hat{\mathbf{k}},\epsilon_n)=f^\dagger(\hat{\mathbf{k}},-\epsilon_n).$$

The self-energy in the p-wave case is specified as

$$\hat{\sigma}=\begin{bmatrix} 0 & \mathbf{\Delta}\cdot\boldsymbol{\sigma}\sigma_2 \\ (\mathbf{\Delta}\cdot\boldsymbol{\sigma}\sigma_2)^\dagger & 0 \end{bmatrix}, \tag{1.2c}$$

where the order parameter $\mathbf{\Delta}\cdot\boldsymbol{\sigma}\sigma_2$ is found from the gap equation

$$\mathbf{\Delta}(\hat{\mathbf{k}};\mathbf{R})\cdot\boldsymbol{\sigma}i\sigma_2=-\lambda T\sum_{\epsilon_n}{}'\int\frac{d^2\hat{\mathbf{k}}'}{4\pi}3\hat{\mathbf{k}}\cdot\hat{\mathbf{k}}'f(\hat{\mathbf{k}}';\epsilon_n:\mathbf{R}). \tag{1.2d}$$

The prime on the summation specifies a high-energy cutoff and the symbol λ denotes the coupling constant. Equations (1.2a)–(1.2d) provide a complete description of bulk systems. Bounded systems, on the other hand, introduce a brand new class of decidedly nontrivial considerations. A reflective wall presents a huge short-ranged potential to incident quasiparticles. The quasiclassical approach, however, presupposes weak and slowly varying external potentials. QCB demonstrated that this impasse may, in fact, be circumvented. The central result of that paper was a boundary condition on $\hat{g}$ that was expressed in the following form. First, we observed that the pertinent properties of any given surface entered the equations through a "surface-scattering t matrix" which we denote $\hat{t}$. This quantity $\hat{t}$ appeared in our final expressions only in the combination of symbols which we denoted by $\Delta\hat{g}$:

$$\Delta\hat{g}=-\frac{i}{v_F\hat{\mathbf{k}}\cdot\hat{\mathbf{n}}}(\hat{\tau}_3\hat{t}\hat{g}-\hat{g}\hat{t}\hat{\tau}_3),$$

where $\hat{\mathbf{n}}$ designates the local surface normal. Finally, the condition obeyed by $\hat{g}$ at a surface in the static limit was given as

$$\Delta\hat{g}(\hat{\mathbf{k}};\mathbf{R}_{\text{surf}}:\epsilon_n)\tau_3\Delta\hat{g}(\hat{\mathbf{k}};\mathbf{R}_{\text{surf}}:\epsilon_n)=0. \tag{1.3}$$

This expression is highly nonlinear in the quantity $\hat{g}$ because $\hat{t}$ is itself a functional of $\hat{g}$. In the following section we discuss the condition (1.3) and its necessary generalization.

II. THE BOUNDARY CONDITIONS

The correct application of Eq. (1.3) is anything but apparent. Nonetheless, a few simple physical considerations facilitate the interpretation of its content and provide motivation for its final form presented below. First, we note that $\hat{g}(\hat{\mathbf{k}};\mathbf{R})$ is the quasiclassical amplitude associated with particles at position $\mathbf{R}$ and with momentum in direction $\hat{\mathbf{k}}$. We expect a scattering boundary condition to relate the amplitude of a particle incident upon a surface with the amplitudes of particles reflected from it. If we let $\hat{\mathbf{n}}$ represent the surface normal direction, then we may classify all $\hat{\mathbf{k}}$ vectors as "outgoing" or "incoming" according to whether $\hat{\mathbf{k}}\cdot\hat{\mathbf{n}}$ is positive or negative. Also, we reserve the symbol $\underline{\hat{\mathbf{k}}}$ to represent the "mirror reflection" of $\hat{\mathbf{k}}$, i.e.,

$$\underline{\hat{\mathbf{k}}}\equiv\hat{\mathbf{k}}-2\hat{\mathbf{n}}\hat{\mathbf{n}}\cdot\hat{\mathbf{k}}.$$

The simplest situation to investigate is that of a specularly reflecting surface. In that case QCB found the surface t matrix to be

$$\hat{t}_{\text{specular}}=-2\pi v_F\,|\,\hat{\mathbf{k}}\cdot\hat{\mathbf{n}}\,|\,[\hat{g}(\hat{\mathbf{k}};\mathbf{R}:\epsilon_n)+\hat{g}(\underline{\hat{\mathbf{k}}};\mathbf{R}:\epsilon_n)]^{-1}. \tag{2.1}$$

By inserting this in (1.3), one may observe that the intuitively plausible condition

$$\hat{g}(\underline{\hat{\mathbf{k}}};\mathbf{R}:\epsilon_n)=\hat{\mathbf{g}}(\hat{\mathbf{k}};\mathbf{R}:\epsilon_n) \tag{2.2}$$

ensures that (1.3) is, indeed, obeyed since then $\Delta\hat{g}=0$. Self-consistent solutions of Eqs. (1.2) and (2.2) have been calculated and they yield eminently satisfactory results.[3] This small success supports the conviction that (2.2) is the correct solution in (1.3) in the specular case, but it should not hide the fact that (2.2) is an additional statement not uniquely implied by (1.3). The following question remains: What should replace (2.2) in the general case? The following clues suggest an answer. Equation (1.3)

must be true for all $\hat{\mathbf{k}}$. This is curious because a scattering boundary condition specifies reflected amplitudes in terms of incoming ones. Therefore a scattering boundary condition should make requirements involving only one-half of all the directions. The fact that (1.3) does this may be seen in the following way. Any 4×4 Fermi Green's function obeys the following symmetry condition (derived from the operator properties used to define it):

$$\hat{g}(\hat{\mathbf{k}}:\epsilon_n)=-\hat{\tau}_1\hat{g}^*(-\hat{\mathbf{k}}:\epsilon_n)\hat{\tau}_1 . \tag{2.3}$$

The matrix $\hat{t}$ obeys an identical condition. With these relations, Eq. (1.3) may be cast as (suppressing unused variables)

$$\Delta\hat{g}(\hat{\mathbf{k}})\hat{\tau}_2\Delta\hat{g}^*(-\hat{\mathbf{k}})=0 ,$$

from which it is clear that demanding

$$\Delta\hat{g}(\hat{\mathbf{k}})=0 \tag{2.4}$$

for only one-half of all $\hat{\mathbf{k}}$ vectors satisfies (1.3) for all $\hat{\mathbf{k}}$ vectors.

We may next demonstrate that (2.4) plus the normalization condition (1.2b) specify a unique generalization to Eq. (2.2). To see this, first rewrite (2.4) in the equivalent form

$$[\hat{t}(\hat{\mathbf{k}})\hat{\tau}_3,\hat{\tau}_3\hat{g}(\hat{\mathbf{k}})]=0 \tag{2.5}$$

for one-half the $\hat{\mathbf{k}}$'s. Now express $\hat{t}$ as the sum of two parts: one having the specular form given in (2.1) denoted $\hat{t}_2$ plus a remainder $\hat{t}_r$, i.e.,

$$\hat{t}\equiv\hat{t}_s+\hat{t}_r$$

which defines $\hat{t}_r$. Now if (2.5) is to be true then $\hat{t}\hat{\tau}_3$ must equal something which commutes with $\hat{\tau}_3\hat{g}$. We write this in complete generality as

$$(\hat{t}_s+\hat{t}_r)\hat{\tau}_3=-2\pi v_F\,|\,\hat{\mathbf{k}}\cdot\hat{\mathbf{n}}\,|\,\{[2\hat{g}(\hat{\mathbf{k}})]^{-1}+\hat{A}\}\hat{\tau}_3 ,$$

where $\hat{A}\hat{\tau}_3$ is "anything" which commutes with $\hat{\tau}_3\hat{g}(\hat{\mathbf{k}})$ (and is yet to be determined). The reason for this decomposition is that (2.2) is recovered in an obvious manner as $\hat{t}_r$ approaches 0. We next use (2.1) to express $\hat{t}_s$ explicitly and then find upon rearranging

$$[\hat{g}(\hat{\mathbf{k}})+\hat{g}(\underline{\hat{\mathbf{k}}})]^{-1}=[2\hat{g}(\hat{\mathbf{k}})]^{-1}+\left[\frac{\hat{t}_r}{2\pi v_F\,|\,\hat{\mathbf{n}}\cdot\hat{\mathbf{k}}\,|}+\hat{A}\right] . \tag{2.6}$$

Now let us, for the moment, call the last term in parentheses simply $\hat{P}$. If we solve for $\hat{g}(\underline{\hat{\mathbf{k}}})$ we find

$$\hat{g}(\underline{\hat{\mathbf{k}}})=[\hat{1}-2\hat{g}(\hat{\mathbf{k}})\hat{P}][\hat{1}+2\hat{g}(\hat{\mathbf{k}})\hat{P}]^{-1}\hat{g}(\hat{\mathbf{k}}) . \tag{2.7}$$

Now we demand the normalization condition (1.2b) shall be true for $\hat{g}(\underline{\hat{\mathbf{k}}})$ if it is true for $\hat{g}(\hat{\mathbf{k}})$. Carrying out the algebra, we find the condition to be $[\hat{P}\hat{\tau}_3,\hat{\tau}_3\hat{g}]_+=0$ which may be reexpressed as

$$\left[\frac{\hat{t}_r\hat{\tau}_3}{2\pi v_F\,|\,\hat{\mathbf{n}}\cdot\hat{\mathbf{k}}\,|},\hat{\tau}_3\hat{g}\right]_+=-2\hat{\tau}_3\hat{g}\,\hat{A}\,\hat{\tau}_3$$

or

$$\hat{A}\hat{\tau}_3=\frac{1}{2\pi^2}\hat{\tau}_3\hat{g}\left[\frac{\hat{t}_r\hat{\tau}_3}{2\pi v_F\,|\,\hat{\mathbf{n}}\cdot\hat{\mathbf{k}}\,|},\hat{\tau}_3\hat{g}\right]_+ .$$

Thus $\hat{A}$ is uniquely determined. Reinserting these results into (2.7) and defining the quantity

$$\hat{t}_-\equiv\frac{[\hat{t}_r\hat{\tau}_3,\hat{\tau}_3\hat{g}]_-}{2\pi v_F\,|\,\hat{\mathbf{n}}\cdot\hat{\mathbf{k}}\,|} \tag{2.8}$$

(which manifestly anticommutes with $\hat{\tau}_3\hat{g}$), we arrive at our final result:

$$\hat{g}(\underline{\hat{\mathbf{k}}})=\hat{\tau}_3(\hat{1}+\hat{t}_-)(\hat{1}-\hat{t}_-)^{-1}\hat{\tau}_3\hat{g}(\hat{\mathbf{k}}) \tag{2.9}$$

for one-half the $\hat{\mathbf{k}}$'s. By construction this expression satisfies (2.5), (1.3), and (1.2b). Equation (2.9) is the central result of this paper and provides the necessary generalization to Eq. (2.2).

III. A MODEL SURFACE EVALUATED

A. The randomly-rippled-wall model

Equation (2.9) completes the basic equations (1.2a)–(1.2d), and together they constitute a closed system. The first step in evaluating these equations in any specific case consists in choosing the boundary surface(s), which is equivalent to specifying the t matrix as a function of $\hat{g}$. Thereafter, the basic equations specify $\hat{g}$ uniquely.

This section presents the results of just such a calculation, namely that of superfluid ^{3}He-B in the presence of a "rough" bounding plane. We chose the randomly-rippled-wall model (RRW) described in QCB as our surface specification. This model provides for local statistical fluctuations or "ripples" in a smooth average wall which we take to be the x-y plane. Let the deviation from the plane at point $\mathbf{R}$ in the surface be denoted $\xi(\mathbf{R})$. All physical quantities will be averaged over an ensemble of possible ripples. The RRW model allows a perturbation expansion in powers of ξ, and in what follows we retain only the leading two powers. Since the average of ξ vanishes by definition, we are left with the second-order cumulant denoted ξ_2, as our model parameter:

$$\xi_2(\mathbf{R}-\mathbf{R}')\equiv\langle\xi(\mathbf{R})\xi(\mathbf{R}')\rangle .$$

Its two-dimensional Fourier transform will be denoted $\bar{\xi}_2(\mathbf{k})$. We assume that ξ_2 is of short range ($\ll\xi_0$) and that $|\xi|k_F<1$. For this paper we chose the simple model of a Gaussian distribution for ξ_2:

$$\xi_2(\mathbf{R}_1-\mathbf{R}_2)=\frac{a^2}{k_F^2}\exp(-|\mathbf{R}_1-\mathbf{R}_2|^2k_F^2/2b^2) . \tag{3.1}$$

The two remaining parameters, a and b, describe the average height and width of bumps in units of k_F. Below, we present our solutions for various input values.

At this point we assemble our basic equations. The

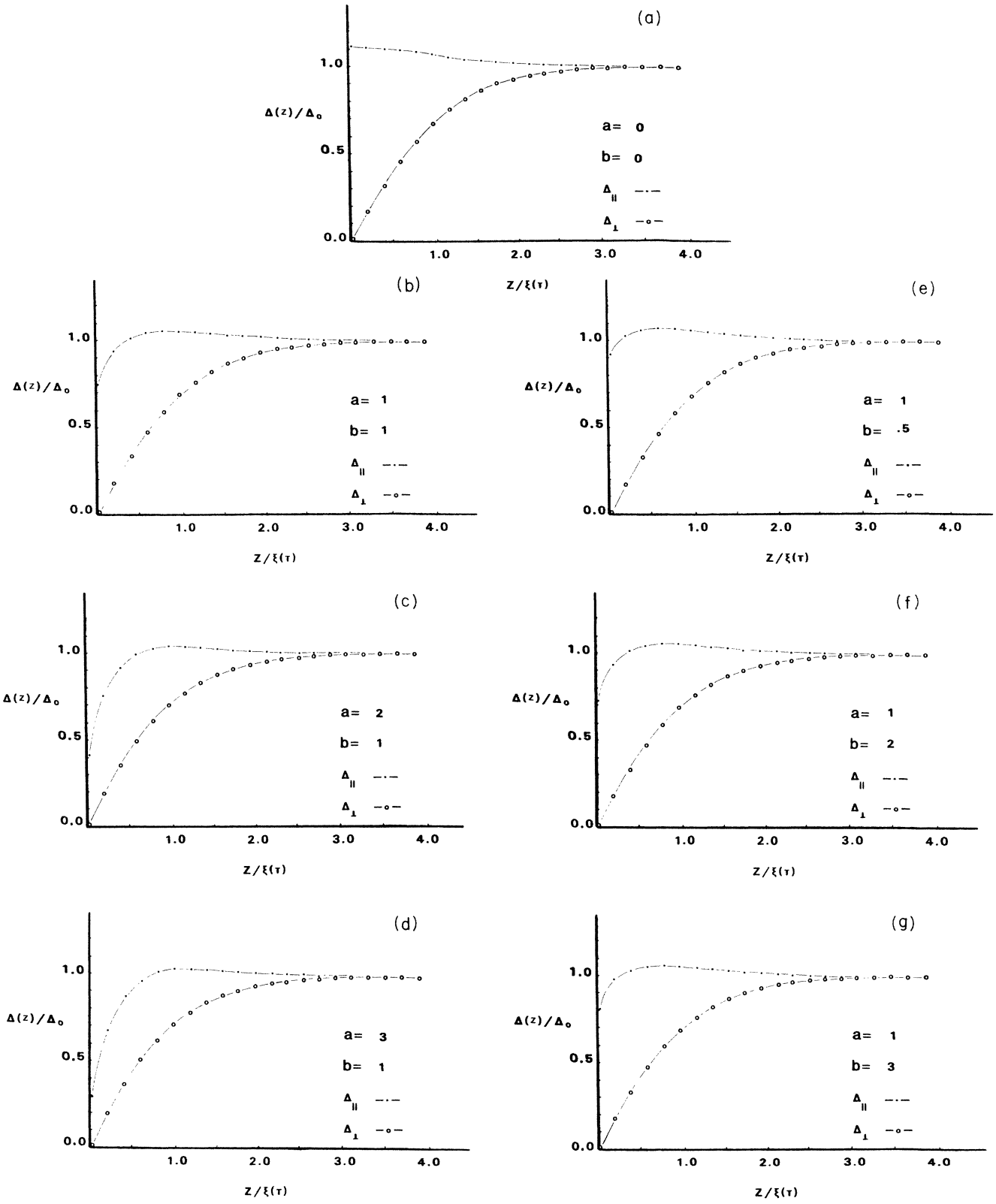

FIG. 1. The sequence of graphs displays the spatial dependence of the order parameters $\Delta_{\|}$ and $\Delta_{\perp}$ for superfluid ^{3}He-B while in proximity with various surfaces. The condition of the "randomly bumpy" surfaces is specified by an "average bump height" equal to ak_F^{-1}, and by an "average bump width" equal to bk_F^{-1}. (a) shows the results for a perfectly flat specular (mirror reflecting) surface, whereas (b), (c), and (d) display the results of successively increasing the bump height and (e), (f), and (g) display the results of successively increasing the bump width. The order parameters are normalized to their common bulk value and distance from the wall is measured in units of the temperature-dependent coherence length $\xi(T)$.

RRW model specifies the surface-scattering t matrix explicitly as a functional of $\hat{g}$. We find, by following the logic of QCB (with small corrections)

$$\hat{t}_r(\hat{\mathbf{k}})=-\frac{v_F}{2}\int\frac{d^2q_{in}}{2\pi}W(\hat{\mathbf{k}},\hat{\mathbf{q}})[\hat{g}^{-1}(\hat{\mathbf{q}})-\hat{g}^{-1}(\hat{\mathbf{k}})\hat{g}(\hat{\mathbf{q}})\hat{g}^{-1}(\hat{\mathbf{k}})]\ , \tag{3.2}$$

where the kernel W stands for

$$W(\hat{\mathbf{k}},\hat{\mathbf{q}})=(\hat{\mathbf{k}}\cdot\hat{\mathbf{n}})^2(\hat{\mathbf{q}}\cdot\hat{\mathbf{n}})^2k_F^4\bar{\xi}_2(\hat{\mathbf{k}}_F(\hat{\mathbf{k}}-\hat{\mathbf{q}}-\hat{\mathbf{n}}\hat{\mathbf{n}}\cdot(\hat{\mathbf{k}}-\hat{\mathbf{q}})))\ .$$

It is important to remember that $\hat{t}$ and $\hat{g}$ are also functions of $\mathbf{R}$ and ϵ_n, but that these are not active in the expression (3.2). The equations we are faced with solving simultaneously are (3.2), (2.9), and (1.2a)–(1.2d), a somewhat daunting prospect to say the least. As a first step it is advantageous to exploit every symmetry at hand. Section III B sketches the method of solution, and a presentation of the numerical results follows.

B. Solving the RRW model

Bulk superfluid ^{3}He-B is characterized by an order parameter of the following form:

$$\Delta(\hat{\mathbf{k}})=\Delta\hat{\mathbf{k}}\ ,$$

i.e., a single number Δ determines the whole order parameter. As we approach a plane bounding surface this number breaks into two in a simple way. If we denote the component of k perpendicular to the surface by $\mathbf{k}_\perp$ and the components of k parallel to the surface by $\mathbf{k}_{||}$, we find that the order parameter becomes

$$\Delta(\hat{\mathbf{k}})=\Delta_{||}\mathbf{k}_{||}+\Delta_\perp\mathbf{k}_\perp\ .$$

The Green's function $\hat{g}$ also simplifies. The problem displays spatial dependence in the z direction only and observes rotation and reflection symmetry about the surface normal. Further, the absence of magnetic fields means time-reversal symmetry also obtains. Employing all of these one may show that $\hat{g}$ depends on just six numbers designated $g_1,\ldots,g_6$:

$$g=-i\pi(g_1+g_2i\hat{\mathbf{k}}_{||}\times\hat{\mathbf{k}}_\perp\cdot\boldsymbol{\sigma})\ ,$$
$$f=-i\pi(g_3\hat{\mathbf{k}}_{||}\cdot\boldsymbol{\sigma}+g_4\hat{\mathbf{k}}_\perp\cdot\boldsymbol{\sigma})\sigma_2\ ,$$
$$\tilde{f}=i\pi\sigma_2(g_5\hat{\mathbf{k}}_{||}\cdot\boldsymbol{\sigma}+g_6\hat{\mathbf{k}}_\perp\cdot\boldsymbol{\sigma})\ ,$$
$$\tilde{g}=i\pi\sigma_2(-g_1+ig_2\hat{\mathbf{k}}_{||}\times\hat{\mathbf{k}}_\perp\cdot\boldsymbol{\sigma})\sigma_2\ .$$

Even these are not all independent since normalization requires that

$$(g_1+g_2i\hat{\mathbf{k}}_{||}\times\hat{\mathbf{k}}_\perp\cdot\boldsymbol{\sigma})^2+(g_3\hat{\mathbf{k}}_{||}\cdot\boldsymbol{\sigma}+g_4\hat{\mathbf{k}}_\perp\cdot\boldsymbol{\sigma})\times(g_5\hat{\mathbf{k}}_{||}\cdot\boldsymbol{\sigma}+g_6\hat{\mathbf{k}}_\perp\cdot\boldsymbol{\sigma})=\hat{\mathbf{1}}\ ,$$

which is actually two conditions, leaving just four independent numbers. The fact that (2.9) adds the correct number of boundary conditions may be seen in the following way. In a semi-infinite geometry the boundary condition on $\hat{g}$ at infinity is that it remains finite. Since $\hat{\sigma}$ approaches a constant value away from the wall the solutions of (1.2a) become exponentials. The condition of finiteness means $\hat{g}$ may contain no components which grow exponentially. Examination of (1.2a) reveals that this imposes two linear conditions among the six numbers forming $\hat{g}(\hat{\mathbf{k}})$. Now notice, however, that $\hat{g}(\hat{\underline{\mathbf{k}}})$ must *also* remain finite and that $\hat{g}(\hat{\underline{\mathbf{k}}})$ and $\hat{g}(\hat{\mathbf{k}})$ are related via (2.9). This imposes two more conditions on $\hat{g}(\hat{\mathbf{k}})$ for a total of six conditions among the six numbers. Once we have specified $\hat{g}(\hat{\mathbf{k}})$ at the surface, Eq. (1.2a) determines $\hat{g}$ everywhere else.

The actual algorithm used to evaluate the equations self-consistently is "fairly involved" and will be published elsewhere. The mutual interdependence of all the equations necessitated an iterative method which, however, converged very rapidly.

C. Numerical results for superfluid ^{3}He-B

Figure 1 displays $\Delta_{||}$ and $\Delta_\perp$, the parallel and perpendicular components of the superfluid order parameter, plotted as a function of distance away from the wall. They are given in units of the bulk value which they approach asymptotically. Distances are expressed in units of the temperature-dependent coherence length $\xi(T)\equiv\hbar v_F/\Delta(T)$ and all calculations were performed for $T/T_c=0.73$. Figure 1(a) displays the results for a perfectly smooth surface. Figures 1(b), 1(c), and 1(d) display situations in which "bump width" (b parameter) is held constant but "bump height" (a parameter) is successively increased. Figures 1(e), 1(f), and 1(g) display situations that have the same bump height but with increasing width. The central trends exposed by these results are fairly intuitive. A flat surface suppresses the perpendicular component completely while actually enhancing the parallel component somewhat. The inclusion of surface roughness tends to suppress the parallel component as well. Increasing bump height corresponds to suppressing $\Delta_{||}$ even further. An increase in bump width while holding the height constant acts to suppress $\Delta_{||}$ until the "width" is between two and three times the height at which an additional increase in width only makes the surface act "smooth." These results are in accord with our expectations about what should happen; yet it is deeply gratifying to see them emerge naturally from such difficult equations.

IV. CONCLUSIONS

The central result of this paper is Eq. (2.9), which solves the previously proposed QCB boundary condition (1.3). The form of Eq. (2.9) lends itself to immediate application, and the results derived from it argue persuasively for its correctness. We hope, as well, that the results of the RRW calculation will prove interesting and useful in their own right. It seems a safe inference to conclude that p-wave Cooper pairing is strongly suppressed by even small amounts of diffuse surface scattering. This should

imply significant consequences for small-channel flow experiments or for any experiment where surface area to volume ratios are large.

Equation (2.9) makes a large number of interesting calculations possible. In future projects we plan to calculate the surface density of states on a rough wall, the effect of magnetic surface impurities, and critical velocities in narrow rough channels.

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