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Possible existence of a Josephson effect in ferromagnets

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When a current density j_x crosses a 180° domain wall in a metallic ferromagnet, the spin **s** of each conduction electron exerts an s-d exchange torque on the localized wall spins. Hence, the wall moment of a Bloch wall is canted out of the wall plane by an angle ψ, given by $j_x=(eC/\hbar)\sin(2\psi)$, where C is the maximum restoring torque at $\psi=45°$. This equation is the exact analog of the dc Josephson effect, and 2ψ is the analog of the superconducting phase difference ϕ across a junction. For $|j_x| > eC/\hbar \simeq 10^6$ A/cm^2, the s-d exchange torque overcomes the restoring torque, and the wall moment precesses with a frequency $\omega=d(2\psi)/dt$. A dc voltage δV is expected to appear across the wall, satisfying the famous ac Josephson relation $2e\delta V=-\hbar\omega$. This wall precession can be described as a translation of Bloch lines, and the Bloch lines are the exact analog of superconducting vortices. The electric current exerts a transverse force on Bloch lines.

I. INTRODUCTION

In ferromagnetic insulators, the dynamics of domain walls has already been studied extensively.[1] In metallic ferromagnets where an electric current crosses a wall, the electric current density constitutes a new dynamic variable giving rise to a novel class of phenomena. In the case of bulk samples, the main interaction between current and wall is hydromagnetic in nature.[2] However, s-d exchange is the dominant[3,4] interaction in films of thickness $w \leq 0.1$ μm, considered in the present paper.

The purpose of the present paper is to show the existence of a complete analogy between the properties of 180° domain walls in a metallic ferromagnet and those of Josephson junctions in a superconductor. The current causes a rotation of the wall moment by an angle ψ; as we will see, 2ψ is the analog of the superconducting phase difference ϕ across a junction.

The s-d exchange interaction[3,5,6] has the approximate form

$$V=g\mu_B[\mathbf{s}\cdot\mathbf{H}_{sd}(x)+H_{sd}/2]\ ,$$
$$\mathbf{H}_{sd}(x)=-2J_{sd}\langle\mathbf{S}(x)\rangle/g\mu_B \tag{1}$$

where g is the gyromagnetic factor, **s** is the spin of a $4s$ conduction electron, $\mathbf{H}_{sd}(x)$ is the intra-atomic s-d exchange field acting on **s** at a location of coordinate x normal to the wall plane, $J_{sd}>0$ is the s-d exchange integral, $\mathbf{S}(x)$ is a localized $3d$ magnetic spin in the wall, and μ_B the Bohr magneton. The z axis is assumed to be parallel to $\mathbf{H}_{sd}(+\infty)$ i.e., antiparallel to $\mathbf{S}(+\infty)$. Our x,y,z coordinates correspond to $-y,x,z$, respectively, in the system of coordinates of Ref. 1. The constant term $H_{sd}/2$ ensures that $V(x=\pm\infty)=0$. The transverse quantum fluctuations of $\mathbf{H}_{sd}$ are neglected in Eq. (1).

Our assumption that the ferromagnetic $3d$ spins $\mathbf{S}(x)$ are localized is probably not strictly correct in metals such as Fe, Ni, Co.[5] However, it is part of the traditional s-d exchange model, it is simple, and it is convenient for our present purpose.

For simplicity, we also assume that the spin-up (i.e., majority-spin) $4s$ electrons have a much higher mobility than the spin-down $4s$ electrons or any other conduction electrons. We assume that only these spin-up $4s$ electrons need to be considered, and that **s** represents the spin of one of them. Finally, these charge carriers are assumed to be electronlike, rather than holelike.

II. EXCHANGE TORQUE

The magnetic spin $\mathbf{S}(x)$ makes an angle $\theta(x)$ with $\mathbf{S}(x=+\infty)$ (Fig. 1). If one crosses the wall by increasing x, the angle θ changes between π and 0. As θ changes, $\mathbf{S}(x)$ is assumed to rotate in a certain plane P (Fig. 1) parallel to the z axis. As an approximation,[1] the orienta-

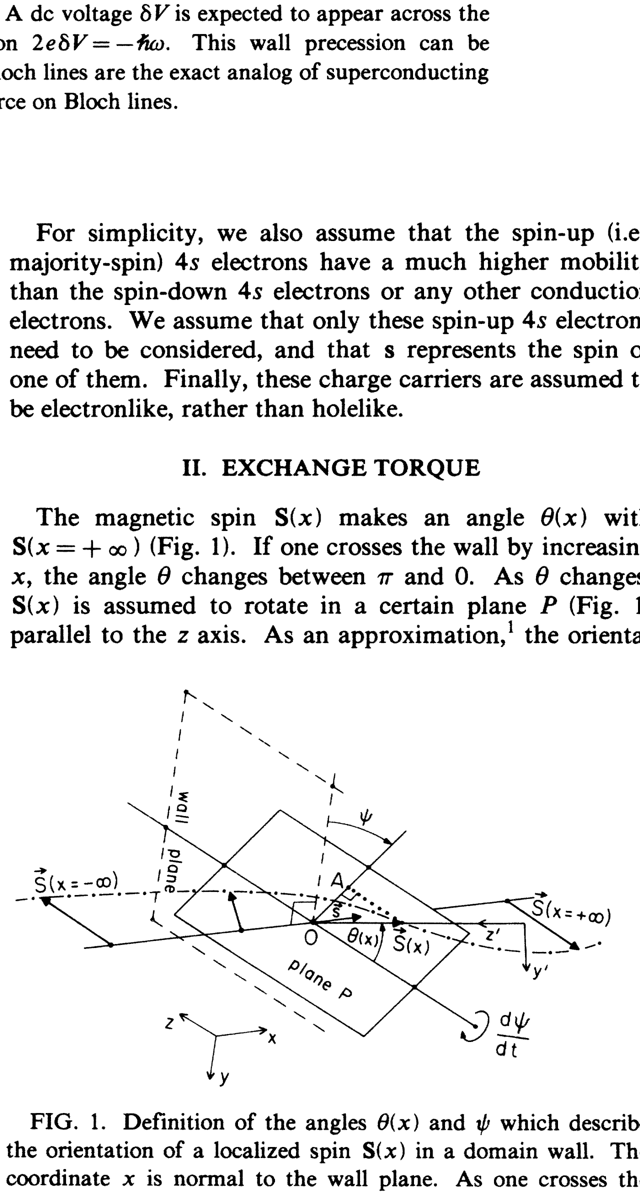

FIG. 1. Definition of the angles $\theta(x)$ and ψ which describe the orientation of a localized spin $\mathbf{S}(x)$ in a domain wall. The coordinate x is normal to the wall plane. As one crosses the wall from $x\simeq-\infty$ to $x\simeq+\infty$, $\mathbf{S}(x)$ turns in the plane P while the angle $\theta(x)$ varies from $\theta=\pi$ to $\theta=0$. In the local system x',y',z', the z' axis is antiparallel to $\mathbf{S}(x)$. Also, the y',z' plane is parallel to the plane P. The plane P makes an angle ψ with the wall plane. The exchange field $\mathbf{H}_{sd}(x)$ is everywhere antiparallel to $\mathbf{S}(x)$. The conduction electron spin **s** is almost parallel to $\mathbf{S}(x)$.

tion of this plane is assumed to be independent of x, y, and z. The projection $0A$ of $\mathbf{S}(x)$ on the xy plane makes a fixed angle ψ with the $-y$ axis. If $\psi=0$ or $\psi=\pi$, we have a Bloch wall. If $\psi=\pi/2$ or $3\pi/2$, we have a Néel wall. The angle ψ constitutes the longitude, and $\theta>0$ the colatitude, in a system of polar coordinates centered around the $-z$ axis. However, our sign convention for ψ is such that ψ is positive, and close to $+\pi/4$, for the case shown in Fig. 1. This sign convention is chosen in such a way that our θ and ψ angles be the same as those of Ref. 1, taking into account the fact that $\mathbf{S}$ is antiparallel to the local magnetization $\mathbf{M}$ used in Ref. 1. Since $J_{sd}>0$, Eq. (1) shows that $\mathbf{S}(x)$ is also antiparallel to $H_{sd}(x)$.

Because of the large H_{sd} value, and because of the large wall thickness Δ, the spin $\mathbf{s}$ of a spin-up conduction electron follows closely[3,7] the local $\mathbf{S}(x)$ direction, as it precesses rapidly around $\mathbf{H}_{sd}(x)$ (Fig. 1). Hence, the $\mathbf{s}$ direction becomes reversed if the electron crosses the 180° wall. Such a flipping of $\mathbf{s}$ in the P plane requires a torque[7] applied by $\mathbf{H}_{sd}(x)$. Inversely, $\mathbf{s}$ creates a reaction torque on $\mathbf{H}_{sd}$ or $\mathbf{S}(x)$. This "s-d exchange torque" τ_{sd} is parallel to the z axis, and tends[7] to change the value of the angle ψ away from its equilibrium value (Fig. 2). At nonzero current density j_x, more electrons cross the wall in one direction than in the other, and a net torque remains. Since each electron has an angular momentum $s_z=+\hbar/2$ on the left side of the wall, we obtain for the total torque on a unit area of wall

$$(\tau_{sd})_z=-\frac{\hbar}{e}j_x-\hbar n_e v_w \ . \tag{2}$$

Here, v_w is the wall velocity in the x direction, $e=+1.6\times10^{-19}$ C, and n_e is the number of electrons per

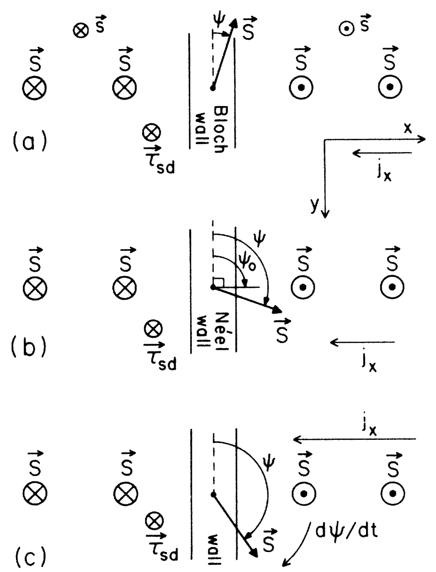

FIG. 2. (a) Canting of the localized spins $\mathbf{S}$ of a Bloch wall, by the s-d exchange torque τ_{sd} generated by a current density j_x crossing the wall. In equilibrium at $j_x=0$, the value of the angle ψ is $\psi_0=0$. (b) Same, in the case of a Néel wall. In equilibrium at $j_x=0$, the value of ψ is $\psi_0=\pi/2$. (c) If $|j_x|$ exceeds a certain critical value j_ψ, the s-d exchange torque τ_{sd} overcomes the restoring torque $-\partial\sigma/\partial\psi$, and the wall spins precess at a rapid rate $d\psi/dt$.

unit volume. In the rest of this section, we will assume $v_w=0$ for simplicity. Our semiclassical treatment of spin dynamics is adequate, since the wall thickness Δ is many Fermi wavelengths and contains many electrons.

The canting effect of τ_{sd} on the wall spins is limited by a restoring torque arising from demagnetizing fields.[8] This torque[1] is $-\partial\sigma/\partial\psi$, where σ is the wall energy per unit area. We assume roughly

$$\begin{aligned}&\sigma\simeq C\sin^2(\psi-\psi_0)+\text{const},\quad C>0\\&-\partial\sigma/\partial\psi\simeq -C\sin[2(\psi-\psi_0)],\quad C>0\ .\end{aligned} \tag{3}$$

Here, ψ_0 is the value of ψ in equilibrium at $j_x=0$. If the wall is of Bloch type in equilibrium, then $\psi_0=0$ or $\psi_0=\pi$ [Fig. 2(a)]. If it is of Néel type in equilibrium, then $\psi_0=\pi/2$ or $3\pi/2$ [Fig. 2(b)].

The maximum $|\partial\sigma/\partial\psi|$ value, obtained at $|\psi-\psi_0|\simeq\pi/4$, is equal to C. The existence of this restoring torque is intimately related to the question of the Döring effective mass[1,8] of a moving wall. And the canting induced by τ_{sd} at $j_x\neq0$ is, of course, similar to the well-known spin canting[1,8] existing in a moving wall.

First, we consider a solution of Eq. (3) with dc current and time-independent ψ. Then, the exchange torque on the wall is balanced by the restoring torque, i.e., $(\tau_{sd})_z-\partial\sigma/\partial\psi=0$. Using Eqs. (2) and (3), this gives

$$j_x=-j_\psi\sin[2(\psi-\psi_0)]\ , \tag{4}$$

$$|j_x|\le j_\psi=\frac{eC}{\hbar}\ . \tag{5}$$

As expected, Eq. (4) shows that the canting angle $|\psi-\psi_0|$ increases as $|j_x|$ is increased [Figs. 2(a) and 2(b)]. When $|j_x|$ approaches the critical value j_ψ, $|\psi-\psi_0|$ approaches $\pi/4$. When $|j_x|$ exceeds j_ψ, Eq. (4) does not have a solution anymore; then, we expect $d\psi/dt\neq0$, i.e., the wall moment precesses rapidly [Fig. 2(c)]. This is similar to the well-known[1] precession happening in magnetic insulators when $|v_w|$ exceeds the so-called Walker limit.

In the case of simple "one-dimensional" walls in strongly uniaxial materials, the demagnetizing field responsible for the maximum restoring torque has[8,1] a value $\mu_0H_x\simeq M_s/\sqrt{2}\simeq0.7$ T. Here, M_s is the saturation magnetization, and μ_0 the vacuum permeability, and we assume $M_s\simeq1$ T. Then, $C\simeq\Delta M_sH_x/2\sqrt{2}$ leads to a value $j_\psi\simeq1\times10^{13}$ A/m^2, by Eq. (5). We have used a value $\simeq30$ nm of the wall thickness. This j_ψ value is very large, and difficult for $|j_x|$ to reach experimentally.

On the other hand, experiments with two-dimensional Bloch walls in Permalloy thin films of thickness 80—250 nm show[9] that a hard-axis field $\mu_0H_x\simeq1\times10^{-4}$ T is sufficient to overcome the demagnetizing field, and to transform a Bloch wall into a Néel wall. This implies $j_\psi\simeq0.4\times10^{10}$ A/m^2, if we assume $M_s\simeq1$ T and $\Delta=100$ nm and use the same formulas. Such a value of the current density is easily realized or exceeded[4] in Permalloy films, by using short current pulses.

Actually, there are reasons to think that wall-moment precession may already occur at $|j_x| < j_\psi$. For example, the easy nucleation[1] of Bloch lines, at certain points close to the surface of the sample, reduces the torque needed for precession.

In a superconducting Josephson junction, two superconductors are separated by a thin insulating film, in such a way that electron pairs can tunnel across the junction. The phase difference between the complex wave functions describing the two superconductors is denoted by ϕ. The dc current density j_x across the junction is related to ϕ by the well-known[10] equation

$$j_x = j_{\max}\sin\phi \ , \tag{6}$$

$$|j_x| \le j_{\max} \ . \tag{7}$$

Equation (6) describes the so-called dc Josephson effect.

Equations (4) and (5) for a ferromagnetic domain wall are mathematically analogous to Eqs. (6) and (7) for a superconducting junction. We see that the quantity $2(\psi-\psi_0)$ is the analog of the superconducting phase difference ϕ. The factor of 2 in the first quantity is expected on the following grounds: While the minimum-energy states of a wall correspond to $\psi-\psi_0 = 0,\pi,2\pi,3\pi,\ldots$, those of a junction correspond to $\phi = 0,2\pi,4\pi,\ldots$. The difference arises from the fact that a 180° wall has two degenerate types of states with opposite chirality,[1] corresponding to $\psi-\psi_0 = 0,2\pi,\ldots$ and to $\psi-\psi_0 = \pi,3\pi,\ldots$, respectively.

Because of this analogy, we will call the phenomena described by Eqs. (4) and (5) the dc ferro-Josephson effect.

III. VISCOUS FORCES

It is well known[1] that viscous damping forces proportional to the wall speed v_w are active on domain walls. Since we consider very thin films, we will assume that the intrinsic (Gilbert) damping is dominant over eddy-current damping. Also, we assume that this intrinsic damping is entirely caused by the s-d exchange interaction of the wall with conduction electrons, rather than by direct interaction with the lattice. This damping force per unit area of wall is the second term of

$$F_x^w = 2M_s\mu_i^{-1}(\beta_1 v_e - v_w) \ , \tag{8}$$

where μ_i is the intrinsic wall mobility.

The first term of Eq. (8) represents[3,4] a viscous force per unit area, exerted by an electric current crossing the wall, which tends to set the wall in motion. Here v_e is the electron drift velocity in the x direction, related to j_x by

$$v_e = R_0 j_x = -j_x/n_e e \tag{9}$$

and $\beta_1 > 0$ is a dimensionless coefficient of order unity. Also, R_0 is the ordinary Hall constant. Like the damping force, this drive force is caused by s-d exchange, in thin films.[3,4]

IV. FORCE CAUSED BY WALL-MOMENT PRECESSION

We introduce local x',y',z' axes, with the y',z' plane parallel to the plane P (Fig. 1). The z' axis is antiparallel to $\mathbf{S}(x)$, and the positive y' axis is the direction corresponding to decreasing θ. These coordinates are the same as the x,y,z used in Ref. 3, where the special case of a Bloch wall was treated.

The total s-d exchange force exerted[3] by the inhomogeneous exchange field $\mathbf{H}_{sd}(x)$ of the wall on the spin $\mathbf{s}$ of a conduction electron is

$$F_x = -g\mu_B H_{sd} s_{y'} d\theta/dx \ . \tag{10}$$

A reaction force of equal magnitude is exerted by the electron on the wall.

A nonzero value of $s_{y'}$ arises from the motion of the electron across the wall when $v_w \neq 0$ or $v_e \neq 0$, and this is[3] the physical origin of the force F_x^w of Eq. (8). A second situation resulting in a nonzero $s_{y'}$ and F_x is that of a precessing wall moment, i.e., $d\psi/dt \neq 0$. Because of the large $\mu_0 H_{sd} \simeq 10^3$ T value, $\mathbf{s}$ precesses with the same angular speed $d\psi/dt$ as the local $\mathbf{S}(x)$ or $\mathbf{H}_{sd}(x)$. The exchange torque τ needed to make $\mathbf{s}$ precess arises from a small angle that $\mathbf{s}$ makes with $-\mathbf{H}_{sd}$, in the y' direction. The equation of motion $\hbar ds_{x'}/dt = \tau_{x'}$ takes the form

$$\frac{\hbar}{2}\sin\theta\frac{d\psi}{dt} = -g\mu_B s_{y'} H_{sd} \ . \tag{11}$$

We substitute Eq. (11) into Eq. (10). Then, we sum F_x over all electrons present inside a unit area of the wall. By changing the sign of the result, we obtain the corresponding reaction force F_x^w exerted on the wall, per unit area

$$F_x^w = -n_e \int_{-\infty}^{+\infty} F_x dx = \hbar n_e \frac{d\psi}{dt} \ . \tag{12}$$

For reasons to be explained in the next section, we call this force the ferro-Josephson force.

V. VOLTAGE ACROSS THE WALL

We consider a steady state of the electron gas, with constant v_e. The force exerted by the wall on the electron gas is $-F_x^w$. We sum the contributions of Eqs. (8) and (12) to F_x^w. We write the balance of external forces acting on the electrons contained (Fig. 3) within a length L of the sample along the x axis, where one wall is present, as

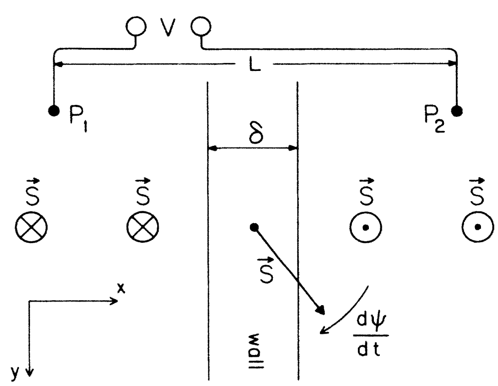

FIG. 3. The voltage V across a wall can be measured with two potential probes P_1 and P_2. One contribution to V is the ferro-Josephson voltage $\delta V = -(\hbar/e)(d\psi/dt)$, associated with precession of the wall moment at a rate $d\psi/dt$.

$$0=-2M_s\mu_i^{-1}(\beta_1 v_e - v_w) - \hbar n_e \frac{d\psi}{dt} - n_e e \int_{-L/2}^{+L/2} E_x(x)dx + n_e e \rho_s L j_x \ . \tag{13}$$

The third term of the equation is the force exerted by the electric field E_x, and the fourth term is the scattering force exerted by impurities or other lattice defects. As before, we are neglecting any hydromagnetic "domain-drag" forces. Also, ρ_s is the electrical resistivity in the absence of walls.

On the other hand, the electrical voltage V between two potential probes P_1 and P_2 located on opposite sides of the wall (Fig. 3), a distance L from each other, is

$$V=V(P_1)-V(P_2) = +\int_{-L/2}^{+L/2} E_x(x)dx = \rho_s L j_x + \delta V \ . \tag{14}$$

Here, $\rho_s L j_x$ is the value of V in the absence of wall, and δV is the "excess voltage"[2] associated with the presence of the wall. We assume $L \gg \Delta$. By combining Eqs. (13), (9), and (14), we finally obtain

$$\delta V = -\frac{\hbar}{e}\frac{d\psi}{dt} + 2M_s R_0 \mu_i^{-1}(\beta_1 v_e - v_w) \ . \tag{15}$$

We estimate the term proportional to v_e in Eq. (15), in the case of Permalloy films[4] with $M_s = 1$ T, $\mu_i = 1.5$ m^2/C, $\beta_1 = 1.8$, $R_0 = -1.4\times10^{-10}$ m^3/C. For $j_x = 1\times10^{11}$ A/m^2, corresponding to $v_e = -14$ m/s by Eq. (9), this term leads to $\delta V \simeq +5$ nV. Since this voltage is $\simeq 10^7$ times smaller than the ordinary Ohmic voltage $\rho_s L j_x$ across a distance L equal to 1 μm, it is very difficult to detect.

Considering the v_w term in Eq. (15), and assuming a wall speed $v_w = 30$ m/s, we also obtain $\delta V \simeq 5$ nV. However, if $v_e = 0$, this voltage will not be masked by Ohmic voltages, and may be observable. A magnetic field H_z can be used to move the wall.

We now turn to the term proportional to $d\psi/dt$ in Eq. (15). One can show that, as soon as $d\psi/dt \neq 0$, this term dominates over the others. For $(d\psi/dt)/2\pi = 30$ MHz, it gives $\delta V = -124$ nV. From now on, we will neglect the other terms in Eq. (15). As before, a field H_z can be used at $v_e = 0$ to cause the wall precession $d\psi/dt$. It turns out that this field generates two additional kinds of forces on the electron gas; however, these forces cancel each other, so that Eq. (15) is still valid at $H_z \neq 0$.

In a superconducting junction at $|j_x| > j_{\max}$, the phase difference ϕ between the two superconductors varies with a rate $d\phi/dt \neq 0$. Under these conditions, a dc voltage δV is observed[10] across the junction, given by

$$\delta V = \frac{\hbar}{2e}\frac{d\phi}{dt} \ . \tag{16}$$

This famous equation describes the so-called ac Josephson effect. There is a close mathematical analogy between the first term of Eq. (15) for a ferromagnetic domain wall, and Eq. (16) for a superconducting junction. This analogy can be made more complete if the first term of Eq. (15) is written in the form

$$\delta V = \frac{-\hbar}{2e}\frac{d[2(\psi-\psi_0)]}{dt} \ . \tag{17}$$

As in the case of the dc Josephson effect [Eqs. (4)—(7)], the quantity $2(\psi-\psi_0)$ is the analog of the superconducting phase difference ϕ. Because of the analogy, we will call the phenomena described by Eq. (17), or by the first term of Eq. (15), the ac ferro-Josephson effect.

Although our derivation of Eq. (15) was based on semiclassical spin dynamics, a simple quantum argument leads to the same result: The precessing wall moment, coupled to the electron gas by s-d exchange, constitutes a time-dependent external perturbation characterized by a frequency $d\psi/dt$. Such a perturbation is expected to transfer an energy quantum $\Delta E = \hbar\, d\psi/dt$ to an electron crossing the wall. In turn, this energy difference across the wall should be balanced by an electrical potential difference δV, such that $-e\delta V = \hbar d\psi/dt$. This reproduces the first term of Eq. (15).

By combining Eqs. (2) and (15), we obtain

$$(\tau_{sd})_z \frac{d\psi}{dt} = (\delta V)(j_x + n_e e v_w) \ . \tag{18}$$

As mentioned before, we are using only the ferro-Josephson voltage, i.e., the first term of Eq. (15). The left-hand side of Eq. (18) represents the work per unit time performed on the wall moment by the electron gas. The right-hand side of Eq. (18) is the work per unit time performed on the electron gas by the electric field associated with δV. Thus, Eq. (18) is a statement that energy is conserved in the electron gas.

The wall moment constitutes a reversible electrical motor, driven at the rate $d\psi/dt$ by the torque $(\tau_{sd})_z$, and generating a counter-electromotive force δV. Actually, the magnetization current, associated with the flow of polarized conduction electrons, is more important to the functioning of this motor than the electrical current density j_x itself.

VI. BLOCH LINES AND GYROSCOPIC FORCES

So far, we have assumed the angle ψ to be uniform over a wall. However, it is well known[1] that a Bloch wall is often divided in regions having different chiralities, corresponding to $\psi=0$ and $\psi=\pi$, respectively (Fig. 4). In the case of Néel walls, the regions would correspond to $\psi=\pi/2$ and $\psi=3\pi/2$. The boundary between two such regions is called a 180° Bloch line (Fig. 4). We will assume that the main results of the present theory, such as Eqs. (12) and (15), still hold when ψ is nonuniform.

Assume that adjacent Bloch lines are separated by a distance d_b, and that they move along the wall with a velocity $\mathbf{v}_b^{\parallel}$ (Fig. 4). Then, since ψ varies by π for each Bloch line passing by a given point

$$\frac{d\psi}{dt} = -\mathbf{v}_b^{\parallel}\cdot\nabla\psi \ , \tag{19}$$

$$|\nabla\psi| = \frac{\pi}{d_b} \ . \tag{20}$$

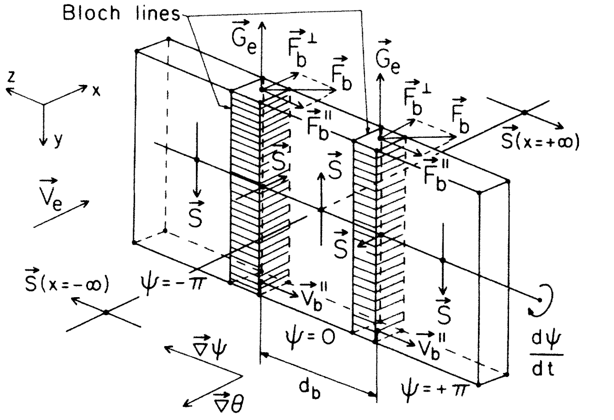

FIG. 4. Bloch lines separate regions of a domain wall with different values of the angle ψ. The velocity $\mathbf{v}_b^{\parallel}$ of a Bloch line is related to the precession rate $d\psi/dt$ of the wall moment, and to the ferro-Josephson force $\mathbf{F}_b^{\perp}$ on the Bloch line. The case shown here is that of a Bloch wall.

Thus, instead of a wall moment precessing with rate $d\psi/dt$, we can use an equivalent language where Bloch lines are moving with velocity $\mathbf{v}_b^{\parallel}$. The torque τ_{sd} on the wall [Eq. (2)] is replaced by a force $\mathbf{F}_b^{\parallel}$ applied to each Bloch line. This force is assumed normal to the Bloch line, and parallel to the wall (Fig. 4). We equate the work per unit time performed on the wall in the two languages

$$(\tau_{sd})_z \frac{d\psi}{dt} = \mathbf{F}_b^{\parallel}\cdot\mathbf{v}_b^{\parallel}/d_b \ . \tag{21}$$

If we combine Eqs. (19), (20), (2), (9), and (21), and use the fact that $\mathbf{v}_b^{\parallel}$, $\mathbf{F}_b^{\parallel}$, and $\nabla\psi$ are all parallel or antiparallel to each other, we find

$$\mathbf{F}_b^{\parallel} = \pi\hbar n_e (v_w - v_e)\nabla\psi / |\nabla\psi| \ . \tag{22}$$

Since $\nabla\theta$ is in the $-x$ direction normal to the wall, this can be rewritten in the form

$$\mathbf{F}_b^{\parallel} = \mathbf{G}_e \times (\mathbf{v}_w - \mathbf{v}_e) \ , \tag{23}$$

$$\mathbf{G}_e = \pi\hbar n_e \hat{\mathbf{t}} \ . \tag{24}$$

Here, $\hat{\mathbf{t}} = \nabla\psi\times\nabla\theta / |\nabla\psi\times\nabla\theta|$ is a unit vector[1] parallel to the Bloch lines, and $\mathbf{G}_e$ is the "*s-d* gyrocoupling vector." Also, $\mathbf{v}_w = v_w\hat{\mathbf{i}}$ and $\mathbf{v}_e = v_e\hat{\mathbf{i}}$, where $\hat{\mathbf{i}}$ is the unit vector along the $+x$ direction. Similarly, we now picture the ferro-Josephson force F_x^w of Eq. (12) as being applied not to a unit area of wall, but rather to the Bloch lines contained in that unit area. Then, the ferro-Josephson force per unit length of Bloch line is

$$\mathbf{F}_b^{\perp} = d_b F_x^w \hat{\mathbf{i}} \ . \tag{25}$$

We combine Eqs. (12), (19), (20), (24) and (25), and obtain

$$\mathbf{F}_b^{\perp} = \mathbf{G}_e \times \mathbf{v}_b^{\parallel} \ . \tag{26}$$

We define a Bloch-line velocity of arbitrary direction by $\mathbf{v}_b = \mathbf{v}_w + \mathbf{v}_b^{\parallel}$. Then, Eqs. (23) and (26) can be added to give the total force $\mathbf{F}_b$ exerted by the electron gas on a Bloch line

$$\mathbf{F}_b = \mathbf{F}_b^{\parallel} + \mathbf{F}_b^{\perp} = \mathbf{G}_e \times (\mathbf{v}_b - \mathbf{v}_e) \ . \tag{27}$$

We see that both the exchange torque and the ferro-Josephson force are described by this one equation.

The concept of a gyrocoupling vector was introduced[11] by Thiele to describe the "dynamic-response force" $\mathbf{F}_b'$ acting on Bloch lines in magnetic insulators. His gyrocoupling vector, $\mathbf{G}_s$, differs from ours

$$\mathbf{F}_b' = \mathbf{G}_s \times \mathbf{v}_b \ ,$$
$$\mathbf{G}_s = \pi\hbar n_s (2S)\hat{\mathbf{t}} \ , \tag{28}$$

where n_s is the number of localized magnetic spins S per unit volume. This force $\mathbf{F}_b'$ couples the Bloch lines to the lattice, not to the electron gas. Since n_e is of the same order of magnitude as n_s, the forces exerted by the electron gas and by the lattice [Eqs. (27) and (28)] are comparable in magnitude.

In the same way that a wall with nonuniform angle ψ can be described in terms of Bloch lines (Fig. 4), so can a superconducting junction with nonuniform phase difference ϕ be described[12] with the concept of superconducting vortex (Fig. 5). While the passage of a 180° Bloch line causes a time variation of ψ by π, the passage of a vortex along the junction corresponds to a variation of ϕ by 2π. As we can see, the mathematical analogy between $2(\psi-\psi_0)$ and ϕ still holds here.

Assume that the vortex is moving with velocity $\mathbf{v}_v$ in a junction traversed by a current density j. Then, an electromagnetic force $\mathbf{F}_v$ is applied[12] to the vortex (Fig. 5).

$$\mathbf{F}_v = \mathbf{G}_e \times (\mathbf{v}_e - \mathbf{v}_v) \ , \tag{29}$$

$$\mathbf{G}_e = \pi\hbar n_e \hat{\mathbf{u}} \ . \tag{30}$$

Here, $\mathbf{v}_e$ is an electron drift velocity in the superconductor, related to j and to the density n_e of superconducting electrons by $\mathbf{j} = -n_e e \mathbf{v}_e$. Also, $\hat{\mathbf{u}}$ is a unit vector parallel to the vortex line. Note the complete analogy between Eqs. (29) and (30) for a superconducting vortex and Eqs. (27) and (24) for a Bloch line.

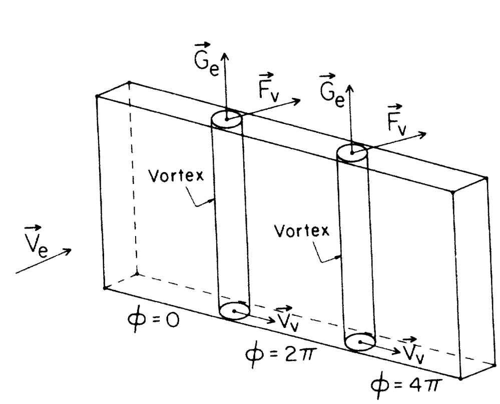

FIG. 5. Vortices separate regions of a superconducting junction with different values of the phase difference ϕ. The velocity $\mathbf{V}_v$ of a superconducting vortex is related to the rate of change $d\phi/dt$ of the phase difference, and to the electromagnetic force $\mathbf{F}_v$ on the vortex.

Just as a longitudinal current interacting with superconducting vortices leads[12] to a transverse dc voltage similar to a Hall voltage, so we expect a current traversing a metallic ferromagnet to generate a small dc voltage in a direction normal to the current and to the Bloch lines. The sign of this voltage would depend on the absolute sense of the vector $\hat{\mathbf{t}}$ describing the chirality of the Bloch lines. This voltage arises from the existence of the force $\mathbf{F}_b^{\parallel}$ of Eq. (23).

VII. OVERALL BALANCE OF FORCES

In order to calculate the dc voltage δV across the wall from Eq. (15), one has to find $d\psi/dt$. For this purpose, we write the balance of external forces applied to the total system containing both the electron gas and the localized spins, projected on the x axis, in the steady state

$$| -n_e e\delta V - g\mu_B(n_e+2Sn_s)H_z + \hbar(2S)n_s d\psi/dt | \leq g\mu_B(n_e+2Sn_s)H_c \ . \quad (31)$$

The first term on the left-hand side is the electrical force on the electron gas, and the second term is the force exerted by a magnetic field H_z parallel to the easy axis. The quantity $-g\mu_B(n_e+2Sn_s)$ is the total magnetization per unit volume. The third term is the Thiele gyroscopic force of Eq. (28), transformed back with the help of Eqs. (19) and (20). The right-hand side is the maximum value of the pinning force on the wall. The coercive field is denoted by H_c. We assume that H_z is large enough that $d\psi/dt \neq 0$, and that the $d\psi/dt$ ferro-Josephson term dominates over the other term in Eq. (15). Then, we eliminate $d\psi/dt$ between Eqs. (15) and (31), and obtain

$$g\mu_B(-H_z-H_c)/e \leq \delta V \leq g\mu_B(-H_z+H_c)/e \ . \quad (32)$$

This general relation is valid for any values of v_e and v_w. The reason why δV does not exceed the bounds given by Eq. (32), at large v_e, is that the wall starts to move in such a way as to prevent that from happening. The s-d exchange forces between electron gas and localized spins are absent from Eq. (31) because they are internal to the system now considered. Assuming $\mu_0|H_z|=1$ mT, $\mu_0 H_c=0.1$ mT, $g=2$, Eq. (32) gives $-127\ \text{nV} \leq \delta V \leq -104$ nV.

As mentioned in Sec. II, plane walls in Permalloy thin films of thickness $\simeq 80$ nm are possible candidates for experiments, because of the relative ease with which the wall moment can be made to precess. Unfortunately, not enough is known about the dynamics of Bloch lines in such samples. Soft, amorphous, thin films with in-plane magnetization, such as Co-Zr, may have similar properties if the thickness is properly chosen.

Metallic bubble materials, such as amorphous Gd-Co, are another possibility. In hard bubbles containing many Bloch lines, any time variation of the bubble diameter causes a free precession of the wall moment, at a rate $d\psi/dt$ which is uniform over the whole wall (see p. 167 of Ref. 1). An ac bias field $\mu_0 H_z \simeq 1$ mT of frequency $\simeq 50$ kHz can be used to modulate the bubble size. Microscopic potential probes are attached inside and outside the bubble, to measure the ferro-Josephson voltage δV across the wall, which has an alternating sign and is given by Eq. (32). Care must be taken to minimize the voltages directly induced by $\mu_0 H_z$ in the voltage leads.

VIII. CONCLUSIONS AND FINAL REMARKS

We have discovered a new exchange force exerted by conduction electrons on a domain wall, in metallic ferromagnets. This "ferro-Josephson" force is proportional to the precession rate $d\psi/dt$ of the wall moment. Correspondingly, there is a "ferro-Josephson" voltage δV across the wall, which satisfies the famous Josephson relation $2e\delta V = -\hbar\omega$. Also, the dynamics of Bloch lines in domain walls in metallic ferromagnets is very similar to the dynamics of vortices in superconductors.

As mentioned earlier, our semiclassical treatment of conduction-electron spins is valid because the wall is rather thick. We can think of the spin s as being averaged over a small volume inside the wall, containing many electrons.

For simplicity, we have considered only the spin-up $4s$ electrons. This one-band model for conduction electrons is not very realistic, and leads to some conceptual difficulties. For example, the spin-up $4s$ band has a nonzero magnetization, so that the separation between magnetic electrons and conduction electrons is incomplete. Calculations with a two-band model still give voltages δV of the same order of magnitude as Eq. (15).

Another limitation of our theory is that we have assumed the walls to be of the traditional one-dimensional type, rather than of the more complex La Bonte two-dimensional type actually found in thin films.

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