Reply to "Comment on 'Annihilation and creation of a Korteweg–de Vries soliton' "

C. Au and P. C. W. Fung
Physics Department, University of Hong Kong, Hong Kong
(Received 8 July 1985)

Previously we analyzed the new solutions of the Korteweg–de Vries equation and interpreted the passage of a soliton through a field as the annihilation and creation of the soliton. Such a solution has a spatial discontinuity. If one disregards this type of discontinuous solution, one may possibly lose certain genuine physical features in the real physical world. We consider it worthwhile to study the solution with discontinuity in detail.

I. INTRODUCTION

Recently,[1] we obtained new solutions to the Korteweg–de Vries (KdV) equation and we found that the solutions contain a number of interesting features. In particular, one set of KdV solutions includes a one-soliton solution plus a field. Carrying out the numerical analysis, we found that the solution describes the creation and annihilation of a soliton as it propagates through a limited spatial region pertaining to the field. In our treatment we do not exclude the singular point in the solution and consider such a solution as physical. This idea is the main source of the comments raised in Ref. 2.

II. REPLY TO COMMENTS

The points made in Ref. 2 are interesting and worthy of discussion. We shall reply to the main issues according to the following items.

(1) In Ref. 2, the author states "Eq. (2) cannot be regarded as solution of the KdV equation valid over the whole x axis," basically because at $x = \pm 1/\sqrt{\lambda}$ the solution is discontinuous. We would emphasize that in some previous cases, workers would not like to consider solutions with discontinuities; however, these discontinuous solutions are indeed solutions to the equation concerned. If we disregard the discontinuous solutions, we may lose some information about certain particular physical phenomena. In the particular case we studied,[1] it is due to the existence of discontinuous solutions that creation and annihilation of solitons could appear.

(2) According to the format of our solution (8) in Ref. 1, we take the absolute value of $(\sqrt{\lambda}x-1)/(\sqrt{\lambda}x+1)$ to ensure that solution u^* is real. We thank the author of Ref. 2 for pointing out correctly that if the absolute value is not taken, the solution u^* is still real.

(3) It is difficult to obtain analytical solutions to nonlinear equations. Various methods may be workable in obtaining solutions. It is too early at this stage to conclude which method is better. Bäcklund transformation[3] is a powerful approach, up to the present time.

(4) The author of Ref. 2 is correct in saying that "the velocity of the soliton varies as a function of x" is not new. In fact, in Ref. 4, such a phenomenon over a very narrow region has been pointed out. However, we emphasize in our work that the velocity of a soliton varies significantly over a wide region in space.

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