Plasmon bands in periodic conducting heterostructures

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A hydrodynamic approach is used to develop a theory of the collective modes of a periodic system consisting of alternate degenerate semiconductor or metallic layers characterized by different equilibrium densities. The plasma modes propagate within the low-density layers, but not in the high-density ones. Due to the periodicity of the system, the modes in different layers interact to form plasmon bands.

Bulk plasmons in a simple metal or degenerate semiconductor of uniform elecron density n satisfy a dispersion relation $\omega^2=\omega_p^2+\beta^2|\mathbf{q}|^2$, where $\omega_p=(4\pi ne^2/m)^{1/2}$ is the plasma frequency, β is a stiffness constant proportional to the Fermi velocity, and $\mathbf{q}$ is the wave vector of the plasmon. In a simple local theory β is set equal to zero and the frequency of a plasmon is independent of its wavelength. For a thin metallic film embedded in a dielectric two types of plasma modes exist. One type, which occurs even in a local theory, consists of the symmetric and antisymmetric linear superpositions of the surface plasmons of each of the metal-dielectric interfaces. The second type consists of bulk plasmons propagating in the thin film. If the film has thickness a in the z direction, these propagating or wave-guide-like plasma modes have frequencies $\omega_n^2=\omega_p^2+\beta^2(q^2+k_n^2)$, where q is the component of the wave vector along the layer and $k_n=n\pi/a$ (for integral n) are the allowed values of the component of the wave vector in the z direction. If the dielectric host is replaced by a metal of high plasma frequency, the boundary conditions at the interfaces are somewhat modified. The density fluctuations which are propagating waves in the low electron density film must decay exponentially with distance into the high electron density host, and this modifies the allowed values of k_n.

For periodic metallic heterostructures the effect of spatial periodicity on the dispersion of plasma modes has been studied previously in the local limit,[1,2] and hence only for the first type of mode mentioned. The different constituents of the superlattice were there described by local dielectric functions. Here we will include nonlocal effects by using a hydrodynamic approach developed by Ying[3] and first applied to the study of plasmons in systems of nonuniform electron density by Eguiluz and Quinn.[4]

We consider a periodic system consisting of alternate layers characterized by different bulk-plasma frequencies ω_A and ω_B and by thicknesses a and b, respectively. The period of the system is $d=a+b$. The equation of motion for the fluctuation $n_1(r,t)=n_1(z)e^{iqy-i\omega t}$ in the number density of electrons is[5]

$$\left[\omega^2-\omega_0^2(z)-\beta^2(z)\left(q^2-\frac{d^2}{dz^2}\right)\right]n_1(z)+\left(\frac{d}{dz}\beta^2(z)\right)\frac{dn_1(z)}{dz}+\frac{1}{4\pi e}E_z^{(1)}(z)\frac{d}{dz}\omega_0^2(z)=0\ , \tag{1}$$

where

$$\omega_0^2(z)=\begin{cases}\omega_A^2, & z\in(md,\ a+md)\ ,\\ \omega_B^2, & z\in(a+md,\ (m+1)d)\ ,\end{cases} \tag{2}$$

$$E_z^{(1)}(z)=4\pi e\int_{-\infty}^{\infty}dz'\,\mathrm{sgn}(z-z')e^{-q|z-z'|}n_1(z')\ . \tag{3}$$

The stiffness β, which is related to the compressibility of the electron gas, is given in the random-phase approximation by

$$\beta^2=\tfrac{3}{5}v_F^2=\frac{3\hbar}{5m}(3\pi^2n_0)^{2/3}\ .$$

Since the last two terms in (1) are different from zero at the interfaces only, one can account for their effects by requiring both n_1 and the current density j_z to be continuous at the interfaces.[5]

j_z is obtained from the equation of continuity, and is given by[4,5]

$$j_z(z)=\frac{1}{i\omega}\left(\frac{\omega_0^2(z)}{4\pi}E_z^{(1)}(z)+e\beta^2(z)\frac{dn_1(z)}{dz}\right)\ . \tag{4}$$

In the light of this, Eq. (1) takes the following form:

$$\left[\omega^2-\omega_A^2-\beta_A^2\left(q^2-\frac{d^2}{dz^2}\right)\right]n_1(z)=0,\quad z\in[0,a]\ , \tag{5a}$$

$$\left[\omega^2-\omega_B^2-\beta_B^2\left(q^2-\frac{d^2}{dz^2}\right)\right]n_1(z)=0,\quad z\in[a,d]\ , \tag{5b}$$

where $n_1(z)$ is assumed to obey Bloch's condition $n_1(z+nd)=e^{iknd}n_1(z)$.

In the regime $\omega_B>\omega>\omega_A$ the solutions of the equations of motion can be written

$$n_1(z)=\begin{cases}Ae^{ip_Az}+Be^{-ip_Az}, & z\in[0,a]\ ,\\ Ce^{-p_Bz}+De^{p_Bz}, & z\in[a,d]\ ,\end{cases} \tag{6}$$

where

$$p_A^2=\beta_A^{-2}(\omega^2-\omega_A^2)-q^2$$

and

$$p_B^2=q^2+\beta_B^{-2}(\omega_B^2-\omega^2).$$

By making use of the Bloch condition and applying the boundary conditions at $z=0$ and $z=a$, one obtains a set of equations for the constants A, B, C, and D. The dispersion relation is then given by the zeros of the determinant of the coefficients. In the expression (4) for j_z, $E_z^{(1)}$ has to be evaluated at $z=a,d$ by using the ansatz (6) in (3). If we introduce the decomposition

$$E_z(z)=E_A(z)A+E_B(z)B+E_c(z)C+E_D(z)D,$$

then the secular equation becomes

$$\begin{vmatrix} e^{ip_A a} & e^{-ip_A a} & -e^{-p_B a} & -e^{p_B(a-d)} \\ e^{ikd} & e^{ikd} & -e^{-p_B d} & -1 \\ \frac{W^2}{4\pi e}E_A(a)-i\beta_A^2 p_A e^{ip_A a} & \frac{W^2}{4\pi e}E_B(a)+i\beta_A^2 p_A e^{-ip_A a} & \frac{W^2}{4\pi e}E_C(a)-\beta_B^2 p_B e^{-p_B a} & \frac{W^2}{4\pi e}E_D(a)e^{-p_B d}+\beta_B^2 p_B e^{p_B(a-d)} \\ \frac{W^2}{4\pi e}E_A(d)-i\beta_A^2 p_A e^{ikd} & \frac{W^2}{4\pi e}E_B(d)+i\beta_A^2 p_A e^{ikd} & \frac{W^2}{4\pi e}E_C(d)-\beta_B^2 p_B d^{-p_B d} & \frac{W^2}{4\pi e}E_D(d)e^{-p_B d}+\beta_B^2 p_B \end{vmatrix}=0 \ , \quad (7)$$

where

$$E_A(a)=-2\pi e e^{-qa}I_{++}S^*+2\pi e e^{qa}I_{-+}(S-1) \ ,$$

$$E_B(a)=-2\pi e e^{-qa}I_{+-}S^*+2\pi e e^{qa}I_{--}(S-1) \ ,$$

$$E_C(a)=-2\pi e e^{-qa}J_{+-}(S^*-1)+2\pi e e^{qa}J_{--}S \ ,$$

$$E_D(a)=-2\pi e e^{-qa}J_{++}(S^*-1)+2\pi e e^{qa}J_{-+}S \ ,$$

$$E_A(d)=-2\pi e e^{-qd}I_{++}S^*+2\pi e e^{qd}I_{-+}(S-1) \ ,$$

$$E_B(d)=-2\pi e e^{-qd}I_{+-}S^*+2\pi e e^{qd}I_{--}(S-1) \ ,$$

$$E_C(d)=-2\pi e e^{-qd}J_{+-}S^*+2\pi e e^{qd}J_{--}(S-1) \ ,$$

$$E_D(d)=-2\pi e e^{-qd}J_{++}S^*+2\pi e e^{qd}J_{-+}(S-1) \ ,$$

$$I_{\pm\pm}=\frac{e^{(\pm q\pm ip_A)a}-1}{\pm q\pm ip_A} \ ,$$

$$J_{\pm\pm}=\frac{e^{(\pm q\pm p_B)d}-e^{(\pm q\pm p_B)a}}{\pm q\pm p_B} \ ,$$

$$S=\frac{e^{qd}-e^{-ikd}}{2(\cosh qd-\cos kd)} \ ,$$

$$W^2=\omega_B^2-\omega_A^2 \ ,$$

and S^* is the complex conjugate of S. If $\omega_B >> \omega_A$, and if b is large, then nearly independent plasmon excitations of each low-density layer can occur at frequencies satisfying

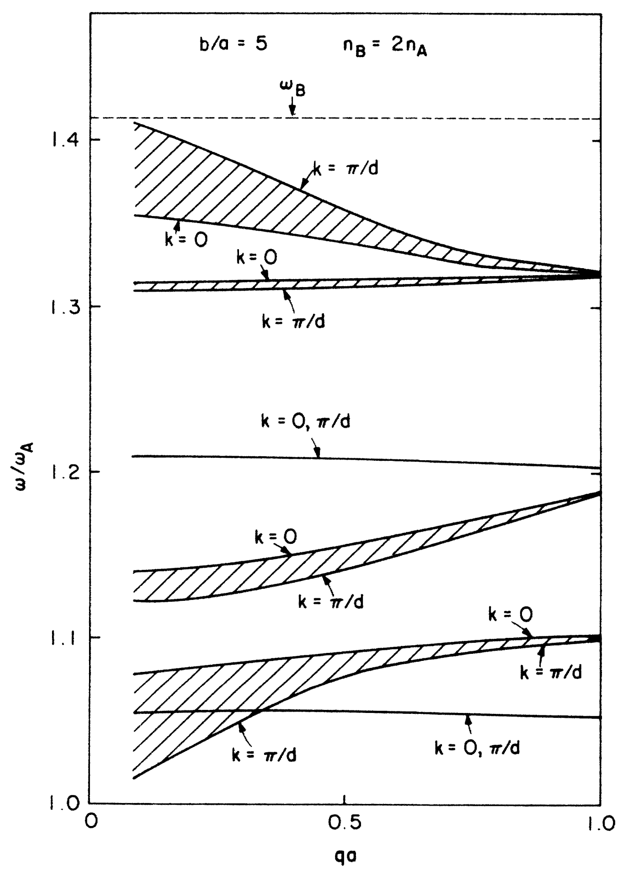

FIG. 1. ω vs q for $k \in [0,\pi/d]$. The parameters used are described in the text.

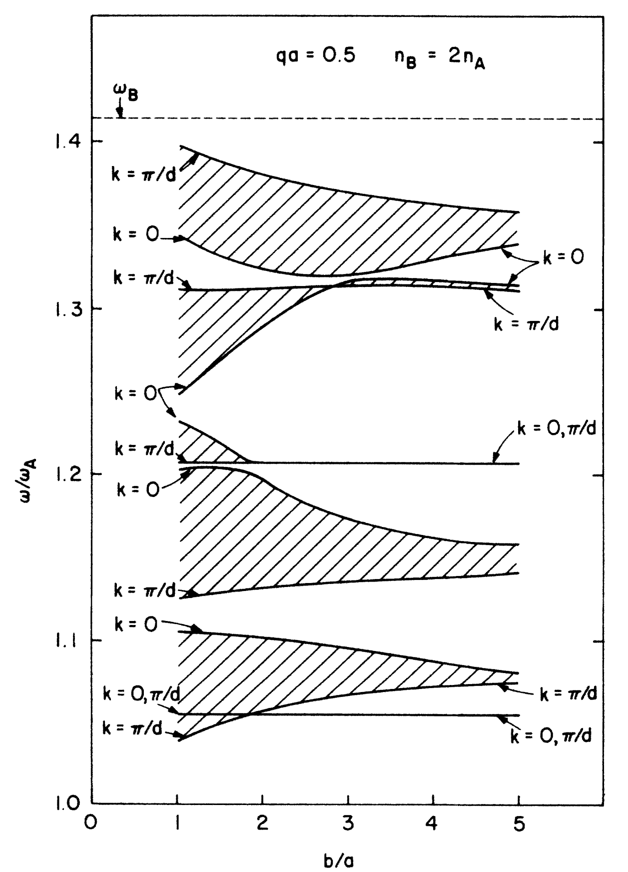

FIG. 2. The frequency of the band edges vs b, the thickness of the high density layers, for $qa=0.5$.

$\omega^2 \cong \omega_A^2 + \beta_A^2 (n\pi/a)^2$, where n is a positive integer. These single-well excitations are just the plasmon waveguide modes discussed earlier; they propagate along the low-density layer with wave vector q, while the wave number in the direction of the superlattice must be roughly $\pi/2$ times an integer. The excitations are confined to the individual wells as long as $\omega << \omega_B$. Due to the periodicity of the system in the superlattice direction, the wave-guide-like modes interact to form bands. Very narrow bands (weak coupling between layers) occur when $qb >> 1$ and $\omega_B >> \omega$. Figure 1 is a plot of ω vs q for $k=0$ and π/d. The system has $n_A = 2.33 \times 10^{17}$ cm^{-3}, $n_B = 2n_A$, $a = 100$ Å, and $b/a = 5$. The bands with very small bandwidth (weak coupling between layers) are single-well excitations obtained by considering a system consisting of a single A layer between two infinite B layers. This weak dispersion will occur when $\omega_B >> \omega$ and the parity of the single-well plasmon is even. The reason for this is that by symmetry the contributions to the electric field due to distant slabs tend to cancel in an even plasmon oscillation. Figure 2 is a plot of the frequency of the band edges as a function of b, the width of the high density layers. For large qb with $\omega << \omega_B$, the bands are very narrow, as expected. The qualitative characteristics of the plasma bands described here should be relatively insensitive to the details of the spatial variation of the periodic electron density. However, metallic or degenerate semiconducting superlattices which come close to the simple model we have assumed can be fabricated. In this situation either electron energy-loss spectroscopy or inelastic light scattering would be possible tools for studying the plasmon bands.

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