Theory of cyclotron-resonance line shape for an electron-phonon system

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(Received 23 May 1985)

Formulas for the linewidth and line shift associated with cyclotron-resonance power-absorption spectral lines are obtained for an electron-phonon system from a self-consistent equation for a self-energy in the case of weak incoherent and strong coherent scattering limits by using the resolvent superoperator method.

I. INTRODUCTION

In 1955, Dresselhaus *et al.*[1] observed cyclotron resonance in germanium. Since then many experiments[2,3] have been carried out to study electronic band structure; the position of the absorption peak gives the value of the effective mass of conduction electrons. However, in recent years special attention has been paid to the study of the shape of a cyclotron-resonance power-absorption line.[4-10] The shape of the line and/or the linewidth and line shift, and their dependence on temperature and magnetic field strength, depend sensitively on the detailed nature of the scattering mechanisms for the charge carriers in solids. Thus, they provide a useful probe for these interactions.

A great number of experimental studies have been done on elemental and other semiconductors, and the experimental results have been analyzed by semiclassical theory[11] or by a more sophisticated theory[12-27] utilizing Green-function methods,[12,13] diagram methods,[15,24,25] or projection-operator methods.[14,16-22] Among these theories, Kubo formalism[28,29] combined with the projection-operator method introduced by Mori[30] and by Argyres and Sigel,[16] and with the diagram method of Lodder and Fujita,[15] appears to be very rigorous and formal. Kawabata[14] adopted Mori's method and derived the formula for the cyclotron-resonance power-absorption line shape (CRLS) for an electron-phonon as well as an electron-impurity system. However, his theory is limited to the incoherent elastic scatterings and therefore cannot be applicable for the strong-interaction case. Argyres and Sigel[16] developed a theory with the use of a similar projection-operator technique and they claimed that the perturbative expansions used in the theories of Kawabata[14] and of Lodder and Fujita[15] are not valid at the peaks of the absorption lines. Recently, Choi and coworkers[19-22] developed a general theory of the CRLS for an electron-phonon system by applying Kawabata's approach utilizing the projection operator and they obtained a formula for a CRLS, including the one-phonon inelastic scattering processes. But their theory is limited to incoherent weak scatterings. In their theory the assert that the theories of Kawabata[14] and of Lodder and Fujita[15] are valid at the peaks of the absorption lines, in contradiction to that of Argyres and Sigel.[16] The origin of the discrepancy among these theories may be traced back to the point where the perturbative expansion is performed in different ways.

In this paper we will utilize a different approach to perturbative expansion,[28] which is developed for an evaluation of the frequency-dependent conductivity of a strongly interacting electron-phonon system, and will show a detailed derivation of the formulas for the linewidth and line shift associated with the cyclotron-resonance transition. This method is much simpler and more direct. Formulas are obtained for a strongly interacting coherent, and a weakly interacting incoherent, scattering case in which one-phonon inelastic scattering processes are included. The conductivity is calculated for a system of free electrons interacting with phonons from Kubo's current correlation integral formula[29] and hence the effects of coherent and incoherent scatterings by phonons can be, in principle, included. We employ a factorization approximation for the equilibrium statistical operator as a grand canonical distribution for the electrons, whereas we employ the canonical distribution for phonons. Then, we can express the conductivity as in Eq. (2.14). Thus, any collision process between an electron and phonons is assumed to take place in the average field of phonons. The effect of such a field is to induce perturbed single-particle energies and to introduce lifetimes for the electron states. The lifetime broadening due to the interaction is responsible for the spectral broadening of line shapes, as will be shown in Sec. III.

In Sec. II the general formula for the frequency-dependent conductivity of an electron-phonon system is given in terms of a simpler effective one-electron resolvent $\mathscr{R}_z^D$. This depends on the self-energy superoperator φ_z^D which is defined at the outset. We will discuss the conductivity for the case of the cyclotron-resonance problems in Sec. III. In Sec. IV the general expression for the temperature-dependent self-energy, which induces the line broadening and line shift in the CRLS, is given. Formulas to give the linewidth and line shift associated with the cyclotron-resonance transition are explicitly given under proper approximations being made. In the Appendix the derivation of a self-consistent equation for a self-energy is given for a general case.

II. FORMULATION OF THE PROBLEM (FORMAL PRELIMINARIES)

The average power absorbed by the electrons when driven by a circularly polarized electric field $\mathbf{E}(t)$ of fre-

quency ω is given by[15]

$$P(\omega)=\tfrac{1}{2}E^2\mathrm{Re}[\sigma_{+-}(\omega)]$$
$$=\tfrac{1}{2}E^2\mathrm{Re}\{\sigma_{xx}(\omega)+\sigma_{yy}(\omega)+i[\sigma_{xy}(\omega)-\sigma_{yx}(\omega)]\}\ , \tag{2.1}$$

where E is the total electric field strength; $\mathrm{Re}[\sigma_{ij}(\omega)]$ is the real part of the complex conductivity tensor. Each element of $\sigma_{ij}(\omega)$, $(i,j=x,y,z)$, is obtained by Kubo's current correlation integral formula[29] and $\sigma_{+-}(\omega)$ can be expressed as

$$\sigma_{+-}(\omega)=\lim_{\delta\to 0^+}\Omega^{-1}\int_0^\infty dt\, e^{-i(\omega-i\delta)t}\times\int_0^\beta d\beta_1\mathrm{Tr}[\rho J_-(-i\hbar\beta_1)\times J_+(t)]\ , \tag{2.2}$$

$$J_\pm=J_x\pm iJ_y\ . \tag{2.3}$$

Here, Ω is the volume of the system, δ is a positive number, $\beta\equiv(k_BT)^{-1}$ is the reciprocal temperature, $J_\pm$ are the transverse components of the total current operator in the many-body formalism, $J(t)$ is the time-dependent total current operator in the Heisenberg picture, and ρ is the grand canonical density operator:

$$\rho=e^{\beta(\zeta N-H)}/\mathrm{Tr}(e^{\beta(\zeta N-H)})\ , \tag{2.4}$$

where ζ is the chemical potential, N is the electron number operator, and H is the Hamiltonian of the system.

We consider an electron-phonon system subject to the static magnetic field $\mathbf{B}$ [perpendicular to $\mathbf{E}(t)$] taken in the z axis. The time-independent Hamiltonian H of the system of free electrons interacting with phonons is

$$H=\sum_l h(\mathbf{r}_l,\mathbf{p}_l)+H_{\mathrm{ph}}\ , \tag{2.5}$$

$$h(\mathbf{r},\mathbf{p})=h_0(\mathbf{r},\mathbf{p})+\eta V(\mathbf{r})\ , \tag{2.6}$$

$$h_0=[\mathbf{p}+e\mathbf{A}(\mathbf{r})]^2/2m\ , \tag{2.7}$$

$$V=\sum_{\mathbf{q}}[\gamma_{\mathbf{q}}(\mathbf{r})b_{\mathbf{q}}+\gamma_{\mathbf{q}}^\dagger(\mathbf{r})b_{\mathbf{q}}^\dagger]\ , \tag{2.8}$$

$$H_{\mathrm{ph}}=\sum_{\mathbf{q}}(b_{\mathbf{q}}^\dagger b_{\mathbf{q}}+\tfrac{1}{2})\hbar\omega_{\mathbf{q}}\ , \tag{2.9}$$

where l in Eq. (2.5) denotes the single-electron index, $b_{\mathbf{q}}$ ($b_{\mathbf{q}}^\dagger$) is the annihilation (creation) operator for a phonon of wave vector $\mathbf{q}$ and energy $\hbar\omega_{\mathbf{q}}$, $\gamma_{\mathbf{q}}(\mathbf{r})$ describes the interaction of the electron and phonon and is of the form $C_{\mathbf{q}}\exp(i\mathbf{q}\cdot\mathbf{r})$, where $C_{\mathbf{q}}$ depends on the type of interaction, $\mathbf{r}$ and $\mathbf{p}$ are, respectively, the position and momentum of a conduction electron with an effective mass m, $\mathbf{A}$ is a vector potential which gives rise to the static field $\mathbf{B}$, and η is a parameter to indicate the order in the expansion, which is set equal to 1 later on.

It is convenient to use the Landau gauge for the vector potential

$$\mathbf{A}(\mathbf{r})=(0,Bx,0)\ . \tag{2.10}$$

The Hamiltonian h_0 of a free electron is then expressed by

$$h_0(\mathbf{r},\mathbf{p})=[p_x^2+(p_y+m\omega_cx)^2+p_z^2]/2m\ , \tag{2.11}$$

and its energy eigenvalues E_λ and eigenstates $|\lambda\rangle=|n,\mathbf{k}\rangle$ are specified by the oscillator quantum number (Landau-level index) $n=0,1,2,\ldots$ and a wave vector $\mathbf{k}=(0,k_y,k_z)$ and are, respectively, given by

$$E_\lambda\equiv E_n(k_z)\equiv(n+\tfrac{1}{2})\hbar\omega_c+\hbar^2k_z^2/2m$$
$$\equiv(n+\tfrac{1}{2})\hbar\omega_c+\epsilon(k_z)\ , \tag{2.12}$$

$$\langle\mathbf{r}\,|\,\lambda\rangle\equiv\langle\mathbf{r}\,|\,n\mathbf{k}\rangle$$
$$=(L_yL_z)^{-1/2}\psi_n(x+\hbar k_y/m\omega_c)\exp(i\mathbf{k}\cdot\mathbf{r})\ . \tag{2.13}$$

Here, $\psi_n(x+\hbar k_y/m\omega_c)$ are the eigenfunctions of a simple harmonic oscillator of frequency $\omega_c\equiv eB/m$, the cyclotron frequency, centered at $-\hbar k_y/m\omega_c$; L_y and L_z are the y- and z-directional normalization lengths. We see from Eq. (2.12) that the motion of the electron is quantized in the x-y plane and that it is quasicontinuous in the z direction. The energy levels form a series of one-dimensional subbands (Landau levels).

Recently, Suzuki and Dunn[28] developed a resolvent superoperator method to evaluate the conductivity tensor $\sigma_{ij}(\omega)$, suitable for a strongly interacting electron-phonon system, starting from Kubo's formula. Applying the method for the electron-phonon system defined by Eqs. (2.5)–(2.9), we can immediately write $\sigma_{+-}(\omega)$ in the single-electron expression

$$\sigma_{+-}(\omega)=-i\hbar\lim_{\delta\to 0^+}\Omega^{-1}tr[\Psi_-(z)j_+]$$
$$\equiv-i\hbar\lim_{\delta\to 0^+}\Omega^{-1}tr[(\mathscr{R}_z^DM_-)j_+]\ , \tag{2.14}$$

where

$$\mathscr{R}_z^D\equiv(\mathscr{L}_0+\mathscr{S}_z^D-z)^{-1}\ , \tag{2.15}$$

$$M_-\equiv\lim_{u\to 0}\frac{\partial}{\partial u}[\langle n_-\rangle_{\mathrm{ph}}+\langle(\mathscr{S}_z^D-\eta\mathscr{V})\mathscr{R}_zn_-\rangle_{\mathrm{ph}}]\ , \tag{2.16}$$

$$n_-\equiv[e^{\beta(h-uj_--\zeta)}+1]^{-1}\ , \tag{2.17}$$

$$j_\pm=j_x\pm ij_y\ , \tag{2.18}$$

$$z\equiv-\hbar(\omega-i\delta)\ . \tag{2.19}$$

Here, n_- is the Fermi operator, u is a classical number, and $j_\pm$ are the transverse components of the one-electron current operators. The resolvent superoperator $\mathscr{R}_z$ and the self-energy superoperator $\mathscr{S}_z^D$ are, respectively, defined by

$$\mathscr{R}_z\equiv(\mathscr{L}_0+\mathscr{H}_{\mathrm{ph}}+\eta\mathscr{V}-z)^{-1}\ , \tag{2.20}$$

$$\mathscr{S}_z^D\equiv-\eta^2\langle\mathscr{V}\mathscr{R}_z\mathscr{V}\rangle_{\mathrm{ph}}\ , \tag{2.21}$$

and can be expressed in terms of an infinite series of the effective one-electron resolvent superoperator $\mathscr{R}_z^D$ defined by

$$\mathscr{R}_z^D\equiv(\mathscr{L}_0+\mathscr{H}_{\mathrm{ph}}+\mathscr{S}_z^D-z)^{-1}\ . \tag{2.22}$$

See Eqs. (2.29) and (2.30) of Ref. 28 for details. It is not-

ed that the many-particle trace (Tr) in Eq. (2.2) is reduced to a single-particle trace (denoted by tr) in Eq. (2.14). We should emphasize that these series expansions for $\mathscr{R}_z$ and $\mathscr{G}_z^D$ are not the conventional perturbation expansions and that, in general, we expect the convergence to be quite rapid, even for strongly interacting systems. The reason for this is that the effects of the interaction are included in $\mathscr{R}_z^D$. This also has the effect that the term-by-term divergences experienced in the usual perturbation theory are circumvented. It is noted that a script letter denotes a superoperator which generates a commutator upon acting on an ordinary quantum operator, such that

$$\mathscr{A}B \equiv [A,B] \equiv AB - BA \ , \tag{2.23}$$

and that the angular brackets $\langle A \rangle_{\rm ph}$ denote the averaging of A over the phonon states:

$$\langle A \rangle_{\rm ph} = {\rm Tr}^{({\rm ph})}(\rho_{\rm ph} A) \ , \tag{2.24}$$

where

$$\rho_{\rm ph} = e^{-\beta H_{\rm ph}} / {\rm Tr}^{({\rm ph})}(e^{-\beta H_{\rm ph}}) \ . \tag{2.25}$$

Equation (2.14) is exact in the thermodynamic limit other than the assumption,[28,31] which is justified for the density operator ρ. It should be noted that the expressions (2.14)—(2.22) are independent of the single-particle representation and hence can be applied to a system subjected to a static magnetic field.

III. GENERAL DISCUSSION OF $\sigma_{+-}(\omega)$

Let us first consider the system of free electrons ($\eta = 0$). Then, the formula (2.14) reduces to

$$\sigma_{+-}^{(0)}(\omega) = -i\hbar \lim_{\delta \to 0^+} \Omega^{-1} {\rm tr}[\Psi_-^{(0)}(z) j_+]$$
$$= -i\hbar \lim_{\delta \to 0^+} \Omega^{-1} {\rm tr}[(\kappa_z^{(0)} M_-^{(0)}) j_+] \ , \tag{3.1}$$

where

$$\kappa_z^{(0)} \equiv (\mathscr{h}_0 - z)^{-1} \ , \tag{3.2}$$

$$M_-^{(0)} \equiv \lim_{u \to 0} \frac{\partial}{\partial u} \langle n_-^{(0)} \rangle_{\rm ph} \ , \tag{3.3}$$

$$n_-^{(0)} \equiv (e^{\beta(\mathscr{h}_0 - u j_- - \zeta)} + 1)^{-1} \ . \tag{3.4}$$

In the Landau representation given by Eq. (2.13), we can express Eq. (3.1) as

$$\sigma_{+-}(\omega) = -i\hbar \lim_{\delta \to 0^+} \Omega^{-1} \sum_{\lambda_1} \sum_{\lambda_2} \langle \lambda_1 | \kappa_z^{(0)} M^{(0)} | \lambda_2 \rangle \langle \lambda_2 | j_+ | \lambda_1 \rangle$$
$$= e(2\hbar\omega_c/m)^{1/2} \Omega^{-1} \lim_{\delta \to 0^+} \sum_{\lambda_1} (n_1+1)^{1/2} \langle \lambda_1 | M_-^{(0)} | \lambda_1 + 1 \rangle \frac{1}{\omega - \omega_c - i\delta} \ . \tag{3.5}$$

As expected, this expression indicates the divergence at the cyclotron frequency $\omega = \omega_c$. The origin of the divergence can be traced without further calculation: it is the energy denominator $(\kappa_z^{(0)})^{-1} = \mathscr{h}_0 - z$, which is responsible. In the following, we shall adopt the notation $|\lambda \pm 1\rangle$ to indicate the state $|n \pm 1, \mathbf{k}\rangle$ if $|\lambda\rangle = |n, \mathbf{k}\rangle$ in the Landau representation.

When electrons are interacting with phonons, the quantity $\sigma_{+-}(\omega)$ is expressed as in Eq. (2.14). One point of difference with Eq. (2.14) is that the energy denominator $(\kappa_z^{(0)})^{-1} = \mathscr{h}_0 - z$ is changed to $(\kappa_z^D)^{-1} = \mathscr{h}_0 + \mathscr{g}_z^D - z$. As we shall see later, the general effect of $\mathscr{g}_z^D$ is the elimination of the divergence near $\omega = \omega_c$. In other words, the self-energy superoperator $\mathscr{g}_z^D$ introduces the resonance broadening and the energy shift. Another marked difference as compared to Eq. (3.1) is the appearance of the term $\langle (\mathscr{g}_z^D - \eta\mathscr{V}) \mathscr{R}_z n_- \rangle_{\rm ph}$ which depends on z (i.e., ω) and which may diverge at $\omega = \omega_c$. However, this term which originates in the inclusion of the interaction term in ρ, contains the energy denominator $(\mathscr{R}_z^D)^{-1} = \mathscr{h}_0 + \mathscr{H}_{\rm ph} + \mathscr{g}_z^D - z$ [see Eq. (2.31) in Ref. 28]. Because of the presence of $\mathscr{H}_{\rm ph}$, this term is unlikely to produce any resonancelike terms. The magnitude of resonance maximum surely depends on the operator n_- and $\langle (\mathscr{g}_z^D - \eta\mathscr{V}) \mathscr{R}_z n_- \rangle_{\rm ph}$. As discussed above, the resonance line broadening (shape) essentially depends on the behavior of the energy denominator $(\kappa_z^D)^{-1} = \mathscr{h}_0 + \mathscr{g}_z^D - z$. Therefore, we may drop $\langle (\mathscr{g}_z^D - \eta\mathscr{V}) \mathscr{R}_z n_- \rangle_{\rm ph}$ and approximate $n_- \simeq n_-^{(0)}$ given by Eq. (3.4). Equation (2.14) is then given by

$$\sigma_{+-}(\omega) \simeq -i\hbar \lim_{\delta \to 0} \Omega^{-1} {\rm tr}[(\kappa_z^D M_-^{(0)}) j_+]$$
$$= -i\hbar \lim_{\delta \to 0^+} \Omega^{-1} \sum_{\lambda_1} \sum_{\lambda_2} \langle \lambda_1 | \Psi_-(z) | \lambda_2 \rangle \times \langle \lambda_2 | j_+ | \lambda_1 \rangle \ , \tag{3.6}$$

where κ_z^D and $M_-^{(0)}$ are, respectively, given by Eqs. (2.15) and (3.3). Therefore, the resonance behavior of the dynamic conductivity is mainly governed by the operator $\Psi_-(z)$ defined by

$$\Psi_-(z) \equiv \kappa_z^D M_-^{(0)} = (\mathscr{h}_0 + \mathscr{g}_z^D - z)^{-1} M_-^{(0)} \tag{3.7}$$

or

$$(\mathscr{h}_0 + \mathscr{g}_z^D - z) \Psi_-(z) = M_-^{(0)} \ . \tag{3.8}$$

Taking the (λ_1, λ_2)th matrix element of both sides of the above equation, we obtain

$$\langle \lambda_1 | (\mathscr{h}_0 + \mathscr{g}_z^D - z) \Psi_-(z) | \lambda_2 \rangle = \langle \lambda_1 | M_-^{(0)} | \lambda_2 \rangle \tag{3.9}$$

or

$$(E_{\lambda_1}-E_{\lambda_2}-z)\langle\lambda_1|\Psi_-(z)|\lambda_2\rangle+\langle\lambda_1|\mathcal{G}_z^D\Psi_-(z)|\lambda_2\rangle = \langle\lambda_1|M_-^{(0)}|\lambda_2\rangle\ , \quad (3.10)$$

where $\langle\lambda_1|M_-^{(0)}|\lambda_2\rangle$ is given by

$$\langle\lambda_1|M_-^{(0)}|\lambda_2\rangle=\lim_{u\to 0}\left\langle\lambda_1\left|\frac{\partial}{\partial u}n_-^{(0)}\right|\lambda_2\right\rangle = \frac{f(E_{\lambda_1})-f(E_{\lambda_2})}{E_{\lambda_1}-E_{\lambda_2}}\langle\lambda_1|j_-|\lambda_2\rangle\ . \quad (3.11)$$

Here, $f(E)$ denotes the Fermi-Dirac distribution function. Therefore, the matrix elements of the operator $\Psi_-(z)$ can be evaluated after an appropriate approximation has been made for the collision term $\mathcal{G}_z^D\Psi(z)$. Let us postulate that

$$\langle\lambda_1|\mathcal{G}_z^D\Psi_-(z)|\lambda_2\rangle\equiv-\hbar\Xi(\omega,\omega_c;\lambda_1{:}\lambda_2)\langle\lambda_1|\Psi_-(z)|\lambda_2\rangle\ , \quad (3.12)$$

where $\hbar\Xi(\omega,\omega_c;\lambda_1{:}\lambda_2)$ is the complex self-energy associated with the transition between states $|\lambda_1\rangle$ and $|\lambda_2\rangle$. Once we obtain a form (3.12), we can formally express the complex conductivity tensor (3.6) as

$$\sigma_{+-}(\omega)=i\hbar\lim_{\delta\to 0^+}\Omega^{-1}\sum_{\lambda_1}\sum_{\lambda_2}\frac{[f(E_{\lambda_1})-f(E_{\lambda_2})]\,|\langle\lambda_1|j_-|\lambda_2\rangle|^2}{(E_{\lambda_1}-E_{\lambda_2})[E_{\lambda_1}-E_{\lambda_2}-\hbar\Xi(\omega,\omega_c;\lambda_1{:}\lambda_2)-z]}\ , \quad (3.13)$$

where we have used Eqs. (3.10), (3.11), (3.12), and the relation

$$\langle\lambda_1|j_+|\lambda_2\rangle=[\langle\lambda_2|j_-|\lambda_1\rangle]^* = ie[2\hbar\omega_c(n_1+1)/m]^{1/2}\delta_{\lambda_2,\lambda_1+1}\ . \quad (3.14)$$

The real and imaginary part of $\Xi(\omega,\omega_c;\lambda_1{:}\lambda_2)$, namely,

$$\Xi(\omega,\omega_c;\lambda_1{:}\lambda_2)\equiv\Delta(\omega,\omega_c;\lambda_1{:}\lambda_2)+i\Gamma(\omega,\omega_c;\lambda_1{:}\lambda_2)\ , \quad (3.15)$$

defines the *line shift* and *linewidth*, respectively, for the electronic transition between states $|\lambda_1\rangle$ and $|\lambda_2\rangle$. Clearly, we need a close look at the term $\langle\lambda_1|\mathcal{G}_z^D\Psi_-(z)|\lambda_2\rangle$, which yields the equation for the self-energy $\hbar\Xi(\omega,\omega_c;\lambda_1{:}\lambda_2)$, as will be shown in the next section. See also the Appendix.

IV. THE SELF-CONSISTENT EQUATION FOR THE SECOND-ORDER SELF-ENERGY

Let us consider the collision term $\langle\lambda_1|\mathcal{G}_z^D\Psi_-|\lambda_2\rangle$, which essentially determines resonance line shape. Hence, hereafter we call this term the cyclotron-resonance line-shape function (CRLSF). We consider only the second-order renormalized self-energy superoperator

$$\mathcal{G}_z^D\simeq-\eta^2\sum_{\mathbf{q}}\langle\mathcal{V}_{\mathbf{q}}\mathcal{R}_z^D\mathcal{V}_{\mathbf{q}}\rangle_{\rm ph}\equiv-\mathcal{L}_z\ , \quad (4.1)$$

where the effective one-electron resolvent superoperators is given by

$$\mathcal{R}_z^D=(\mathcal{L}_0+\mathcal{H}_{\rm ph}-\mathcal{L}_z-z)^{-1}\ . \quad (4.2)$$

For cyclotron resonance we only need the matrix element $\langle\lambda_1|\Psi_-(z)|\lambda_1+1\rangle$ because of the factor

$$\langle\lambda_2|j_+|\lambda_1\rangle=[\langle\lambda_1|j_-|\lambda_2\rangle]^* = ie(2\hbar\omega_c n_2/m)^{1/2}\delta_{\lambda_2,\lambda_1+1}$$

appearing in conductivity tensor (3.6). Therefore, using Eqs. (4.1) and (4.2) and applying Ψ_- from the right on both sides of Eq. (3.20) of Ref. 28, we obtain the CRLSF

$$\langle\lambda_1|\mathcal{G}_z^D\Psi_-(z)|\lambda_1+1\rangle$$

in the Landau representation as follows:

$$\begin{aligned}\langle\lambda_1|\mathcal{G}_z^D\Psi_-(z)|\lambda_1+1\rangle&\simeq\langle\lambda_1|\mathcal{L}_z\Psi_-(z)|\lambda_1+1\rangle\\ &=\eta^2\sum_{\mathbf{q}}\sum_{\lambda_3}\sum_{\lambda_4}(N_{\mathbf{q}}+1)\{[\langle\lambda_1|\gamma_{\mathbf{q}}|\lambda_3\rangle\langle\lambda_3|\gamma_{\mathbf{q}}^\dagger|\lambda_4\rangle\langle\lambda_4|\Psi_-(z)|\lambda_1+1\rangle\\ &\quad-\langle\lambda_1|\gamma_{\mathbf{q}}|\lambda_3\rangle\langle\lambda_3|\Psi_-(z)|\lambda_4\rangle\langle\lambda_4|\gamma_{\mathbf{q}}^\dagger|\lambda_1+1\rangle]\\ &\quad\times[E_{\lambda_3}-E_{\lambda_1}-\hbar\omega_c+\hbar\omega_{\mathbf{q}}-b_z(\lambda_3,\lambda_1+1)-z]^{-1}\\ &\quad+[\langle\lambda_1|\Psi_-(z)|\lambda_4\rangle\langle\lambda_4|\gamma_{\mathbf{q}}|\lambda_3+1\rangle\langle\lambda_3+1|\gamma_{\mathbf{q}}^\dagger|\lambda_1+1\rangle\\ &\quad-\langle\lambda_1|\gamma_{\mathbf{q}}|\lambda_4\rangle\langle\lambda_4|\Psi_-(z)|\lambda_3+1\rangle\langle\lambda_3+1|\gamma_{\mathbf{q}}^\dagger|\lambda_1+1\rangle]\\ &\quad\times[E_{\lambda_1}-E_{\lambda_3}-\hbar\omega_c-\hbar\omega_{\mathbf{q}}-b_z(\lambda_1,\lambda_3+1)-z]^{-1}\}\end{aligned}$$

$$
\begin{aligned}
&+\eta^2\sum_{\mathbf{q}}\sum_{\lambda_3}\sum_{\lambda_4}N_{\mathbf{q}}\{[\langle\lambda_1|\gamma_{\mathbf{q}}^{\dagger}|\lambda_3\rangle\langle\lambda_3|\gamma_{\mathbf{q}}|\lambda_4\rangle\langle\lambda_4|\Psi_-(z)|\lambda_1+1\rangle \\
&\qquad -\langle\lambda_1|\gamma_{\mathbf{q}}^{\dagger}|\lambda_3\rangle\langle\lambda_3|\Psi_-(z)|\lambda_4\rangle\langle\lambda_4|\gamma_{\mathbf{q}}|\lambda_1+1\rangle] \\
&\qquad \times[E_{\lambda_3}-E_{\lambda_1}-\hbar\omega_c-\hbar\omega_{\mathbf{q}}-b_z(\lambda_3,\lambda_1+1)-z]^{-1} \\
&\qquad +[\langle\lambda_1|\Psi_-(z)|\lambda_4\rangle\langle\lambda_4|\gamma_{\mathbf{q}}^{\dagger}|\lambda_3+1\rangle\langle\lambda_3+1|\gamma_{\mathbf{q}}|\lambda_1+1\rangle \\
&\qquad\quad -\langle\lambda_1|\gamma_{\mathbf{q}}^{\dagger}|\lambda_4\rangle\langle\lambda_4|\Psi_-(z)|\lambda_3+1\rangle\langle\lambda_3+1|\gamma_{\mathbf{q}}|\lambda_1+1\rangle] \\
&\qquad \times[E_{\lambda_1}-E_{\lambda_3}-\hbar\omega_c+\hbar\omega_{\mathbf{q}}-b_z(\lambda_1,\lambda_3+1)-z]^{-1}\}\ , \qquad (4.3)
\end{aligned}
$$

where

$$
N_{\mathbf{q}}=\mathrm{Tr}^{(\mathrm{ph})}(\rho_{\mathrm{ph}}b_{\mathbf{q}}^{\dagger}b_{\mathbf{q}})=(e^{\beta\hbar\omega_{\mathbf{q}}}-1)^{-1}\ . \qquad (4.4)
$$

It should be noted that the matrix elements of $\mathscr{L}_z$ in the energy denominators, which are responsible for the higher-order effects of the interactions, are approximated by taking the diagonal parts of $\mathscr{L}_z$. Let us carry out the λ_4 summation. Although there are many terms arising from the λ_4 summation, we keep only the dominant terms. Such dominant terms are determined by the following rough estimation: let us consider the first term of Eq. (4.3). From Eqs. (3.10), (3.11), and (3.14), we see

$$
\langle\lambda_4|\Psi_-(z)|\lambda_1+1\rangle\sim\langle\lambda_4|M_-^{(0)}|\lambda_1+1\rangle/(E_{\lambda_4}-E_{\lambda_1+1}-z)\propto\delta_{\lambda_4,\lambda_1}\ . \qquad (4.5)
$$

That is, $\langle\lambda_4|\Psi_-(z)|\lambda_1+1\rangle$ contributes resonantly only when $\lambda_4=\lambda_1$. By making a similar estimation for other terms, we can pick out the most dominant terms from the λ_4 summation and we obtain the CRLSF as

$$
\begin{aligned}
\langle\lambda_1|\mathscr{L}_z\Psi_-(z)|\lambda_1+1\rangle&=\eta^2\sum_{\mathbf{q}}\sum_{\lambda_3}(N_{\mathbf{q}}+1)\{(|\langle\lambda_1|\gamma_{\mathbf{q}}|\lambda_3\rangle|^2-\langle\lambda_1|\gamma_{\mathbf{q}}|\lambda_3\rangle\langle\lambda_3+1|\gamma_{\mathbf{q}}^{\dagger}|\lambda_1+1\rangle\Phi) \\
&\qquad\times[E_{\lambda_3}-E_{\lambda_1}-\hbar\omega_c+\hbar\omega_{\mathbf{q}}-b_z(\lambda_3,\lambda_1+1)-z]^{-1} \\
&\qquad +(|\langle\lambda_1+1|\gamma_{\mathbf{q}}|\lambda_3+1\rangle|^2-\langle\lambda_1|\gamma_{\mathbf{q}}|\lambda_3\rangle\langle\lambda_3+1|\gamma_{\mathbf{q}}^{\dagger}|\lambda_1+1\rangle\Phi) \\
&\qquad\times[E_{\lambda_1}-E_{\lambda_3}-\hbar\omega_c-\hbar\omega_{\mathbf{q}}-b_z(\lambda_1,\lambda_3+1)-z]^{-1}\}\langle\lambda_1|\Psi_-(z)|\lambda_1+1\rangle \\
&+\eta^2\sum_{\mathbf{q}}\sum_{\lambda_3}N_{\mathbf{q}}\{(|\langle\lambda_1|\gamma_{\mathbf{q}}^{\dagger}|\lambda_3\rangle|^2-\langle\lambda_1|\gamma_{\mathbf{q}}^{\dagger}|\lambda_3\rangle\langle\lambda_3+1|\gamma_{\mathbf{q}}|\lambda_1+1\rangle\Phi) \\
&\qquad\times[E_{\lambda_3}-E_{\lambda_1}-\hbar\omega_c-\hbar\omega_{\mathbf{q}}-b_z(\lambda_3,\lambda_1+1)-z]^{-1} \\
&\qquad +(|\langle\lambda_1+1|\gamma_{\mathbf{q}}^{\dagger}|\lambda_3+1\rangle|^2-\langle\lambda_1|\gamma_{\mathbf{q}}^{\dagger}|\lambda_3\rangle\langle\lambda_3+1|\gamma_{\mathbf{q}}|\lambda_1+1\rangle\Phi) \\
&\qquad\times[E_{\lambda_1}-E_{\lambda_3}-\hbar\omega_c+\hbar\omega_{\mathbf{q}}-b_z(\lambda_1,\lambda_3+1)-z]^{-1}\}\langle\lambda_1|\Psi_-(z)|\lambda_1+1\rangle \\
&\equiv\hbar\Xi^{(2)}(\omega,\omega_c;\lambda_1:\lambda_1+1)\langle\lambda_1|\Psi_-(z)|\lambda_1+1\rangle\ , \qquad (4.6)
\end{aligned}
$$

where

$$
\Phi\equiv\frac{\langle\lambda_3|\Psi_-(z)|\lambda_3+1\rangle}{\langle\lambda_1|\Psi_-(z)|\lambda_1+1\rangle}=\frac{E_{\lambda_1}-E_{\lambda_1+1}-\hbar\Xi^{(2)}(\omega,\omega_c;\lambda_1:\lambda_1+1)-z}{E_{\lambda_3}-E_{\lambda_3+1}-\hbar\Xi^{(2)}(\omega,\omega_c;\lambda_3:\lambda_3+1)-z}\,\frac{\langle\lambda_3|M_-^{(0)}|\lambda_3+1\rangle}{\langle\lambda_1|M_-^{(0)}|\lambda_1+1\rangle}\ . \qquad (4.7)
$$

We have used Eqs. (3.10) and (3.12) to obtain Eq. (4.7). Here the second-order renormalized self-energy $\hbar\Xi^{(2)}$ is defined as in Eq. (3.12). It is interesting to note that if we neglect the term with cross-matrix elements and b's in the energy denominators, Eq. (4.6) tends to the usual expression which arises in the transition rate calculation within the Born approximation. However, this approximation (only the diagonal part of b is taken) cannot be justified since the contribution from the cross-matrix elements, which arise from the nondiagonal part of b, is neglected. Hence, even for a weakly

interacting system, this approximation is not valid. To obtain reliable information it is necessary to take into account the effects arising from the terms with the cross-matrix elements. These cross-matrix elements are the so-called "vertex correction" terms[32] which have either been missed or neglected by earlier workers.[18] In the following evaluations we keep these terms, together with b's in the energy denominators, to take into account the higher effects of the interaction. Therefore, the result obtained can be applicable to the strongly interacting system. Noting that

$$b_z(\lambda_3,\lambda_1+1)\equiv\hbar\Xi^d(\omega,\omega_c;\lambda_3:\lambda_1+1)\ ,$$

etc., we obtain the exact self-consistent equation for complex self-energy associated with cyclotron-resonance transition:

$$\begin{aligned}\hbar\Xi^{(2)}(\omega,\omega_c;\lambda_1:\lambda_1+1)=\eta^2\sum_{\mathbf{q}}\sum_{\lambda_3}(N_{\mathbf{q}}+1)\{\,|\langle\lambda_1|\gamma_{\mathbf{q}}|\lambda_3\rangle|^2-\langle\lambda_1|\gamma_{\mathbf{q}}|\lambda_3\rangle\langle\lambda_3+1|\gamma_{\mathbf{q}}^\dagger|\lambda_1+1\rangle\\ \times[(n_3+1)/(n_1+1)]^{1/2}\Theta\}/[E_{\lambda_3}-E_{\lambda_1}-\hbar\omega_c+\hbar\omega_{\mathbf{q}}-\hbar\Xi^d(\omega,\omega_c;\lambda_3:\lambda_1+1)-z]\\ +\{\,|\langle\lambda_1+1|\gamma_{\mathbf{q}}|\lambda_3+1\rangle|^2-\langle\lambda_1|\gamma_{\mathbf{q}}|\lambda_3\rangle\langle\lambda_3+1|\gamma_{\mathbf{q}}^\dagger|\lambda_1+1\rangle\\ \times[(n_3+1)/(n_1+1)]^{1/2}\Theta\}/[E_{\lambda_1}-E_{\lambda_3}-\hbar\omega_c-\hbar\omega_{\mathbf{q}}\\ -\hbar\Xi^d(\omega,\omega_c;\lambda_1:\lambda_3+1)-z])\\ +\eta^2\sum_{\mathbf{q}}\sum_{\lambda_3}N_{\mathbf{q}}(\{\,|\langle\lambda_1|\gamma_{\mathbf{q}}^\dagger|\lambda_3\rangle|^2-\langle\lambda_1|\gamma_{\mathbf{q}}^\dagger|\lambda_3\rangle\langle\lambda_3+1|\gamma_{\mathbf{q}}|\lambda_1+1\rangle\\ \times[(n_3+1)/(n_1+1)]^{1/2}\Theta\}/[E_{\lambda_3}-E_{\lambda_1}-\hbar\omega_c-\hbar\omega_{\mathbf{q}}\\ -\hbar\Xi^d(\omega,\omega_c;\lambda_3;\lambda_1+1)-z]\\ +\{\,|\langle\lambda_1+1|\gamma_{\mathbf{q}}^\dagger|\lambda_3+1\rangle|^2-\langle\lambda_1|\gamma_{\mathbf{q}}^\dagger|\lambda_3\rangle\langle\lambda_3+1|\gamma_{\mathbf{q}}|\lambda_1+1\rangle\\ \times[(n_3+1)/(n_1+1)]^{1/2}\Theta\}/[E_{\lambda_1}-E_{\lambda_3}-\hbar\omega_c+\hbar\omega_{\mathbf{q}}\\ -\hbar\Xi^d(\omega,\omega_c;\lambda_1:\lambda_3+1)-z])\ ,\end{aligned}\tag{4.8}$$

where Θ is given by

$$\Theta\equiv\frac{f(E_{\lambda_3})-f(E_{\lambda_3}+\hbar\omega_c)}{f(E_{\lambda_1})-f(E_{\lambda_1}+\hbar\omega_c)}\,\frac{\hbar\omega_c+\hbar\Xi^{(2)}(\omega,\omega_c;\lambda_1:\lambda_1+1)+z}{\hbar\omega_c+\hbar\Xi^{(2)}(\omega,\omega_c;\lambda_3:\lambda_3+1)+z}\tag{4.9}$$

and use has been made of Eqs. (3.11), (3.14), and (4.7).

Lodder and Fujita[15] obtained a similar expression of self-energy associated with cyclotron-resonance transition by using a diagram approach. More recently, Ryu and Choi[22] obtained the expression by applying Kawabata's approach[14] which is based on the Kubo formalism[29] and the Mori method[30] of calculation. In fact, if we set $\Theta=1$ and neglect the higher-order effect of the interaction, i.e., Ξ^d, in the energy denominators, Eq. (4.8) reduces to that of Lodder and Fujita [Eq. (6.11)] (Ref. 15) and Ryu and Choi [Eq. (4.6)] (Ref. 22); if we further neglect the phonon energy $\hbar\omega_{\mathbf{q}}$ and set $\omega=\omega_c$, our result leads to Eqs. (3.13) and (3.14) of Kawabata.[14] Neglecting the terms with Θ corresponds to the approximation made by Shin *et al.*,[18] although their theory is made for an electron-impurity system. It should be noted that Eq. (4.8) is not limited to a weakly interacting system, unlike other theories.

Upon substituting the interaction operator and making use of the Landau state given by Eq. (2.13), Eq. (4.8) can be expressed in a more convenient form:

$$\begin{aligned}\hbar\Xi^{(2)}(\omega,\omega_c;n_1,k_1:n_1+1,k_1)=\eta^2\sum_{\mathbf{q}}(N_{\mathbf{q}}+1)\,|\,C_{\mathbf{q}}\,|^2\sum_{n_3=0}^{\infty}(\{K_1(n_1,n_3,t)-[(n_3+1)/(n_1+1)]^{1/2}K_2(n_1,n_3,t)\Theta\}\\ \times[E_{n_3}(k_{1z}-q_z)-E_{n_1}(k_{1z})+\hbar\omega_q-\hbar\omega_c\\ -\hbar\Xi^d(\omega,\omega_c;n_3,k_1-q:n_1+1,k_1)-z]^{-1}\end{aligned}$$

$$+\{K_1(n_1+1,n_3+1,t)-[(n_3+1)/(n_1+1)]^{1/2}K_2(n_1,n_3,t)\Theta\}$$

$$\times[E_{n_1}(k_{1z})-E_{n_3}(k_{1z}-q_z)-\hbar\omega_q-\hbar\omega_c$$

$$-\hbar\Xi^d(\omega,\omega_c;n_1,k_1:n_3+1,k_1-q)-z]^{-1})$$

$$+\eta^2\sum_q N_q\,|\,C_q\,|^2\sum_{n_3=0}^{\infty}(\{K_1(n_1,n_3,t)-[(n_3+1)/(n_1+1)]^{1/2}K_2(n_1,n_3,t)\Theta\}$$

$$\times[E_{n_3}(k_{1z}-q_z)-E_{n_1}(k_{1z})-\hbar\omega_q-\hbar\omega_c$$

$$-\hbar\Xi^d(\omega,\omega_c;n_3,k_1-q:n_1+1,k_1)-z]^{-1}$$

$$+\{K_1(n_1+1,n_3+1,t)\ -[(n_3+1)/(n_1+1)]^{1/2}K_2(n_1,n_3,t)\Theta\}$$

$$\times[E_{n_1}(k_{1z})-E_{n_3}(k_{1z}-q_z)+\hbar\omega_q-\hbar\omega_c$$

$$-\hbar\Xi^d(\omega,\omega_c;n_1,k_1:n_3+1,k_1-q)-z]^{-1})\,, \tag{4.10}$$

where we have carried the summation over the $\mathbf{k}_3$ state, and K_1 and K_2 are, respectively, given by

$$K_1(n,n',t)\equiv(n!/n'!)t^{n'-n}e^{-t}[L_n^{n'-n}(t)]^2\,, \tag{4.11}$$

$$K_2(n,n',t)\equiv\sqrt{n!/n'!}\sqrt{(n+1)!/(n'+1)!}\,t^{n'-n}\times e^{-t}L_n^{n'-n}(t)L_{n+1}^{n'-n}(t)\,. \tag{4.12}$$

Here, $L_n^m(t)$ is the associated Laguerre polynomial[33]

$$L_n^m(t)=\frac{\exp(t)}{n!}t^{-m}\frac{d^n}{dt^n}[t^{n+m}\exp(-t)] \tag{4.13}$$

and t is defined by

$$t\equiv(q_x^2+q_y^2)\hbar/2m\omega_c\,. \tag{4.14}$$

Θ is given by

$$\Theta=\frac{f(E_{n_3}(k_{1z}-q_z))-f(E_{n_3}(k_{1z}-q_z)+\hbar\omega_c)}{f(E_{n_1}(k_{1z}))-f(E_{n_1}(k_{1z})+\hbar\omega_c)}\times\frac{\hbar\omega_c+\hbar\Xi^{(2)}(\omega,\omega_c;n_1,k_1:n_1+1,k_1)-z}{\hbar\omega_c+\hbar\Xi^{(2)}(\omega,\omega_c;n_3,k_1-q:n_3+1,k_1-q)-z}\,. \tag{4.15}$$

Disregarding Ξ^d and $\hbar\omega_q$ in the energy denominators of Eq. (4.10), we can see that those terms with $n_3=n_1$ in the n_3 summation contribute resonantly for small q_z. This approximate diagonality with respect to the oscillator quantum number can be considered as good at high-resonance frequency.[15] Therefore, we may write Eq. (4.10) as

$$\hbar\Xi^{(2)}(\omega,\omega_c;n,k:n+1,k)=\eta^2\sum_q(N_q+1)\,|\,C_q\,|^2\{[K_1(n,n,t)-K_2(n,n,t)\Theta][\epsilon(k_z-q_z)-\epsilon(k_z)+\hbar\omega_q-\hbar\omega_c$$

$$-\hbar\Xi^d(\omega,\omega_c;n,k-q:n+1,k)-z]^{-1}$$

$$+[K_1(n+1,n+1,t)-K_2(n,n,t)\Theta][\epsilon(k_z)-\epsilon(k_z-q_z)-\hbar\omega_q-\hbar\omega_c$$

$$-\hbar\Xi^d(\omega,\omega_c;n,k:n+1,k-q)-z]^{-1}\}$$

$$+\eta^2\sum_q N_q\,|\,C_q\,|^2\{[K_1(n,n,t)-K_2(n,n,t)\Theta][\epsilon(k_z-q_z)-\epsilon(k_z)-\hbar\omega_q-\hbar\omega_c$$

$$-\hbar\Xi^d(\omega,\omega_c;n,k-q:n+1,k)-z]^{-1}$$

$$+[K_1(n+1,n+1,t)-K_2(n,n,t)\Theta]$$

$$\times[\epsilon(k_z)-\epsilon(k_z-q_z)+\hbar\omega_q-\hbar\omega_c-\hbar\Xi^d(\omega,\omega_c;n,k:n+1,k-q)-z]^{-1}\}\,, \tag{4.16}$$

where

$$\Theta \simeq \frac{\hbar\omega_c + \hbar\Xi^{(2)}(\omega,\omega_c;n,k:n+1,k) - z}{\hbar\omega_c + \hbar\Xi^{(2)}(\omega,\omega_c;n,k-q:n+1,k-q) - z} . \tag{4.17}$$

Equation (4.16) with Eq. (4.17) forms the basis for further evaluation of self-energy associated with the cyclotron-resonance transition $\lambda \rightarrow \lambda+1$. In a cruder approximation both $\Xi^d(\omega,\omega_c;n,k-q:n+1,k)$ and $\Xi^d(\omega,\omega_c;n,k:n+1,k-q)$ may be replaced by $\Xi^{(2)}(\omega,\omega_c;n,k:n+1,k)$, noting that the most dominant terms, which contribute resonantly, are characterized by small q. Then we can obtain the approximated self-consistent equation for $\Xi^{(2)}(\omega,\omega_c;n,k:n+1,k)$:

$$\begin{aligned}\hbar\Xi^{(2)} = \sum_{\mathbf{q}} (N_{\mathbf{q}}+1)\,|\,C_{\mathbf{q}}\,|^2 \{ &[K_1(n,n,t)-K_2(n,n,t)][\epsilon(k_z-q_z)-\epsilon(k_z)+\hbar\omega_{\mathbf{q}}-\hbar\omega_c-\hbar\Xi^{(2)}-z]^{-1} \\ &+[K_1(n+1,n+1,t)-K_2(n,n,t)]\,[\epsilon(k_z)-\epsilon(k_z-q_z)-\hbar\omega_{\mathbf{q}}-\hbar\omega_c-\hbar\Xi^{(2)}-z]^{-1}\} \\ + \sum_{\mathbf{q}} N_{\mathbf{q}}\,|\,C_{\mathbf{q}}\,|^2 \{ &[K_1(n,n,t)-K_2(n,n,t)][\epsilon(k_z-q_z)-\epsilon(k_z)-\hbar\omega_{\mathbf{q}}-\hbar\omega_c-\hbar\Xi^{(2)}-z]^{-1} \\ &+[K_1(n+1,n+1,t)-K_2(n,n,t)][\epsilon(k_z)-\epsilon(k_z-q_z)+\hbar\omega_{\mathbf{q}}-\hbar\omega_c-\hbar\Xi^{(2)}-z]^{-1}\} ,\end{aligned} \tag{4.18}$$

where the K_1 and K_2 functions are, respectively, given by Eqs. (4.11) and (4.12). Solving Eq. (4.18) for Ξ self-consistently, we can determine the linewidth and lineshift associated with resonance transition $\lambda_1 \rightarrow \lambda_2 = \lambda_1 + 1$ by making use of Eq. (3.15). In practice, since for weak coupling we are only interested in frequencies which satisfy $\omega - \omega_c \ll \omega_c$, the higher-order dressed expressions for the self-energy (4.18) will be an order of magnitude smaller than the undressed self-energy. Therefore, we may drop Ξ in the energy denominators. In this case we obtain the equation for the linewidth $\Gamma(\omega,\omega_c;n,k_z:n+1,k_z)$ as

$$\begin{aligned}\mathrm{Im}\Xi^{(2)}(\omega,\omega_c;n,k_z:n+1,k_z) &\equiv \Gamma(\omega,\omega_c;n,k_z:n+1,k_z) \\ &= \frac{\pi}{\hbar}\sum_{\mathbf{q}} (N_{\mathbf{q}}+1)\,|\,C_{\mathbf{q}}\,|^2 \{[K_1(n,n,t)-K_2(n,n,t)]\,\delta(\epsilon(k_z-q_z)-\epsilon(k_z)+\hbar\omega_{\mathbf{q}}-\hbar\omega_c+\hbar\omega) \\ &\quad +[K_1(n+1,n+1,t)-K_2(n,n,t)] \\ &\quad \times\delta(\epsilon(k_z-q_z)-\epsilon(k_z)+\hbar\omega_{\mathbf{q}}+\hbar\omega_c-\hbar\omega)\} \\ &\quad + \frac{\pi}{\hbar}\sum_{\mathbf{q}} N_{\mathbf{q}}\,|\,C_{\mathbf{q}}\,|^2 \{[K_1(n,n,t)-K_2(n,n,t)]\,\delta(\epsilon(k_z-q_z)-\epsilon(k_z)-\hbar\omega_{\mathbf{q}}-\hbar\omega_c+\hbar\omega) \\ &\quad +[K_1(n+1,n+1,t)-K_2(n,n,t)] \\ &\quad \times\delta(\epsilon(k_z-q_z)-\epsilon(k_z)-\hbar\omega_q+\hbar\omega_c-\hbar\omega)\} ,\end{aligned} \tag{4.19}$$

from which we can calculate the shift $\Delta(\omega,\omega_c;n,k_z:n+1,k_z)$ as

$$\mathrm{Re}\Xi^{(2)}(\omega,\omega_c;n,k_z:n+1,k_z) \equiv \Delta(\omega,\omega_c;n,k_z:n+1,k_z) = \frac{1}{\pi}\mathrm{P}\int \frac{\Gamma(\omega',\omega_c;n,k_z:n+1,k_z)}{\omega-\omega'}d\omega' , \tag{4.20}$$

where Γ is given by Eq. (4.19). To obtain Eqs. (4.19) and (4.20), we have used the Dirac identity

$$\lim_{\delta\rightarrow 0^+}\frac{1}{x\pm i\delta} = \mathrm{P}\left[\frac{1}{x}\right] \mp i\pi\delta(x) , \tag{4.21}$$

where P denotes Cauchy's principle-value integral.

For strong coupling, i.e., $\omega - \omega_c < \omega_c$, the higher-order dressed expressions for the self-energy may become the same order of magnitude as the undressed self-energy. Therefore, we cannot neglect Ξ in the energy denominators. We have to solve the following coupled equations for $\Gamma(\omega,\omega_c;n,k_z:n+1,k_z)$ and $\Delta(\omega,\omega_c;n,k_z:n+1,k_z)$:

$$\Gamma=\sum_{\mathbf{q}}(N_{\mathbf{q}}+1)\,|\,C_{\mathbf{q}}\,|^2\left[\frac{[K_1(n,n,t)-K_2(n,n,t)]\Gamma}{[\epsilon(k_z-q_z)-\epsilon(k_z)+\hbar(\omega_{\mathbf{q}}-\omega_c+\omega-\Delta)]^2+\hbar^2\Gamma^2}+\frac{[K_1(n+1,n+1,t)-K_2(n,n,t)]\Gamma}{[\epsilon(k_z-q_z)-\epsilon(k_z)+\hbar(\omega_{\mathbf{q}}+\omega_c-\omega+\Delta)]^2+\hbar^2\Gamma^2}\right]$$
$$+\sum_{\mathbf{q}}N_{\mathbf{q}}\,|\,C_{\mathbf{q}}\,|^2\left[\frac{[K_1(n,n,t)-K_2(n,n,t)]\Gamma}{[\epsilon(k_z-q_z)-\epsilon(k_z)-\hbar(\omega_{\mathbf{q}}+\omega_c-\omega+\Delta)]^2+\hbar^2\Gamma^2}+\frac{[K_1(n+1,n+1,t)-K_2(n,n,t)]\Gamma}{[\epsilon(k_z-q_z)-\epsilon(k_z)-\hbar(\omega_{\mathbf{q}}-\omega_c+\omega-\Delta)]^2+\hbar^2\Gamma^2}\right], \tag{4.22}$$

$$\hbar\Delta=\sum_{\mathbf{q}}(N_{\mathbf{q}}+1)\,|\,C_{\mathbf{q}}\,|^2\left[\frac{[K_1(n,n,t)-K_2(n,n,t)][\epsilon(k_z-q_z)-\epsilon(k_z)+\hbar(\omega_q+\omega_c-\omega+\Delta)]}{[\epsilon(k_z-q_z)-\epsilon(k_z)+\hbar(\omega_{\mathbf{q}}+\omega_c-\omega+\Delta)]^2+\hbar^2\Gamma^2}-\frac{[K_1(n+1,n+1,t)-K_2(n,n,t)][\epsilon(k_z-q_z)-\epsilon(k_z)+\hbar(\omega_{\mathbf{q}}+\omega_c-\omega+\Delta)]}{[\epsilon(k_z-q_z)-\epsilon(k_z)+\hbar(\omega_{\mathbf{q}}+\omega_c-\omega+\Delta)]^2+\hbar^2\Gamma^2}\right]$$
$$+\sum_{\mathbf{q}}N_{\mathbf{q}}\,|\,C_{\mathbf{q}}\,|^2\left[\frac{[K_1(n,n,t)-K_2(n,n,t)][\epsilon(k_z-q_z)-\epsilon(k_z)-\hbar(\omega_{\mathbf{q}}+\omega_c-\omega+\Delta)]}{[\epsilon(k_z-q_z)-\epsilon(k_z)-\hbar(\omega_{\mathbf{q}}+\omega_c-\omega+\Delta)]^2+\hbar^2\Gamma^2}-\frac{[K_1(n+1,n+1,t)-K_2(n,n,t)][\epsilon(k_z-q_z)-\epsilon(k_z)+\hbar(\omega_{\mathbf{q}}+\omega_c-\omega+\Delta)]}{[\epsilon(k_z-q_z)-\epsilon(k_z)-\hbar(\omega_{\mathbf{q}}-\omega_c+\omega-\Delta)]^2+\hbar^2\Gamma^2}\right]. \tag{4.23}$$

In terms of Γ and Δ calculated from Eqs. (4.19) and (4.20) for a weak coupling or Eqs. (4.22) and (4.23) for a strong coupling, we can express $\mathrm{Re}[\sigma_{+-}(\omega)]$ as

$$\mathrm{Re}[\sigma_{+-}(\omega)]=\frac{2e^2}{m\Omega}\sum_{\mathbf{k}}\sum_{n=0}^{\infty}(n+1)[f(E_n)-f(E_{n+1})]\times\frac{\Gamma(\omega,\omega_c;n,k_z:n+1,k_z)}{[\omega-\omega_c-\Delta(\omega,\omega_c;n,k_z:n+1,k_z)]^2+[\Gamma(\omega,\omega_c;n,k_z:n+1,k_z)]^2}, \tag{4.24}$$

where we have used Eqs. (3.13)–(3.15). Equation (4.24) is the formula for the power-absorption spectral line shape due to the cyclotron-resonance transition $n\rightarrow n+1$.

Utilizing the formulas obtained above along with the specific interaction potential form of $C_{\mathbf{q}}$, we can evaluate experimentally measurable quantities, the linewidth and line shift, by calculating Γ and Δ, respectively, and hence the spectral line shape associated with cyclotron-resonance transition. Application of the present theory to specific problems is planned to be reported in future publications.

ACKNOWLEDGMENTS

Part of the present investigation was made while I was at the J. J. Thomson Physical Laboratory, University of Reading. I wish to thank Dr. Dennis Dunn for useful conversations during my stay at Reading.

APPENDIX: DERIVATION OF A SELF-CONSISTENT EQUATION FOR SELF-ENERGY

To obtain the complex conductivity $\sigma_{rs}(z)$, we have to solve the quantum Boltzmann-like equation of the form [cf. Eq. (3.10)]

$$(E_{\lambda_1}-E_{\lambda_2}-z)\langle\lambda_1\,|\,\Psi_s(z)\,|\,\lambda_2\rangle+\langle\lambda_1\,|\,\mathscr{D}_2^D\Psi_s(z)\,|\,\lambda_2\rangle=\langle\lambda_1\,|\,M_s\,|\,\lambda_2\rangle\,. \tag{A1}$$

In the above expression, the collision (self-energy) operator, which is a function of superoperators, is not in general diagonal. Therefore, an exact solution of Eq. (A1) for $\langle\lambda_1\,|\,\Psi_s(z)\,|\,\lambda_2\rangle$ is not feasible, but we would like to have its solution of the form

$$\langle\lambda_1\,|\,\Psi_s(z)\,|\,\lambda_2\rangle=[E_{\lambda_1}-E_{\lambda_2}-\hbar\Xi(\omega,\omega_c;\lambda_1:\lambda_2)-z]^{-1}\langle\lambda_1\,|\,M_s\,|\,\lambda_2\rangle \tag{A2}$$

by postulating the following form [cf. Eq. (3.12)]:

$$\langle\lambda_1\,|\,\mathscr{D}_z^D\Psi_s(z)\,|\,\lambda_2\rangle\equiv-\hbar\Xi(\omega,\omega_c;\lambda_1:\lambda_2)\langle\lambda_1\,|\,\Psi_s(z)\,|\,\lambda_2\rangle\,. \tag{A3}$$

Equation (A3) defines the self-energy $\hbar\Xi(\omega,\omega_c;\lambda_1{:}\lambda_2)$ associated with the transition between the electron states $|\lambda_1\rangle$ and $|\lambda_2\rangle$ due to collisions. Our problem is now to find the algebraic equation for Ξ, assuming that the solution of Eq. (A1) is given by the form (A2); that is, instead of solving Eq. (A1) for $\langle\lambda_1|\Psi_s(z)|\lambda_2\rangle$, we would like to obtain the equation for Ξ and then solve the resultant equation for Ξ. We introduce a tetradic notation for expressing the matrix elements of a superoperator since $\mathcal{G}_z^D$ is a function of superoperators.

Let us write the collision term of Eq. (A1) as a sum of its diagonal and nondiagonal parts:

$$\begin{aligned}\langle\lambda_1|\mathcal{G}_z^D\Psi_s(z)|\lambda_2\rangle &= \sum_{\kappa_1}\sum_{\kappa_2}\mathcal{G}^D_{z\lambda_1\lambda_2:\kappa_1\kappa_2}\langle\kappa_1|\Psi_s(z)|\kappa_2\rangle \\ &= \mathcal{G}^D_{z\lambda_1\lambda_2:\lambda_1\lambda_2}\langle\lambda_1|\Psi_s(z)|\lambda_2\rangle \\ &\quad + \sum_{\substack{\kappa_1\\(\neq\lambda_1,\lambda_2)}}\sum_{\kappa_2}\mathcal{G}^D_{z\lambda_1\lambda_2:\kappa_1\kappa_2}\langle\kappa_1|\Psi_s(z)|\kappa_2\rangle ,\end{aligned} \tag{A4}$$

where $|\lambda\rangle$ and $|\kappa\rangle$ are the eigenstates of h_0. By making use of Eq. (A2), we can write the nondiagonal part of Eq. (A4) as

$$\begin{aligned}\sum_{\substack{\kappa_1\\(\neq\lambda_1,\lambda_2)}}\sum_{\kappa_2}\mathcal{G}^D_{z\lambda_1\lambda_2:\kappa_1\kappa_2}\langle\kappa_1|\Psi_s(z)|\kappa_2\rangle &= \sum_{\substack{\kappa_1\\(\neq\lambda_1,\lambda_2)}}\sum_{\kappa_2}\mathcal{G}^D_{z\lambda_1\lambda_2:\kappa_1\kappa_2}(E_{\kappa_1}-E_{\kappa_2}-\hbar\Xi(\omega,\omega_c;\kappa_1{:}\kappa_2)-z)^{-1}\langle\kappa_1|M_s|\kappa_2\rangle \\ &= \sum_{\substack{\kappa_1\\(\neq\lambda_1,\lambda_2)}}\sum_{\kappa_2}\mathcal{G}^D_{z\lambda_1\lambda_2:\kappa_1\kappa_2}\frac{(E_{\lambda_1}-E_{\lambda_2}-\hbar\Xi(\omega,\omega_c;\lambda_1{:}\lambda_2)-z)}{(E_{\kappa_1}-E_{\kappa_2}-\hbar\Xi(\omega,\omega_c;\kappa_1{:}\kappa_2)-z)}\frac{\langle\kappa_1|M_s|\kappa_2\rangle}{\langle\lambda_1|M_s|\lambda_2\rangle}\langle\lambda_1|\Psi_s(z)|\lambda_2\rangle .\end{aligned} \tag{A5}$$

From Eqs. (A1), (A4), and (A5), we can write the solution of Eq. (A1) in terms of Ξ;

$$\langle\lambda_1|\Psi_s(z)|\lambda_2\rangle = \left[E_{\lambda_1}-E_{\lambda_2}+\mathcal{G}^D_{z\lambda_1\lambda_2:\lambda_1\lambda_2}+\sum_{\substack{\kappa_1\\(\neq\lambda_1,\lambda_2)}}\sum_{\kappa_2}\mathcal{G}^D_{z\lambda_1\lambda_2:\kappa_1\kappa_2}\frac{(E_{\lambda_1}-E_{\lambda_2}-\hbar\Xi(\omega,\omega_c;\lambda_1{:}\lambda_2)-z)}{(E_{\kappa_1}-E_{\kappa_2}-\hbar\Xi(\omega,\omega_c;\kappa_1{:}\kappa_2)-z)}\times\frac{\langle\kappa_1|M_s|\kappa_2\rangle}{\langle\lambda_1|M_s|\lambda_2\rangle}-z\right]^{-1}\langle\lambda_1|M_s|\lambda_2\rangle . \tag{A6}$$

By comparing Eq. (A2) with Eq. (A6), we find the self-consistent equation for Ξ:

$$\hbar\Xi(\omega,\omega_c;\lambda_1{:}\lambda_2)=\mathcal{G}^D_{z\lambda_1\lambda_2:\lambda_1\lambda_2}+\sum_{\substack{\kappa_1\\(\neq\lambda_1,\lambda_2)}}\sum_{\kappa_2}\mathcal{G}^D_{z\lambda_1\lambda_2:\kappa_1\kappa_2}\frac{(E_{\lambda_1}-E_{\lambda_2}-\hbar\Xi(\omega,\omega_c;\lambda_1{:}\lambda_2)-z)}{(E_{\kappa_1}-E_{\kappa_2}-\hbar\Xi(\omega,\omega_c;\kappa_1{:}\kappa_2)-z)}\frac{\langle\kappa_1|M_s|\kappa_2\rangle}{\langle\lambda_1|M_s|\lambda_2\rangle} . \tag{A7}$$

Instead of solving Eq. (A1) for $\langle\lambda_1|\Psi_s(z)|\lambda_2\rangle$, our problem is reduced to solving Eq. (A7) for Ξ self-consistently. Equation (A7) is the desired self-consistent equation for Ξ in the general case.

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