Electron-phonon scattering in superlattices

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The total scattering rate and momentum relaxation rate are evaluated for an ideal, periodic superlattice by taking matrix elements of the electron-phonon interaction between basis states which are plane-wave combinations of finite quantum-well wave functions. This is done for quasielastic deformation-potential scattering by acoustic and nonpolar optical phonons. The intrawell scattering rate is of the form found by other workers for a quasi-two-dimensional electron gas and superlattice. In addition, contributions due to interwell transfer and due to the interference of intrawell and interwell processes are obtained. The respective form factors for these processes are identified and evaluated, and the dependence on density of final states is found.

I. INTRODUCTION

Man-made heterostructures with precise interfacial and dimensional control made possible by molecular-beam-epitaxy (MBE) have been the subject of much current interest.[1] Considerable work has been done on the electrical and optical properties of single quantum wells and multiple quantum-well structures (MQW), in which the individual wells do not interact with one another and all physical phenomena are simply additive. However, the physically interesting case is that of an ideal periodic array of wells separated by barriers sufficiently thin that the overlap of well wave function is appreciable—a superlattice. As is well known, the additional long-range order of the superlattice results in a reduced Brillouin-zone scheme of minibands separated in energy by miniband gaps. Indeed, the original predictions of negative differential resistance (NDR) and Bloch oscillations by Esaki and Tsu[2] were based on the acceleration of Bloch electrons to the negative-mass regions and Brillouin-zone boundaries of the minizone structures, respectively. Predictions have also been made of large nonlinear optical susceptibilities[3] due to mobile carriers in the highly nonparabolic minibands, and the use of low-temperature thermoelectric power[4] measurements to map out the superlattice band structure. However, while the long-range order of the superlattice has been observed experimentally in zone folding of LA phonon spectra in GaAs as seen in Raman scattering,[5] the same has not been unequivocally observed in the electronic spectra, in spite of some early claims to the contrary.[6] It would appear that the electrons are much more sensitive than are phonons to structural variations from well to well, compositional fluctuations (e.g., Al content in the $Al_xGa_{1-x}As$ barriers), and interfacial disorder. These random fluctuations would be expected to smear out the superlattice band structure at the very least, or possibly lead to localization and hopping transport between individual wells.[7] (The question of disorder in superlattices is a separate problem in its own right which will not be addressed here.) However, with the continual progress in growth and interfacial quality made possible by MBE, the superlattice behavior cited above is likely to be realized. The present paper is a study of electron-phonon scattering in such ideal systems.

Electron-phonon scattering in quasi-two-dimensional semiconductor quantum-well structures has been studied for a variety of scattering mechanisms.[8–10] In these studies, the electronic wave function is taken to be confined in the direction perpendicular to the layers. The dependence of the total scattering rate and momentum relaxation time on energy, well size, and density of states (constant for this case) has been obtained.[8] In the present work, these results are extended to a superlattice, as previously defined. Here, the discrete levels of the quantum well are broadened into a series of minibands of narrow width into which the electron can scatter with momenta perpendicular to the layers. Our approach is to take matrix elements of the electron-phonon Hamiltonian using basis states which are plane-wave combinations of individual, quantum-well eigenfunctions. In analogy with the tight-binding approximation of energy-band theory, such a state strictly applies in the narrow-band limit appropriate to weakly coupled wells in which only nearest neighbor overlap of well wave functions is appreciable. In addition to the intrawell scattering already obtained for the isolated quantum well, interwell electron-phonon transitions are identified in which absorption or emission of phonons with momenta perpendicular to the layers is accompanied by transfer of the electron from one well to an adjacent well. As might be expected, the amplitude for this process is proportional to the overlap factor $\exp(-2aK)$, with $2a$ the barrier thickness and K the inverse decay length. The interwell transfer is evaluated for the case where transfer occurs within the same subband and between adjacent subbands. The total scattering rate and momentum relaxation times are evaluated for both the intrawell and interwell processes for the case of deformation potential scattering by acoustic and optical phonons, which, in the usual approximation, is independent of phonon wave vector Q. In addition, there is an interference between the intrawell and interwell amplitude which is larger by a factor of $\exp(aK)$ than the interwell process alone.

An earlier study[11] of electron-phonon scattering in superlattices for elastic, deformation-potential scattering was based on a tight-binding approximation of the electronic dispersion perpendicular to the layers. Only the intrawell contribution, proportional to the superlattice density of states, was obtained. This work was subsequently generalized to the Kronig-Penney model and also to the case of polar-optic scattering.[12] In the present study, not only is the form of the dispersion left arbitrary, but, more important, the explicit form of the Bloch state of the superlattice [cf. Eq. (5a)] gives rise to the additional interwell and interference terms referred to previously.

II. FORMULATION

The normalized wave function for a quasi-two-dimensional quantum well with infinite potential barriers is[8]

$$\psi(r,z)=e^{i\mathbf{k}\cdot\mathbf{r}}\left[\frac{2}{V}\right]^{1/2}\sin(k_n^{(0)}z)\ , \tag{1}$$

where

$$k_n^{(0)}=n\pi/L,\ \ n=1,2,\ldots \tag{2}$$

$L=2b$ is the well width in the z direction [see Fig. 1(a)], V is the well volume, and $\mathbf{K}=(\mathbf{k},k_n^{(0)})$ is the total wave vector, with $\mathbf{k}$ and $k_n^{(0)}$ the components in the (x,y) plane and z direction, respectively. A similar notation applies to the position vector of the electron $(\mathbf{r},z)$ and phonon wave vector $\mathbf{Q}=(\mathbf{q},q_z)$. The energy eigenvalues corresponding to $\psi(\mathbf{r},z)$ in the effective-mass approximation are

$$\begin{aligned}E(\mathbf{K})&=E(\mathbf{K})+E_n^{(0)}\\&=\hbar^2k^2/2m^*+n^2(\hbar^2\pi^2/2m^*L^2)\ .\end{aligned} \tag{3}$$

There are two basic modifications of Eq. (1). First, since the barriers of the superlattice are necessarily finite, we replace

$$\left[\frac{2}{L}\right]^{1/2}\sin(k_n^{(0)}z)\rightarrow\phi_n(z)\ , \tag{4}$$

where $\phi_n(z)$ is a normalized solution of the isolated finite well referring to the nth band, and includes a portion which decays exponentially into the barrier region [see Fig. (1)]. The energy eigenvalues $E_n^{(1)}$ of ϕ_n are the solutions of the finite well problem.

Second, because of the long-range order of the superlattice, the total wave function must satisfy Bloch's theorem. In the limit of weakly interacting quantum wells, it is written as a plane-wave superposition of well eigenfunctions given by Eq. (1) with the modification given by Eq. (4),

$$|\,\mathbf{k},\mathbf{k}_z,n\,\rangle=(AN_w)^{-1/2}\sum_j e^{ik_zz_j}e^{i\mathbf{k}\cdot\mathbf{r}}\phi_n(z-z_j)\ , \tag{5a}$$

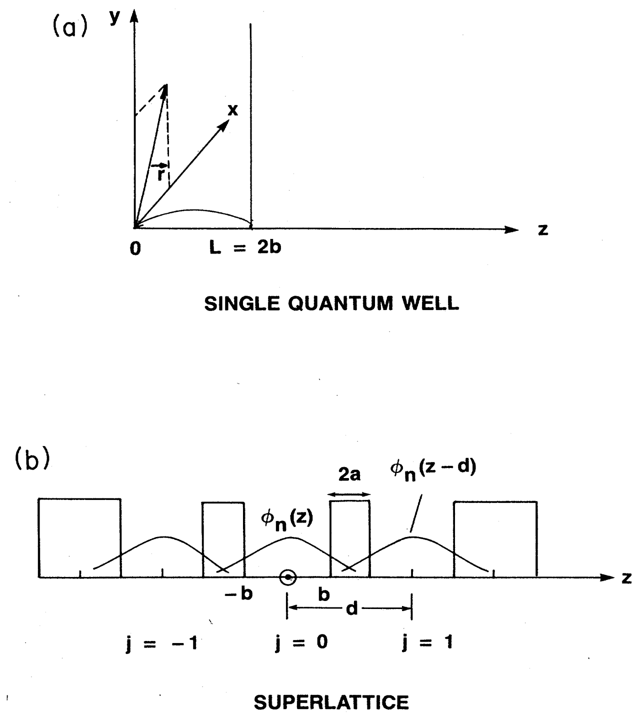

FIG. 1. Geometry and wave functions for quantum well (a) and superlattice (b).

where k_z is the quasimomentum in the z direction, N_w is the number of wells in the sample, and A is the projection (on the xy plane) of the area of each well. The centroid of the jth well is

$$z_j=jd,\ \ j=0,\pm1,\pm2,\ldots \tag{5b}$$

where $d=2a+2b$ is the period of the superlattice [see Fig. 1(b)]. The energies corresponding to the state of Eq. (5a) are

$$E(\mathbf{K},n)=E(\mathbf{k})+E_n(k_z)=(\hbar^2k^2/2m^*)+E_n(k_z)\ , \tag{6}$$

where $E_n(k_z)$ gives the dispersion along z for the nth miniband and is not further specified at this stage. As stated in the Introduction, Eq. (5a) strictly applies in the narrow-band limit of small, nearest-neighbor overlap, though it would be expected to provide a good extrapolation to larger overlap as does conventional tight-binding theory. Moreover, the results of the present theory (interwell and interference effects) would be expected to be general features of any band scheme adopted.

In the basis of Eq. (5a), the matrix elements of the electron-phonon Hamiltonian are

$$\langle\,\mathbf{k}',k_z',n\,|\,H_{e-\mathrm{ph}}\,|\,\mathbf{k},k_z,n\,\rangle=(AN_w)^{-1}\sum_{j'}\sum_j e^{-ik_z'z_{j'}}e^{ik_zz_j}\int dr\int dz\,\phi_{n'}^*(z-z_{j'})e^{-i\mathbf{k}'\cdot\mathbf{r}}H_{e\text{-ph}}e^{i\mathbf{k}\cdot\mathbf{r}}\phi_n(z-z_j)\ . \tag{7}$$

The integral of Eq. (7) is treated analogously to the case of the isolated quantum well given in Ref. 8. We get

$$R(K,K')\delta_q N_w^{-1}\sum_{j'}\sum_{j} e^{ik_z' z_{j'}} e^{ik_z z_j}\int dz\,\phi_{n'}^*(z-z_{j'})\phi_n(z-z_j)e^{iq_z z}\ , \tag{8a}$$

where

$$R(\mathbf{K}',\mathbf{K})=(\hbar/2MN\omega_Q)^{1/2}C(Q)(N_Q+\tfrac{1}{2}\pm\tfrac{1}{2})^{1/2}\ , \tag{8b}$$

where M is the ionic mass, ω_Q is the angular frequency of the mode of momentum $\mathbf{Q}$, $C(Q)$ is the coupling strength, and $\delta_q=\delta_{k',k+q}$ gives momentum conservation in the (x,y) plane.

With regard to the overlap integral of Eq. (8a), in the approximation of Eq. (5a) this is appreciable only if z_j and $z_{j'}$ are identical or refer to the centroids of adjacent wells, the orbital overlap of more distant pairs of wells taken to be negligible,

$$z_{j'}-z_j=0 \text{ or } \pm d\ .$$

Thus Eq. (8a) becomes

$$R(\mathbf{K}',\mathbf{K})\left[N_w^{-1}\sum_j e^{-i(k_z'-k_z-q_z)z_j}\right]\delta_q\left[\int dz\,\phi_{n'}^*(z)\phi_n(z)e^{iq_z z}+\sum_{\pm} e^{\pm ik_z' d}\int dz\,\phi_{n'}^*(z\mp d)\phi_n(z)e^{iq_z z}\right] \tag{9}$$

for the intrawell and interwell contributions, respectively. The second factor gives momentum conservation in the z direction. Thus, momentum conservation, which was smeared out in the case of the single well, is here recovered because of the long-range periodicity of the superlattice. Equation (9) is then written

$$\langle \mathbf{k}',k_z',n'\,|\,H_{e-\text{ph}}\,|\,\mathbf{k},k_z,n\rangle=R(\mathbf{K},\mathbf{K}')\delta_q\delta_{q_z}\{G_{n'n}^{(0)}(q_z)+[e^{-ik_z' d}G_{n'n}^{(+)}(q_z)+e^{ikz'd}G_{n'n}^{(-)}(q_z)]\}\ , \tag{10}$$

where

$$G_{n'n}^{(0)}(q_z)=\int dz\,\phi_{n'}^*(z)\phi_n(z)e^{iq_z z}\ , \tag{11}$$

$$G_{n'n}^{(\pm)}(q_z)=\int dz\,\phi_{n'}^*(z\mp d)\phi_n(z)e^{iq_z z}\ , \tag{12}$$

and

$$\delta_{q_z}\equiv\delta_{k_z'-k_z-q_z}\ .$$

Equation (11) is identical to the intrawell form factor obtained elsewhere,[8] but with $\sin(k_n^{(0)}z)$ appropriate to infinite potential barriers here replaced by $\phi_n(z)$. In that case, with $k_n^{(0)}=n\pi/L$, it becomes a sum of functions of the form $(\sin x)/x$, each giving momentum conservation $q_z=\pm(k_{n'}\pm k_n)$ to within $\Delta q_z=L^{-1}$, with L the well width. In the momentum-conservation-approximation (MCA) adopted by Ridley,[8] these are replaced by delta functions ($L\to\infty$), the error of which is estimated for various cases of interest. In the present case, k_n is given by $k_n=(2mE_n/\hbar^2)^{1/2}$, where $E_n=E_n(k_z)$ denotes the nth miniband. Thus, $(k_z n)\to(k_z',n')$ denotes intersubband transitions, while $(k_z,n)\to(k_z',n)$ denotes intrasubband transitions.

A procedure identical to that in Ref. 8 gives for Eq. (11) [cf. Eq. (11) of Ref. 1]

$$G_{n'n}^{(0)}(q_z)=\frac{1}{2}\frac{\sin\{[q_z\pm(k_n'\pm k_n)]b\}}{[q_z\pm(k_n'\pm k_n)]b}\times\exp\{i[q_z\pm(k_n'\pm k_n)]b\}\ , \tag{13}$$

where one of the terms is a maximum when

$$q_z=0 \text{ or } \pm(k_n'\pm k_n)\ . \tag{14}$$

There is also an exponentially small contribution arising from the tails of ϕ_n that will be considered when the intrawell contribution is evaluated; it makes a contribution comparable to the interwell term considered below [Eq. (23) *et seq.*] However, both of these terms are smaller than the interference term [Eqs. (33) and (34)].

With regard to the interwell form factor given by Eq. (12), it is easily shown that

$$G_{n'n}^{(+)}(q_z)=G_{n'n}^{(-)}(-q_z) \tag{15}$$

if $\phi_{n'}$ and ϕ_n are both even or both odd, while

$$G_{n'n}^{(+)}(q_z)=-G_{n'n}^{(-)}(-q_z) \tag{16}$$

if $\phi_{n'}$ is even and ϕ_n odd, or vice versa.

Initially, intrasubband scattering ($n'=n$) will be considered, so that Eq. (15) is obeyed, and, in Eq. (10),

$$e^{-ik_z' d}G_{nn}^{(+)}(q_z)+e^{ik_z' d}G_{nn}^{(-)}(q_z)=2\cos(k_z' d)G_{nn}^{(+)}(q_z)\ . \tag{17}$$

The dominant contribution to the interwell form factor, Eq. (12), comes from the overlap of the exponential tails of well eigenfunctions in the barrier region. Thus, for the well centered at the origin in Fig. 1(b),

$$\phi_n(q)=C_n e^{-K_n z},\quad b\le z\le b+2a \tag{18}$$

where

$$\hbar K_n=[2m_b^*(V_0-E_n)]^{1/2}$$

and where m_b^* is the effective mass in the barrier. Integrating over the barrier, we obtain

$$G_{nn}^{(+)}(q_z)=|\,C_n\,|^2 e^{-K_n d}e^{iq_z d/z}2a\frac{\sin(q_z a)}{q_z a}\ , \tag{19}$$

where the normalization constant

$$|C_n|^2=\cos^2(k_nb)\left[b\left[\frac{\sin(2k_nb)}{2k_nb}+1+\frac{\cos^2(k_nb)}{K_nb}\right]\right]^{-1}. \quad (20)$$

Now k_n and K_n are functions of E_n and even though $n'=n$, $E_n(k_z)\neq E_n(k_{z'})$ because $k_z'\neq k_z$. However, we shall neglect this dependence here and take k_n and K_n to be given by $E_n^{(1)}$, the energy of the isolated finite well, and hence independent of k_z or k_z'.

The absolute square of the transition matrix element, Eq. (10), is

$$|\langle \mathbf{k}',k_z',n'|H_{e-\mathrm{ph}}|\mathbf{k},k_z,n\rangle|^2 = |R(\mathbf{K}',\mathbf{K})|^2\delta_{\mathbf{k}',\mathbf{k}+\mathbf{q}}\delta_{k_z',k_z+q_z}\{|G_{nn}^{(0)}(q_z)|^2+4\cos^2(k_z'd)|G_{nn}^{(+)}(q_z)|^2+[2\cos(k_z'd)G_{nn}^{(0)}(q_z)G_{nn}^{(+)}(q_z)+\text{c.c.}]\}, \quad (21)$$

where $R(K',K)$, is given by Eq. (8b).

The first term in the curly brackets of Eq. (21) is the intrawell contribution studied in Ref. 8 and elsewhere, the second term is the interwell contribution, while the last term is an interference term between the intrawell and interwell processes. The total scattering rate is given by

$$W_{\mathbf{K}}=\frac{2\pi}{\hbar}\int|\langle \mathbf{K}'|H_{e-\mathrm{ph}}|\mathbf{K}\rangle|^2\delta(E_{\mathbf{K}'}-E_{\mathbf{K}}\pm\hbar\omega_Q)d\mathbf{K}'.$$

We will consider deformation-potential scattering by acoustic or optical phonons, where, in the usual approximations,

$$|R(\mathbf{K}',\mathbf{K})|^2=R^2=\begin{cases}\dfrac{C_0^2k_BT}{2C_LV}, & \text{acoustic phonons}\\[2ex] \dfrac{\hbar D_0^2}{2\rho\omega_0V}[N(\omega_0V)+\tfrac{1}{2}\mp\tfrac{1}{2}], & \text{optical phonons}\end{cases} \quad (22)$$

where C_0 is the acoustic deformation potential, D_0 the optical deformation potential, C_L the elastic constant, and ρ the mass density. For this case, $|R(\mathbf{K}',\mathbf{K})|^2=R^2$ is independent of $Q=|\mathbf{Q}|=|\mathbf{K}'-\mathbf{K}|$.

For the interwell scattering rate given by the second term in the curly brackets of Eq. (21),

$$W_{\mathbf{k}}^{(\mathrm{inter})}=\frac{2\pi}{\hbar}(R^2V)4a^2|C_n|^4e^{-2K_nd}\left[\frac{2}{(2\pi)^3}\int\int\int\left[\frac{\sin(q_za)}{q_za}\right]^2 4\cos^2(k_z'd)k'dk'd\theta'dk_z'\,\delta(E_{\mathbf{K}}-E_{\mathbf{K}'})\right], \quad (23)$$

having taken the deformation-potential scattering to be elastic. Integrating over θ' and k', the quantity in large square brackets becomes

$$\frac{m^*}{2\pi^2\hbar^2}\int_{-k_z(E)}^{k_z(E)}dk_z'\left[\frac{\sin[(k_z'-k_z)a]}{(k_z'-k_z)a}\right]^2 4\cos^2(k_z'd),$$

provided that energy is conserved

$$E(\mathbf{K}',n)=E(\mathbf{K},n)\equiv E$$

with $E(k,n)$ given by Eq. (6). Here, the limiting values of k_z for the given energy E is the solution of

$$E=E_n(k_z(E)). \quad (24a)$$

For example, in the tight-binding approximation

$$E_n=t_n[1-\cos(k_zd)], \quad (24b)$$

$$k_z(E)=\frac{1}{d}\cos^{-1}\left[\frac{t_n-E}{t_n}\right].$$

The magnitude of $k_z(E)$ for typical superlattice doping levels will be estimated at the end of this section.

To evaluate the integral of Eq. (23), it is noted that since $d=2a+2b$, $a/d=\frac{1}{6}$ for $a=(\frac{1}{2})b$, and $a/d=\frac{1}{4}$ for $a=b$. Then the first (form) factor in the integral is quite broad and nearly constant over the integration range, and approximately independent of k_z. Then Eq. (23) becomes

$$W_{\mathbf{k}}^{(\mathrm{inter})}\simeq\frac{8}{\pi}\mu^2a^2\frac{m^*}{\hbar^3d}|C_n|^4e^{-2K_nd}\{\sin[2k_z(E)d]+2k_z(E)d\}, \quad (25a)$$

with $\mu^2=R^2V$, and $k_z(E)$ may be related to the density of states per spin:

$$\rho(E)=\frac{1}{(2\pi)^3}\int d^3K'\delta(E-E_{\mathbf{K}}')$$
$$=\frac{1}{(2\pi)^3}\int dk_z'\int dk'k'\int d\theta'\delta\left[E-\frac{\hbar^2k'^2}{2m^*}-E(k_z')\right]$$
$$=\frac{m^*}{4\pi^2\hbar^2}\int_{-k_z(E)}^{k_z(E)}dk_z'$$
$$=\frac{m^*}{2\pi^2\hbar^2}k_z(E)\ . \quad (25b)$$

It is noted that, because of the different length scales involved, this approximation for the superlattice is opposite the momentum-conservation-approximation (MCA) for the isolated quantum well.[8] There, the $(\sin x)/x$ factors are taken to act as delta functions.

The second term in the curly brackets of Eq. (25a) gives the proportionality to the density of final states; the first term is an additional oscillatory contribution due to scattering into forward and backward quantum wells.

Specifically, in view of Eq. (22), Eq. (25a) becomes

$$W_{\mathbf{k}}^{(\text{inter})}\simeq\frac{8}{\pi}\frac{C_0^2k_BT}{2C_L}a^2\frac{m^*}{\hbar^3d}\,|\,C_n\,|^4e^{-2K_na}\{\sin[2k_z(E)d]+2k_z(E)d\}\quad\text{(acoustic phonons)} \quad (26)$$

$$\simeq\frac{8}{\pi}\frac{\hbar D_0^2}{2\rho\omega_0}a^2\frac{m^*}{\hbar^3d}\,|\,C_n\,|^4e^{-2K_na}(N(\omega_0)\{\sin[2k_z(E)d]+2k_z(E)d\}$$
$$+[N(\omega_0)+1]\{\sin[2k_z(E+\hbar\omega_0)d]+2k_z(E+\hbar\omega_0)d\})$$
$$\text{(nonpolar optical phonons)} \quad (27)$$

where, in Eq. (27) for scattering by nonpolar optical phonons via the deformation potential, the inelasticity has been taken into account in the phonon population and density-of-states factors, but not in the matrix element, following standard procedures.[8]

The magnitude of $k_z(E)$ compared to the size of the reduced minizone (π/d) is easily estimated for various doping concentrations. The carriers introduced into the superlattice form an anisotropic degenerate electron gas at low temperatures. Taking the dispersion to be of tight-binding form for this example [cf. Eq. (24b)], the density of carriers in the lowest miniband ($n=1$) corresponding to Fermi energy E, is[4]

$$N=\frac{m^*t_1}{\pi^2\hbar^2d}\left\{-\left[\frac{E}{t_1}\right]\cos^{-1}\left[1-\frac{E}{t_1}\right]+\left[2\left[\frac{E}{t_1}\right]-\left[\frac{E}{t_1}\right]^2\right]^{1/2}\right\}. \quad (28)$$

For a typical GaAs-$Ga_{1-x}Al_xAs$ superlattice with $2b=50$ Å, $2a=25$ Å ($d=75$ Å), $m^*=0.067m_e$, and conduction-band offset of 160 meV ($x\simeq0.15$), Kronig-Penney calculations give a bandwidth

$$E_1(k_z=\pi/d)-E_1(k_z=0)\equiv2t_1=65\text{ meV}\ ,$$

and the prefactor of Eq. (28) is

$$N_1=\frac{m^*t_1}{\pi^2\hbar^2d}=3.9\times10^{17}\text{ cm}^{-3}\ .$$

For $N=N_1$, $E=t_1=33$ meV, and $k_z(E)\simeq\pi/2d$. Since this is a relatively high doping level, $k_z(E)<\pi/2d$ over which the form factor varies very little, confirming the approximation made in obtaining Eq. (25a).

As for the isolated quantum well,[8] the momentum relaxation rate τ_K^{-1} is identical with the total scattering rate $W_{\mathbf{K}}$ for a superlattice in the case of deformation-potential scattering. This is shown explicitly in the Appendix.

From Eqs. (13) and (21), the intrawell contribution may be evaluated in the same approximations as before. As for other treatments for deformation-potential scattering, we find the rates proportional to the density of final states,

$$W_{\mathbf{k}}^{(\text{intra})}=(\tau_{\mathbf{k}}^{-1})^{(\text{intra})}=\frac{2\pi}{\hbar}\mu^2\rho(E)\ . \quad (29)$$

There is an additional contribution to the intrawell scattering arising from the tails of the ϕ_n, which is comparable to the interwell term given by Eq. (25a). Inserting the exponential wave function of Eq. (18) into Eq. (11), the integral over the range $b\leq|z|<\infty$ is readily carried out, with the result that to Eq. (13) gets added a "wing" term

$$\Delta G_{nn}^{(0)}(q_z)=2\,|\,C_n\,|^2e^{-2K_nb}b\frac{2(K_nb)\cos(q_zb)-(q_zb)\sin(q_zb)}{(2K_nb)^2+(q_zb)^2}\ . \quad (30)$$

In evaluating the first term of the curly brackets of Eq. (21), there is now an additional cross term between Eqs. (13) and (30) above, giving

$$W_{\mathbf{K}}^{\text{intrawing}}=\frac{2\pi}{h}R^2\frac{V}{(2\pi)^3}\int\int\int dk'k'd\theta'dk'_z\frac{1}{2}\sin\frac{[\,|(k_z'-k_z)\pm 2k_n\,|b]}{[\,|(k_z'-k_z)\pm 2k_n\,|b]}e^{i[\,|(k_z'-k_z)\pm 2k_n\,|b]}$$
$$\times 2\,|C_n|^2 e^{-K_nb}b\frac{2(K_nb)\cos[(k_z'-k_z)b]-(k_z'-k_z)b\sin[(k_z'-k_z)b]}{(2K_nb)^2+(q_2b)^2}\delta(E_k-E_{k'})+\text{c.c.} \tag{31}$$

Again, for low doping levels, $k_z(E_F)d \ll 1$, the principal contribution comes from the vicinity of $q_z=k_z'-k_z=0$, with the result

$$W_{\mathbf{K}}^{\text{intrawing}}=\frac{m^*\mu^2}{\hbar^3}\,|C_n|^2\frac{e^{-2K_nb}}{(K_nb)}\ . \tag{32}$$

Finally, we write down the interference contribution to $W_{\mathbf{K}}$, given by the last term of Eq. (21). Using Eqs. (13) and (19), we get

$$W_{\mathbf{K}}^{(\text{interf})}=\frac{2m^*a}{\pi\hbar^3}\mu^2\,|C_n|^2e^{-K_nd}\delta_{q_z,k_z'-k_z}\sum_{\pm}\int_{-k_z(E)}^{k_z(E)}dk_z'\cos(k_z'd)\left[\frac{\sin[q_z\pm(k_n'\pm k_n)]b}{[q_z\pm(k_n'\pm k_n)]b}\right]$$
$$\times\frac{\sin(q_za)}{q_za}\cos\left[q_z\left[\frac{d}{2}+b\right]\pm(k_n'\pm k_n)b\right]\ . \tag{33}$$

It is noted that, in order of magnitude, this interference term is larger by a factor $\exp(K_na)$ than the interwell contribution given by Eq. (25a). In general, this expression is not readily integrable. However, it may be evaluated in the low-density limit $k_z(E_F)d \ll 1$. For the example given previously [Eq. (28) *et. seq.*], this would apply for $N<10^{17}$ cm^{-3}. In this case, the trigonometric factors in the integral are expanded in a Taylor series valid for $k_z(E_F)d \ll 1$. Since $|q_z|\le 2k_z(E_F)$, only the $q_z=0$ term of Eq. (13) contributes. The result is

$$W_{\mathbf{k}}^{(\text{interf})}=\frac{2m^*a}{\pi\hbar^3}\mu^2\,|c_n|^2e^{-K_nd}\left[\left[1-\frac{k_z^2d^2}{2}\right]2k_z(E)-\frac{(\bar d)^2}{9}[k_z(E)]^3\right] \tag{34}$$

where

$$(\bar d)^2=3d^2+a^2+b^2+3\left[\frac{d}{2}+b\right]^2$$

and $k_z(E)$ is given Eqs. (24) and (25b). Again, using Eq. (22), this can be written explicitly for deformation-potential scattering by acoustic and optical phonons in analogy with Eqs. (26) and (27) for the interwell contribution.

III. INTERSUBBAND TRANSITIONS

Finally, we shall briefly examine the interwell transition rate for intersubband transitions ($n'\neq n$) for the case of deformation-potential scattering. We would expect this to correspond to the phonon-induced transition from the lowest $n=1$ subband of a given well to the $n'=2$ subband of the neighboring well. For this case, $\phi_{n'}$ and ϕ_n would be expected to have opposite parity so Eq. (16) holds, and, instead of Eq. (17), we have

$$e^{-ik_z'd}G_{n'n}^{(+)}(q_z)+e^{ik_z'd}G_{n'n}^{(-)}(q_z)=-2i\sin(k_z'd)G_{n'n}^{(+)}(q_z)\ . \tag{35}$$

For the well centered at $z=0$, the wave function in the barrier region is given by Eq. (18), while for the well centered at $z=d$,

$$q_{n'}(z)\simeq C_n'e^{K_n'(z-d)},\quad b\le z\le b+2a\ .$$

From Eq. (12), we get

$$G_{nn'}^{(+)}(q_z)=C_nC_{n'}e^{-K_{n'}d}\frac{e^{\Delta Ka}-e^{-\Delta Ka}}{\Delta K}\ , \tag{36}$$

where $\Delta K=K_{n'}-K_n+iq_z$. For $K_{n'}\simeq K_n$, we get

$$G_{nn'}^{(+)}\simeq C_nC_{n'}e^{-K_{n'}d}e^{(K_n-K_{n'})d/2}e^{iq_zd/2}$$
$$\times\frac{\sin(q_za)+(K_{n'}-K_n)a\cos(q_za)}{q_za}\ . \tag{37}$$

Assuming as before that the interband form factor is constant over the integration range of k_z', the dependence of the interband transition rate on the density of states is found to be

$$W_{\mathbf{k}}^{(\text{inter})}(n\to n')\sim\{-\sin[2k_z(E)d]+[2k_z(E)d]\}$$
$$\approx\tfrac{4}{3}[k_z(E)d]^3,\quad k_z(E)d\ll 1 \tag{38}$$

in contrast to that given by Eq. (25a) for the intraband transition rate. Again, $k_z(E)$ is given in terms of the density of states (per spin) by Eq. (25b).

IV. SUMMARY

We have evaluated the total scattering rate and momentum relaxation rate for an ideal, periodic superlattice by taking matrix elements of the electron-phonon interaction Hamiltonian using basis states which are plane-wave combinations of finite quantum-well eigenfunctions. In addition to the contribution due to scattering within individual wells, terms due to interwell transfer and to the interference between the intrawell and interwell processes are identified. This has been done for deformation-potential scattering by acoustic and nonpolar optical phonons. The form factors for the two processes are approximated in the present treatment. Further work will focus on incorporating these factors more exactly, and treating scattering mechanisms with explicit wave-vector dependences: polar optical scattering and piezoelectric scattering.

APPENDIX: EQUALITY OF MOMENTUM RELAXATION RATE AND TOTAL SCATTERING RATE FOR A SUPERLATTICE WITH DEFORMATION-POTENTIAL SCATTERING

To obtain the momentum relaxation rate τ_k^{-1} rather than the total scattering rate $W_{\mathbf{K}}$, Eq. (21) must be multiplied by $1-\cos[\gamma(\mathbf{K}',\mathbf{K})]$, where $\gamma(\mathbf{K}',\mathbf{K})$ is the scattering angle between total momenta $\mathbf{K}$ and $\mathbf{K}'$. In view of the somewhat complex dependence on the wave vector, it is not obvious that $\tau_{\mathbf{K}}^{-1}=W_{\mathbf{K}}$ for deformation-potential scattering in this case; however, we see immediately below that it is.

With $\mathbf{K}=\mathbf{k}+\hat{\mathbf{z}}\mathbf{k}_z$, and $\mathbf{k}$ taken along the x direction ($\hat{\mathbf{z}}$ is a unit vector along z),

$$\cos[\gamma(\mathbf{K}',\mathbf{K})]=\frac{\mathbf{K}'\cdot\mathbf{K}}{|\mathbf{K}'||\mathbf{K}|}=\frac{kk'\cos\theta'+k_z'k_z}{(k'^2+k_z'^2)^{1/2}(k^2+k_z^2)^{1/2}}\ ,$$

where θ' is the polar angle that appears in Eq. (23). Integrating over θ', the first term above vanishes. Carrying out the integration over k' by making use of the energy-conserving delta function, the integral of Eq. (23) becomes

$$\frac{k_z}{(k^2+k_z^2)^{1/2}}\int_{-k_z(E)}^{k_z(E)}dk_z'\left[\frac{\sin(q_za)}{q_za}\right]^2 4\cos^2(k_z'd)\times\frac{k_z'}{\left[\frac{2m^*}{\hbar^2}[E-E(k_z')]+k_z'^2\right]}\ .$$

Again taking $[\sin(q_za)/q_za]^2$ to be approximately constant over the integration range, the integrand is odd in k_z and vanishes. Thus,

$$(\tau_{\mathbf{k}}^{-1})^{(\mathrm{inter})}=W_{\mathbf{k}}^{(\mathrm{inter})}\ .$$

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