RAPID COMMUNICATIONS

PHYSICAL REVIEW B VOLUME 32, NUMBER 12 15 DECEMBER 1985

Single-particle relaxation time versus scattering time in an impure electron gas

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(Received 30 August 1985)

Relative magnitudes of the single-particle relaxation time and the scattering time that enters in conductivity are given for two- and three-dimensional electron gases in the presence of random distributions of charged Coulomb scattering centers. We find that for accessible electron densities in the usual three-dimensional metallic systems the scattering time is at most a factor of ~ 2 larger than the single-particle relaxation time, whereas in high-mobility GaAs-based heterojunctions the spatial separation between the impurities and the carriers gives rise to scattering times which can be as much as two orders of magnitude larger than the corresponding single-particle relaxation times.

In the transport theory of normal metallic systems, one must deal with two different characteristic times—a single-particle relaxation time τ_s and a scattering time τ_t.[1,2] The single-particle relaxation time is related to the imaginary part Γ_s of the single-particle self-energy function by

$$\Gamma_s = \hbar/2\tau_s \ . \tag{1}$$

It is a measure of the time for which an electronic momentum eigenstate can be defined even in the presence of scattering. The scattering time is related to the dc conductivity σ by

$$\sigma = ne^2\tau_t/m \ , \tag{2}$$

where n is the density of carriers and e and m are the electronic charge and effective mass, respectively. In this Rapid Communication we discuss the quantitative difference between these two times and show that this difference can become very large for electrons in some high-mobility heterojunctions.

From a many-body-theory viewpoint the single-particle relaxation time τ_s is related to the one-electron Green's function of the coupled electron-impurity system, whereas the scattering time τ_t is related to the two-electron correlation function that defines the conductivity in the system.[3] Even though τ_t is more directly related to experiment, τ_s enters in an important way into many theoretical calculations. In particular, the single-particle level broadening Γ_s determines the modifications of the electronic density of states due to the electron-impurity interaction.[3] It also determines the modification of screening of an electron gas due to the presence of impurities, as discussed for three dimensions by de Gennes[4] and for two dimensions by Ando[5] and Das Sarma.[6]

In this paper we consider a two- (2D) or three-dimensional (3D) electron gas at absolute zero in the presence of charged impurity centers and calculate τ_t and τ_s by assuming the interaction between an electron and an impurity to be a linearly screened Coulomb potential. For the screening function we use the static random-phase-approximation screening, as given for three dimensions by Lindhard[7] and for two dimensions by Stern.[8] The characteristic times in three dimensions are given by

$$\frac{1}{\tau} = \frac{2\pi m N_i}{\hbar^3} \int \frac{d^3k'}{(2\pi)^3} f(\theta) \left| u\left(2k_F \sin\left[\frac{\theta}{2}\right]\right)\right|^2 \frac{\delta(k'-k_F)}{k'} \ , \tag{3}$$

where $f(\theta) = 1-\cos\theta$ for obtaining τ_t and $f(\theta)=1$ for obtaining τ_s.[9] In Eq. (3), N_i is the concentration of impurities, k_F is the Fermi wave vector, and $u(q)$ is the screened electron-impurity interaction calculated[3,7,8] in the static random-phase approximation. A slightly modified expression gives the characteristic times for two-dimensional electron systems, as noted below.

The relaxation time and the scattering time are equal for short-range (δ-function) scattering, for which the scattering cross section is independent of angle and for which the average value of $\cos\theta$ vanishes. However, if the scattering is strongly peaked in the forward direction, τ_t can be considerably greater than τ_s, as we show below.

Figure 1 shows the ratio τ_t/τ_s for a three-dimensional electron gas as a function of $y \equiv k_F/q_{\rm TF}$, where $q_{\rm TF}$ is the three-dimensional Thomas-Fermi screening constant.[10] The solid curve gives the results calculated with the full wave-vector-dependent polarizability function,[7] whereas the dashed curve is calculated using the long-wavelength limit for the screening. For $y \ll 1$ one expects and finds $\tau_t/\tau_s \sim 1$ because the scattering is nearly isotropic when the screening is strong (the ratio is slightly less than 1 if the full wave-vector-dependent screening, corresponding to the solid curve in Fig. 1, is used because the screening decreases with increasing wave vector, favoring backward scattering over forward scattering). On the other hand, τ_t/τ_s diverges for large y. Expressions relating k_F and $q_{\rm TF}$ to the density for both two- and three-dimensional free-carrier systems are given in Table I. For metals y is between 0.5 and 1.0, implying that $\tau_t \simeq \tau_s$ for charged impurity scattering at low temperatures. A wider range of values is possible in degenerate semiconductors.

For two-dimensional electron systems—even in the idealized case in which the electrons are confined to a plane and the scatterers lie in a plane parallel to that of the

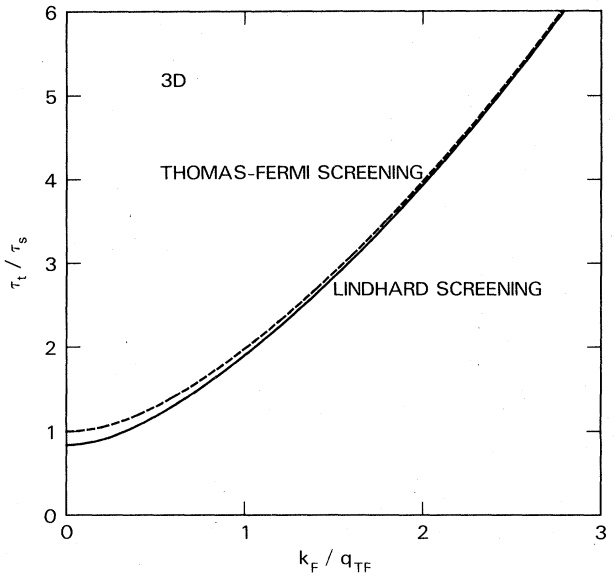

FIG. 1. Calculated ratio of the scattering time τ_t to the single-particle relaxation time τ_s in a three-dimensional electron gas at absolute zero in the presence of Coulomb scattering vs the ratio of the Fermi wave vector k_F to the Thomas-Fermi screening parameter q_{TF}. The solid curve and the dashed curve are obtained using the full Lindhard screening and the long-wavelength Thomas-Fermi screening, respectively.

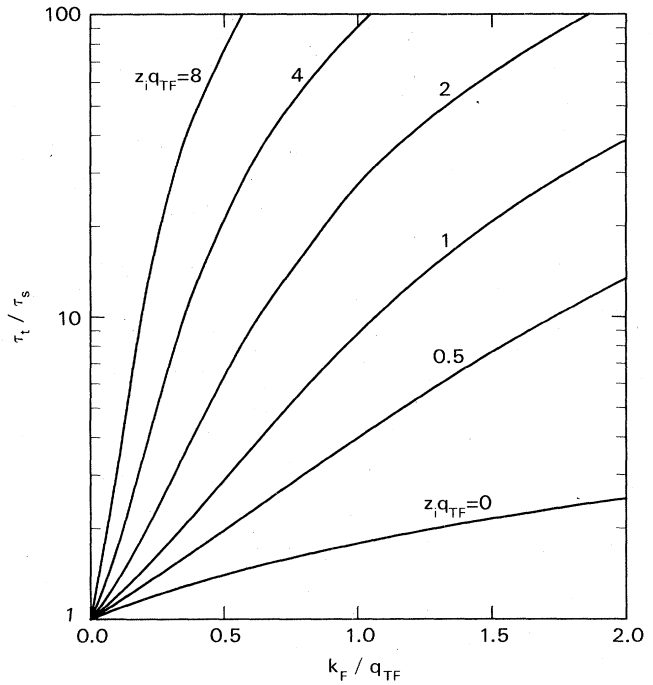

FIG. 2. Same as Fig. 1, but for an ideal two-dimensional electron gas with six different values of the separation z_i between the electron layer and the impurity layer. Note that the random phase approximation and the long-wavelength limit give the same results here. The value of q_{TF} for two-dimensional electrons in GaAs is 2.0×10^6 cm^{-1} and the corresponding value for a Si(001) inversion layer is 1.9×10^7 cm^{-1} if the average dielectric constant of Si and SiO_2 is used.

electrons—there is an additional parameter characterizing the scattering, namely, the distance z_i between the scatterers and the electron plane. This leads to an additional factor of $\exp[-4k_F z_i \sin(\theta/2)]$ inside the integral in the two-dimensional analog of Eq. (3).[11] Figure 2 shows τ_t/τ_s vs k_F/q_{TF} for six values of z_i. All the values converge to $\tau_t/\tau_s=1$ for $k_F/q_{TF}\to 0$ because for two-dimensional systems the screening is independent of wave vector for the entire accessible range of scattering wave vectors ($0\leq q\leq 2k_F$) in an electron gas with isotropic effective mass at absolute zero. For large values of y, on the other hand, the scattering becomes increasingly peaked in the forward direction, especially for large values of z_i, and τ_t increases rapidly while τ_s is affected much less.

Figure 3 shows calculated values of τ_t and τ_s for $Al_{0.3}Ga_{0.7}As$-GaAs heterojunctions with two different acceptor doping levels in the GaAs as a function of spacer thickness d_{sp}, which determines the channel electron density.[12] The calculation proceeds along standard lines,[11] using the Fang-Howard variational function to approximate the spatial distribution of electrons in the GaAs.

It is clear from Fig. 3 that for high mobility GaAs heterojunctions τ_t is substantially larger than τ_s and that the single-particle relaxation time cannot be estimated from the mobility. In lower-mobility samples, for which the scattering may arise from impurities closer to the channel, the difference between the two characteristic times becomes smaller.

On the other hand, in a Si(001) inversion layer with $N_s=10^{12}$ cm^{-2}, $k_F/q_{TF}\simeq 0.1$, making $\tau_t\simeq\tau_s$. This conclusion is not changed when interface roughness scattering is included. Thus it is a reasonable approximation to extract the single-particle relaxation time from the mobility for a Si(001) inversion layer.

Support for these conclusions comes from comparison of the scattering time deduced from the mobility with the time deduced from the magnitude of Shubnikov–de Haas oscillations in both GaAs-based heterojunctions and silicon inversion layers. Paalanen, Tsui, and Hwang[13] find large differences in the two times for electrons in heterojunctions and point out the importance of long-range scattering in this system. Harrang *et al.*[14] find the two times to be very close for the silicon case, as had been shown by Fang, Fowler, and Hartstein,[15] but substantially different for the GaAs case. Terwilliger and Higgins[16] have shown that the integration over scattering angle in Eq. (3) has a lower cutoff when the relaxation time for the Landau levels is calculated. They argue that scattering that has little or no effect on a cyclotron orbit will not contribute to the level broadening. Thus, the

TABLE I. Expressions for the Fermi wave vector k_F and the Thomas-Fermi screening parameter q_{TF} in terms of the electron density n (in three dimensions) or N_s (in two dimensions). Parameters that enter are Planck's constant $2\pi\hbar$, the electronic charge e and effective mass m, the background dielectric constant κ, and the valley degeneracy g_v, which equals 1 for simple metals and for electrons in GaAs and equals 2 for electrons in Si(001) inversion layers. All values are in cgs units.

	k_F	q_{TF}
3D	$(3\pi^2 n/g_v)^{1/3}$	$(2e/\hbar)(m/\kappa)^{1/2}g_v^{1/3}(3n/\pi)^{1/6}$
2D	$(2\pi N_s/g_v)^{1/2}$	$2g_v me^2/\kappa\hbar^2$

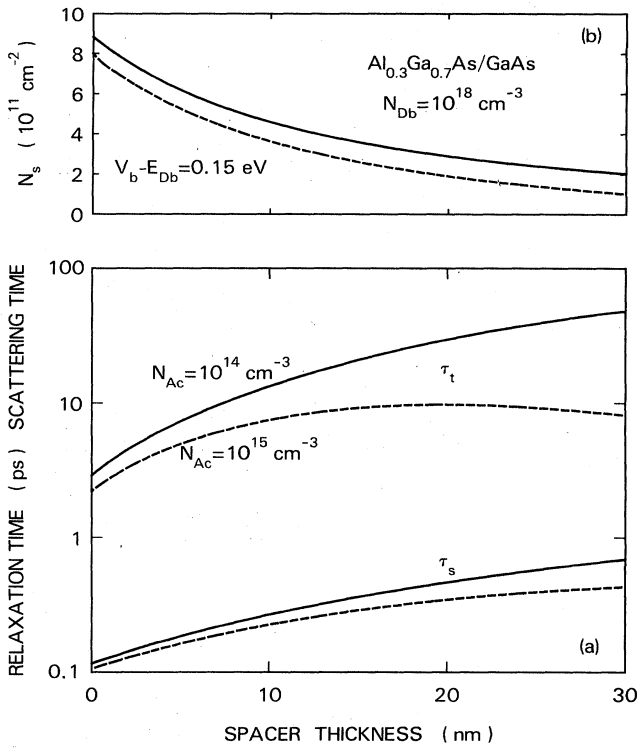

FIG. 3. (a) Calculated values of τ_t and τ_s vs spacer thickness for electrons in $Al_{0.3}Ga_{0.7}As$-GaAs heterojunctions with GaAs acceptor doping levels N_{Ac} of 10^{14} cm^{-3} (full curves) and 10^{15} cm^{-3} (dashed curves) and a donor doping level of 10^{18} cm^{-3} in the $Al_{0.3}Ga_{0.7}As$. Note that the electron mobility is $\mu(\text{in cm}^2/\text{V s}) = 2.5\times10^4\tau_t$ (in ps) and that the level broadening is Γ (in meV) $= 0.33/\tau_s$ (in ps). (b) Corresponding values of the channel electron density N_s. The difference between the conduction band offset, V_b, and the donor binding energy in the barrier, E_{Db}, is taken to be 0.15 eV. All values are for absolute zero.

effective relaxation time they determine will be intermediate between our τ_t and τ_s. In strong magnetic fields where the carrier motion and the screening are strongly perturbed, the simple considerations that lead to Eq. (3) are no longer valid. More generally, the imaginary part of the self-energy is a frequency- and wave-vector-dependent quantity[3] and our calculated τ_s refers to the so-called "mass shell" approximation in which the electron is assumed to be on the Fermi surface. Characterizing the imaginary part of the self-energy by a constant, independent of frequency and wave vector, as done in our present work, is an oversimplification and can only suggest some qualitative trends that require much more careful treatment.

In summary, we have obtained the relative magnitude of the single-particle relaxation time and the scattering time that enters in conductivity of two- and three-dimensional electron gases with Coulomb scattering by a random distribution of static, point-charged impurity centers. We find that τ_t/τ_s can be very large in high-mobility GaAs heterojunctions. Our conclusions may be relevant to a number of calculations for two-dimensional systems in which the level broadening that enters in the single-particle Green's function plays a role, including calculations involving cyclotron resonance[17,18] and magnetoresistance.[19] The relevance of these considerations to plasmon effects[20,21] is less clear, but it appears that the time that enters there is closer to the scattering time. Our results may also be relevant to the broadening of Landau levels seen in recent measurements of magnetic susceptibility[22] and specific heat,[23] although for large magnetic fields the simple considerations used here must be replaced by more detailed calculations.[24]

The work of one of the authors (S.D.S.) at the University of Maryland is supported in part by the U.S. Office of Naval Research, and by the National Science Foundation. We are indebted to P. B. Allen, T. Ando, J. P. Harrang, and C. Kallin for giving us copies of their work prior to publication, and to P. B. Allen, M. Ya. Azbel, A. Gold, A. Pinczuk, and R. E. Prange for comments.

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