PHYSICAL REVIEW B VOLUME 32, NUMBER 12 15 DECEMBER 1985

Spikes in the orbital magnetic susceptibility of a two-dimensional electron gas

I. D. Vagner
Department of Physics and Astronomy, Tel Aviv University, Ramat Aviv, 69978 Tel Aviv, Israel

Tsofar Maniv
Department of Chemistry and the Solid State Institute, Technion–Israel Institute of Technology, Technion City, 32000 Haifa, Israel

(Received 29 November 1984)

It is shown that the height of the spikes predicted to appear in the orbital magnetic susceptibility of a two-dimensional electron gas at low temperatures depends exponentially on inverse temperature. The spikes are significant only within exponentially narrow regions of the magnetic induction where the chemical potential moves rapidly between adjacent Landau levels. It is pointed out that in systems where the chemical potential is pinned within a magnetic gap (quantum Hall-effect systems) these spikes are expected to disappear.

It is well known[1–3] that in a pure infinite two-dimensional (2D) electron gas the orbital magnetic susceptibility (OMS) at zero temperature exhibits spikes as a function of the magnetic induction B. These spikes are the manifestation of the de Hass–van Alphen effect in 2D conductors. Although the appearance of such spikes was predicted a long time ago, the experimental observation of this phenomenon was reported only very recently in silicon inversion layer[4] and in stage-2 Br_2-graphite intercalated compound.[5] The de Haas–van Alphen effect has been also investigated in a modulation-doped $GaAs/Ga_xAl_{2-x}As$ heterostructure.[6] These systems[4,6] exhibit dips in their magnetoresistance and plateaus in their Hall conductance known as the quantum Hall effect (QHE).[7–18] The QHE is known to be associated with a pinning of the chemical potential within a magnetic gap.[9,10]

In this paper we investigate the spikes in the OMS of a 2D electron gas at low temperatures and show that in a pure infinite system the height of the spikes depends exponentially on inverse temperature. These spikes are significant only within exponentially narrow regions of B, where the chemical potential moves rapidly between adjacent Landau levels. In these regions the spikes in the OMS are a direct measure of the chemical potential derivative with respect to the magnetic induction B. In the presence of localized states within the magnetic energy gap the "motion" of the chemical potential between adjacent Landau levels is slowed down and the spikes are smeared out. We estimate the value of the density of localized states within the magnetic gap for which the spikes in the OMS disappear and the exponentially narrow plateaus in the Hall conductivity start to appear.

At zero temperature the energy $E(B)$ of a pure 2D electron gas is a piecewise quadratic function of B. At discrete values of $B=B_n^*$, corresponding to the situations where the first n_F Landau levels are completely filled while all the other levels are completely empty, the energy $E(B, T=0)$ exhibits downward cusps. The OMS is therefore constant between any adjacent values $B_{n_F}^*$, where the value of this constant decreases with increasing n_F. At $B_{n_F}^*$ the OMS is undefined. At finite temperatures the cusps in the free energy are rounded so that the OMS is well defined there.

To find the analytical form of the OMS in these regions we use the definition of the magnetization $M(B)$ in terms of the Gibbs thermodynamical potential Ω:

$$M(B)=-(\partial\Omega/\partial B)_{\mu,T}\ , \tag{1}$$

where

$$\Omega(B)=-k_BTg(B)\sum_{n=0}^{\infty}\ln(1-f_n)\ . \tag{2}$$

Here, $f_n=\{1+\exp[(E_n-\mu)/k_BT]\}^{-1}$ is the Fermi distribution, $E_n=\hbar\omega_c(n+\frac{1}{2})$ is the energy of the nth Landau level, and $g(B)=eB/hc$ is the degeneracy of the Landau levels per unit area.

It can be shown that for this form of Ω the differential relationship (1) between $M(B)$ and Ω becomes an algebraic one; that is,

$$M(B)=-\Omega(B)/B-E(B)/B\ . \tag{3}$$

Differentiating M with respect to B and using the well-known relation

$$\Omega(B)=F(B)-n_{\square}\mu(B)\ , \tag{4}$$

where $F(B)$ is the Helmholtz free energy and $n_{\square}$ is the electron density per unit area, we obtain

$$\begin{aligned}\chi(B)&=-[\partial^2F(B)/\partial B^2]_{n_{\square},T}\\&=[n_{\square}(\partial\mu/\partial B)-(\partial E/\partial B)]/B\ .\end{aligned} \tag{5}$$

The chemical potential $\mu(B)$ is defined by the condition

$$g(B)\sum_{n=0}^{\infty}f_n=n_{\square}\ , \tag{6}$$

which can be readily solved to yield[13]

$$\mu(B)=\hbar\omega_cn_F-k_BT\left\{\frac{A\cosh\alpha+(1+A^2\sinh^2\alpha)^{1/2}}{1-A}\right\}\ , \tag{7}$$

where $A=n_F-n/g$. Equation (7) shows that around the value $B=B^*$, where $|A|\exp\alpha<1$, the chemical potential is

approximately

$$\mu \simeq \mu^* - k_B T A e^{\alpha}/2 \ , \tag{7a}$$

where $\mu^* \simeq \hbar\omega_c^* n_F$; namely, the chemical potential is located around the middle of a magnetic energy gap (a situation which resembles an intrinsic semiconductor). The derivative of μ with respect to B in this region is, therefore, exponentially large:

$$(\partial\mu/\partial B)_{B\simeq B^*} \simeq -(k_B T n_{\square}\Phi_0/2B^{*2})\exp\alpha \ . \tag{8}$$

The large exponential value of the chemical potential derivative exhibited in Eq. (8) can be understood by the following consideration. Around $B = B^*$ the number of electrons N_e^* in the level just above μ and the number of "holes" N_h^* in the level just below are exponentially small: $N_e^* = N_h^* = g^* \exp(-\alpha)$. Reducing the magnetic induction B^* by a small amount δB, the degeneracy $g(B)$ of each level is reduced by the amount $\delta g \simeq g^* \delta B/B^*$. As a result, the number of electrons in the upper level increases by this amount. The number of holes in the lower level, however, remains unchanged. Now the location of the chemical potential is determined by the ratio between the number of electrons in the upper level and the number of holes in the lower level:

$$\exp(\delta\mu/k_B T)_{\mu^*} \simeq N_e/N_h = 1 + \delta g/N_h \ . \tag{6a}$$

Therefore, for infinitesimally small δB one gets

$$(\partial\mu/\partial B)_{B=B^*} \simeq -k_B T g^*/B^* N_h = -(k_B T n_{\square}\Phi_0/2B^{*2})\exp\alpha \ ,$$

which is our Eq. (8).

The region ΔB, where $(\partial\mu/\partial B)$ is exponentially large, is determined by the condition $|A|e^{\alpha} < 1$, which yields

$$\Delta B/B^* \simeq e^{-\alpha}/n_F \ . \tag{9}$$

The second term in Eq. (5) involves the derivative of the energy $E(B)$ with respect to B. Unlike the first term, this one does not diverge in the zero temperature limit, since $E(B)$ is a continuous function of B at $T = 0$. Therefore, the dominant contribution to the spikes in the OMS originates in the chemical potential derivative with respect to B.

Thus, within this narrow region the OMS is a direct measure of the derivative of the chemical potential μ with respect to B, so that by measuring experimentally the magnetic susceptibility one can readily obtain the value of $(\partial\mu/\partial B)$.

This result has been derived under ideal conditions. In real systems, however, the presence of localized[14–16] electronic states within the magnetic energy gap such as, for example, impurity states or edge states,[18] may lead to a significant slowing down in the variation of the chemical potential with respect to B, or even to the pinning[9,10] of μ (a QHE situation), thus substantially reducing the height of the spikes in OMS. Since the intrinsic width of the spikes, Eq. (9), is exponentially small, the effect of such localized states may be important even for very small concentrations of the extrinsic states.

To study the effect of localized states we assume, for the sake of simplicity, a given localized state density $D_{\rm loc}(E)$ within the magnetic energy gap. The chemical potential of the system is now determined by the following equation:

$$N = g(n_F - 1) + g/\{1 + \exp[(E_{n_F-1} - \mu)/k_B T]\} + g/\{1 + \exp[(E_{n_F} - \mu)/k_B T]\} + \int D_{\rm loc}(E)\,dE/\{1 + \exp[(E-\mu)/k_B T]\} \ . \tag{10}$$

Differentiating Eq. (10) with respect to B and assuming the $D_{\rm loc}(E)$ to be a slowly varying function of E around the middle of the magnetic energy gap, one gets

$$(\partial\mu/\partial B)[2g^* e^{-\alpha}/k_B T + D_{\rm loc}(E = \mu^*)] = (n_F - 1)\partial g/\partial B \ , \tag{11}$$

which yields

$$(\partial\mu/\partial B)_{B=B^*} \simeq (\partial\mu/\partial B)^{\rm free}_{B=B^*}/\{1 + [D_{\rm loc}(\mu^*)/4D_{\rm 2D}^{\rm free}](e^{\alpha}/\alpha)\} \ , \tag{11a}$$

where $(\partial\mu/\partial B)^{\rm free}$ is defined by Eq. (8) and $D_{\rm 2D}^{\rm free} \equiv m/2\pi\hbar^2$ is the density of states of a 2D electron gas.

The condition under which $(\partial\mu/\partial B)$ retains its intrinsic behavior (i.e., $\propto e^{\alpha}/\alpha$) is that the density of localized states within the magnetic energy gap should be exponentially small with respect to the density of states of the 2D free-electron gas:

$$D_{\rm loc}(E) < 4\alpha e^{-\alpha} D_{\rm 2D}^{\rm free} \ . \tag{12}$$

To estimate the height of the spikes in the OMS, Eq. (5), consider a model 2D electron gas with $n_{\square} = 10^{12}\ \mathrm{cm}^{-2}$, $m_c = 0.1 m_0$, $E_F = 10^{-13}$ erg, $B = 3.10^4$ g, $d = 10^{-6}$ cm, $k_B T = 1$ K. Those parameters are typical (by an order of magnitude) to those observed in real 2D conductors such as the inversion layers in Si metal-oxide-semiconductor field-effect transistors[4] (MOSFET's) and the modulation-doped $GaAs/Ga_xAl_{1-x}As$ heterostructures.[6] In that case, Eqs. (5) and (9) yield $\chi^* = 10^{-2}/\mathrm{cm}^2$ and $\Delta B = 1$ g.

If the density of localized states within the magnetic energy gap is sufficiently large such that condition (12) is no longer satisfied, the chemical potential becomes a smooth function of B and the spikes disappear. Under these circumstances, one expects that the magnetoresistance would show pronounced dips around B^*, where the Hall conductance exhibits plateaus of finite width (QHE).

In systems exhibiting strong magnetic interactions[1,13] (MI) where diamagnetic domains are formed, the spikes are also expected to disappear due to the pinning of the chemical potential within a magnetic gap.[13,19] Markiewicz, Meskoob, and Zahopoulos[5] reported recently the first direct observation of such giant MI in a stage-2 Br_2 graphite intercalation compound. Measuring the magnetic suscepibility χ as a function of the magnetic field, they observed a smooth field dependence below a threshold value at about 4 T and a structure of giant spikes above. This seems to agree with our estimated threshold magnetic field for diamagnetic domain formation in graphite intercalation compounds.[13] However, one should notice that their measurement of χ has been performed by field-modulation technique, which may not be interpreted in a straightforward manner by our static theory. Furthermore, inhomogeneity in the magnetic field or in the electron density[6,20] also leads to a significant smearing of the spikes.

We would like to thank Professor A. Aharony, Professor E. Ehrenfreund, and Professor D. Schoenberg for their interest in this work, and Professor M. Ya. Azbel' for many valuable discussions which stimulated this paper.

[1]D. Schoenberg, *Magnetic Oscillations in Metals* (Cambridge Univ. Press, Cambridge, England, 1984).
[2]L. E. Gurevich and A. Ya. Shik, Zh. Eksp. Teor. Fiz. **54**, 1873 (1968) [Sov. Phys. JETP **27**, 1006 (1968)].
[3]A. Isihara and Y. Shiwa, in *High Field Magnetism,* edited by M. Date (1983), p. 127; A. Isihara and D. Y. Kojima, Phys. Rev. B **19**, 846 (1979).
[4]F. F. Fang and P. J. Stiles, Phys. Rev. B **28**, 6992 (1983).
[5]R. S. Markiewicz, M. Meskoob, and C. Zahopoulos, Phys. Rev. Lett. **54**, 1436 (1985).
[6]H. L. Stormer, T. Haavasoja, V. Narajanamurti, A. G. Gossard, and W. Wiegman, J. Vac. Sci. Technol. B **1**, 423 (1983); T. Havasoja, H. L. Stormer, V. Narajanamurti, A. G. Gossard, and W. Wiegman (unpublished).
[7]K. v. Klitzing, G. Dorda, and M. Pepper, Phys. Rev. Lett. **45**, 494 (1980).
[8]D. S. Tsui, H. L. Stormer, and A. G. Gossard, Phys. Rev. Lett. **48**, 1559 (1982).
[9]T. Ando, A. B. Fowler, and F. Stern, Rev. Mod. Phys. **54**, 437 (1982).
[10]B. I. Halperin, Helv. Phys. Acta **56**, 75 (1983).
[11]I. M. Lifshitz, M. Ya. Azbel', and M. I. Kaganov, *Electron Theory of Metals* (Consultants Bureau, New York, 1973).
[12]R. E. Prange and Tsu-Wei Nee, Phys. Rev. **168**, 779 (1968); B. I. Halperin, Phys. Rev. B **25**, 2185 (1982); A. H. MacDonald and P. Streda (unpublished); M. Ya. Azbel' and Marc Brodsky (unpublished); G. F. Giuliani, J. J. Quinn and S. C. Ying, Phys. Rev. B **28**, 2969 (1983).
[13]I. D. Vagner, Tsofar Maniv, and E. Ehrenfreund, Phys. Rev. Lett. **51**, 1700 (1983).
[14]R. Laughlin, Phys. Rev. B **23**, 5632 (1982).
[15]Y. Imry, J. Phys. C **16**, L221 (1982).
[16]R. F. Kazarinov and S. Luryi, Phys. Rev. B **25**, 7626 (1982); S. A. Trugman, *ibid.* **27**, 7539 (1983).
[17]M. Ya. Azbel', Phys. Rev. B **26**, 3430 (1982).
[18]M. Ya. Azbel', Solid State Commun. **53**, 147 (1985).
[19]A. B. Pippard (private communication).
[20]D. Shoenberg, J. Low Temp. Phys. **56**, 417 (1984).