Edge magnetoplasmons in a bounded two-dimensional electron fluid

Alexander L. Fetter
Institute of Theoretical Physics, Department of Physics, Stanford University, Stanford, California 94305
(Received 29 April 1985)

A hydrodynamic model is used to study the magnetoplasma modes of a two-dimensional electron fluid confined to a half-plane. In addition to the usual bulk modes, an approximate theory predicts new localized edge modes that propagate along the boundary, with a frequency proportional to B^{-1} in high magnetic fields. These modes provide a good fit to those seen in recent experiments of Mast and Dehm.

I. INTRODUCTION

Recent experiments on electrons trapped on the surface of liquid ^{4}He have found two sorts of two-dimensional (2D) magnetoplasma modes.[1-3] One set may be identified with the usual bulk modes, whose squared frequency in the presence of a perpendicular magnetic field increases linearly with the squared cyclotron frequency. The second (anomalous) set had not been predicted previously. Its frequency varies inversely with the magnetic field B in the large-field limit, which differs qualitatively from the behavior of the bulk modes of a 2D electron fluid. This unexpected mode has been identified as an edge mode that propagates along the boundary of the 2D charged layer (the analog of a surface wave in three dimensions). The present paper describes the calculations that support this view.

Three-dimensional (3D) surface plasmons are well known and have been studied extensively.[4] The effect of a magnetic field on these modes has been less well investigated, with fewer experiments to guide the theory. Nevertheless, calculations[5,6] suggest the existence of two surface magnetoplasmons that reduce to the usual surface plasmons in the zero-field limit. For large fields, the frequency of one mode increases indefinitely and the other tends to zero inversely with the field.[7] In view of the similarity to the observed behavior for the 2D electron layer on liquid He,[1,2] it is natural to suggest a similar explanation for the recently observed anomalous modes.

For many purposes, 2D systems are simpler than 3D ones, because the more restricted phase space often allows exact solutions to nontrivial statistical-mechanical problems. This situation does not immediately apply to electrostatic phenomena, since screening in a 2D charged system is qualitatively different from that in 3D. In the particular case of electrons confined to a single bounded layer, the screening is less complete, and much of the effective interaction between localized charge elements is mediated by the fringing fields in the surrounding medium.[8-10] As a result, the relation between the charge density and the electrostatic potential becomes an intrinsically nonlocal integral one. Here, the problem is formulated in detail for the special case of a half-plane; although an exact solution would become complicated, a tractable approximation[11] produces a set of model equations that are exactly soluble.

Since the present approach relies on the comparison with the 3D surface plasmons, in Sec. II we will review this standard solution in some detail. In Sec. III we then formulate the corresponding 2D problem exactly and provide an approximate solution that appears to fit the experimental observations.[1,2] The effect of additional grounded planes above and below the charged layer is treated in Sec. IV.

II. BULK AND SURFACE MAGNETOPLASMONS IN THREE DIMENSIONS

To understand the behavior of magnetoplasma modes in the presence of boundaries, it is natural to rely first on simple geometries, and the present work will consider only a half-space (3D) and a half-plane (2D). In addition, I use a hydrodynamic model of a weakly damped, compressible charged-electron fluid placed in a rigid, neutralizing positive background.[4] This model includes dispersion along with the proper Lorentz force, but it omits retardation and the corresponding propagating electromagnetic waves. In 3D it provides an exact model solution for the bulk and surface modes because Poisson's equation is explicitly local. In 2D, however, a given point in the plane experiences nonlocal electrostatic restoring forces arising from the fields in the surrounding medium, which complicates the situation considerably. Since the 3D problem will serve as a basis for the approximate 2D solution, its properties will be presented in some detail.

Consider a rigid positive background with charge density en_0 and a compressible electron fluid with number density n_0+n, where n is a small perturbation. Conservation of matter implies the usual linearized equation of continuity

$$\frac{\partial n}{\partial t}+n_0\nabla\cdot\mathbf{v}=0\ , \qquad (1)$$

where $\mathbf{v}$ is the electron's velocity. Euler's equation then specifies the dynamics:

$$\frac{\partial \mathbf{v}}{\partial t}+\frac{\mathbf{v}}{\tau}=-\frac{s^2}{n_0}\nabla n+\frac{e}{m}\nabla\Phi+\omega_c\hat{\mathbf{z}}\times\mathbf{v}\ . \qquad (2)$$

Here, τ is an effective collision time, $s^2=m^{-1}(\partial p/\partial n)$ characterizes the speed of compressional waves in the absence of electrostatic effects, Φ is the usual electrostatic potential, and $\omega_c = eB/mc$ is the cyclotron frequency associated with the static magnetic field $B\hat{\mathbf{z}}$. Finally, Poisson's equation relates the potential to the net charge density

$$\nabla^2\Phi = 4\pi en \ , \qquad (3)$$

where the dielectric constant has been set equal to 1 for simplicity (the relevant modifications are elementary for the present 3D case).

In the absence of boundaries, this set of equations has plane-wave solutions of the form $e^{i(\mathbf{q}\cdot\mathbf{r}-\omega t)}$, where $\mathbf{q}$ is a real three-dimensional wave vector. For the present purpose it is sufficient to restrict $\mathbf{q}$ to the x-y plane perpendicular to $\mathbf{B}$, since that is the appropriate configuration for the corresponding two-dimensional motion (Sec. III). Direct substitution readily yields the dispersion relation

$$\frac{\omega}{\tilde{\omega}}(\tilde{\omega}^2-\omega_c^2) = \Omega_p^2 + s^2q^2 \ , \qquad (4a)$$

where $\tilde{\omega} = \omega + i/\tau$ and

$$\Omega_p = (4\pi n_0 e^2/m)^{1/2}$$

is the bulk 3D plasma frequency. In the collisionless limit ($\omega\tau \gg 1$), this equation reduces to the familiar result

$$\omega^2 = \Omega_p^2 + \omega_c^2 + s^2q^2 \ . \qquad (4b)$$

In particular, the squared frequency of all bulk magnetoplasmons with $\mathbf{q}\perp\mathbf{B}$ increases with increasing magnetic field, and the dispersive corrections involving s are small in the long-wavelength limit ($qs/\Omega_p \ll 1$).

In the absence of a magnetic field, the motion is purely longitudinal, but a nonzero magnetic field induces a transverse component through the $\mathbf{v}\times\mathbf{B}$ term in the Lorentz force. The corresponding trajectory of an undamped electron becomes elliptical in the plane perpendicular to $\mathbf{B}$, with the major axis oriented along $\mathbf{q}$, the ratio of semiminor to semimajor axes given by ω_c/ω and the sense of rotation determined by the right-hand rule. Equation (4) shows that this motion becomes circular in the high-field limit.

The analogous magnetoplasma modes in a half-space are less familiar, although the limiting case of zero field has been studied extensively. For comparison with the behavior in two dimensions, the charged medium will be taken to occupy the region $x<0$, with the magnetic field $B\hat{\mathbf{z}}$ lying in the plane of the surface. Translational invariance allows plane waves propagating in the y-z plane, and only the case $\mathbf{q}=q\hat{\mathbf{y}}$ will be considered here (thus the wave and $\mathbf{B}$ both lie in the surface, at right angles). As a result, all quantities take the form $f(x)e^{i(qy-\omega t)}$, where the amplitudes $f(x)$ remain to be determined.

Inside the material ($x<0$), Eqs. (1)—(3) apply, whereas only the potential is defined in the exterior region ($x>0$). Direct substitution yields the coupled differential equations

$$-i\omega n + n_0(\partial v_x + iqv_y) = 0 \ , \qquad (5a)$$

$$-i\tilde{\omega} v_x + (s^2/n_0)\partial n - (e/m)\partial\Phi_< + \omega_c v_y = 0 \ , \qquad (5b)$$

$$-i\tilde{\omega} v_y + iq(s^2/n_0)n - iq(e/m)\Phi_< - \omega_c v_x = 0 \ , \qquad (5c)$$

$$(\partial^2 - q^2)\Phi_< = 4\pi en \qquad (5d)$$

in the interior, and the single equation

$$(\partial^2 - q^2)\Phi_> = 0 \qquad (6)$$

in the exterior, where ∂ denotes ∂_x. This set must be solved subject to the boundary conditions that Φ and $\partial\Phi$ are continuous at $x=0$, and that v_x vanishes there, along with suitable bounded behavior as $|x|\to\infty$.

For $x>0$, the only solution is

$$\Phi_>(x) = \Phi_0 e^{-qx} \ , \qquad (7)$$

which reflects the omission of retardation. For $x<0$ the functions may be assumed to have an exponential form, and it is not difficult to obtain the two independent solutions e^{qx} and $e^{\kappa x}$, where

$$\kappa^2 = q^2 + s^{-2}[\Omega_p^2 + \omega(\omega_c^2 - \tilde{\omega}^2)/\tilde{\omega}] \ . \qquad (8)$$

Evidently, κ depends explicitly on both q and ω. Here and henceforth, it is simplest to consider a wave propagating in the $+y$ direction, so that q is intrinsically positive; on the other hand, ω_c can take either sign, depending on the direction of $\mathbf{B}$.

Equation (8) has two types of solutions. If κ^2 is negative ($=-k^2$, say), it becomes precisely the dispersion relation found in Eq. (4) for an unbounded medium. The corresponding solutions then represent propagating waves in the x-y plane that are reflected (with a phase shift) by the boundary at $x=0$. They are analogous to continuum states in quantum mechanics and behave very much like the bulk magnetoplasma modes. In particular, for a collisionless medium, their squared frequency increases linearly with ω_c^2, as in Eq. (4b).

The other type of solution corresponds to positive κ^2, so that the amplitudes all decay exponentially away from the boundary at $x=0$; such surface modes are analogous to bound states in quantum mechanics. In this case, the interior potential has the form

$$\Phi_<(x) = \Phi_1 e^{qx} + \Phi_2 e^{\kappa x} \ , \qquad (9a)$$

and the corresponding normal component of the velocity becomes

$$4\pi e n_0 v_x = \frac{iq\Omega_p^2\Phi_1}{\tilde{\omega}-\omega_c} e^{qx} + i(\kappa\tilde{\omega} + q\omega_c)\frac{\omega}{\tilde{\omega}}\Phi_2 e^{\kappa x} \ . \qquad (9b)$$

Imposing the boundary conditions leads to a set of homogeneous equations in the amplitudes Φ_i ($i=0,1,2$), and the vanishing of the determinant gives the dispersion relation, which can be written in either of two equivalent forms[5]

$$2(\kappa\tilde{\omega} + q\omega_c) = \frac{\tilde{\omega}}{\omega}\frac{\Omega_p^2(q+\kappa)}{\tilde{\omega}-\omega_c} = \frac{\Omega_p^2}{s^2}\frac{\tilde{\omega}-\omega_c}{q+\kappa} \ . \qquad (10)$$

For the present work, damping will be neglected entirely ($\tilde{\omega}=\omega$), which simplifies Eq. (10) considerably. In the limit $sq \ll \Omega_p$, a straightforward reduction yields the well-known relation[5]

$$\omega^2-\omega\omega_c-\tfrac{1}{2}\Omega_p^2=0\ , \tag{11}$$

which immediately reproduces the zero-field surface plasmon with frequency $\pm\Omega_p/\sqrt{2}$. Since the sum of the roots of Eq. (11) is ω_c but the product is independent of ω_c, the two roots shift with increasing field; in particular, they must vary linearly and inversely with ω_c in the high-field limit. It is convenient to label them as

$$\omega_+=\tfrac{1}{2}\mathrm{sgn}\omega_c[(2\Omega_p^2+\omega_c^2)^{1/2}+|\omega_c|]\ , \tag{12a}$$

$$\omega_-=-\tfrac{1}{2}\mathrm{sgn}\omega_c[(2\Omega_p^2+\omega_c^2)^{1/2}-|\omega_c|]\ , \tag{12b}$$

so that $|\omega_-|$ shrinks with increasing B, in contrast to all other long-wavelength magnetoplasma modes of the bounded electron fluid. In addition, the high-field limit of ω_- is proportional to Ω_p^2 and hence to the unperturbed electron density.[1] These characteristic field and density dependences will be seen to persist in the case of a 2D electron gas; they play a central role in the experimental detection and identification of such "anomalous" modes.[1,2]

It is instructive to compare these *surface magnetoplasmons* with the bulk modes discussed below Eq. (4b). Consider the electrons' motion at a distance x (<0) from the boundary. For general values of q, the amplitude (9b) contains two characteristic decay lengths q^{-1} and κ^{-1}; the first of these diverges in the long-wavelength limit, whereas the second remains finite but depends specifically on the choice of mode (namely ω_+ or ω_-). For definiteness, only the limit $sq\ll\Omega_p$ will be considered. It is not difficult to see that the motion (confined to the x-y plane) is again elliptical, with the semimajor axis a parallel to the surface (along $\hat{\mathbf{y}}$) and the semiminor axis b along the normal $\hat{\mathbf{x}}$:

$$a=1-(\omega_c/\omega)e^{\kappa x}\ , \tag{13a}$$

$$b=1-e^{\kappa x}\ . \tag{13b}$$

These amplitudes depend explicitly on the distance x (<0) from the surface. As expected from the boundary condition on v_x, b vanishes at the surface, where the motion is plane-polarized. Deep in the sample, the orbit becomes circular, but the present long-wavelength limit neglects the eventual exponential decay with length q^{-1}. For the $+$ mode in Eq. (12a), it is easy to see that the decay constant κ_+ is given by $(2s)^{-1}|\omega_-|$, which becomes small in the high-field limit ($\omega_c/\Omega_p\gg1$). Correspondingly, a also becomes small near the surface in this limit, so that the $+$ magnetoplasma surface mode will probably not couple strongly to an external electromagnetic field. The situation is reversed for the $-$ mode, whose second decay constant becomes $\kappa_-=(2s)^{-1}|\omega_+|$. This quantity increases dramatically with increasing field, ultimately producing a magnetic screening of length s/ω_c, proportional to B^{-1}. Furthermore, the ratio ω_c/ω_- is now negative and large for high fields, enhancing the longitudinal amplitude a relative to the transverse one b. This behavior suggests that an incident electromagnetic wave will couple preferentially to the $-$ magnetoplasma surface mode.

The dispersion relation (10) can also be inverted analytically for all q to give the two roots ($q>0$ here)

$$\omega_+(q)=\tfrac{1}{2}\mathrm{sgn}\omega_c\{[2\Omega_p^2+(|\omega_c|-qs)^2]^{1/2}+|\omega_c|+qs\}\ , \tag{14a}$$

$$\omega_-(q)=-\tfrac{1}{2}\mathrm{sgn}\omega_c\{[2\Omega_p^2+(|\omega_c|+qs)^2]^{1/2}-|\omega_c|+qs\}, \tag{14b}$$

which obviously reduce to those found previously in the long-wavelength limit. A detailed calculation shows that Eq. (14b) yields a real positive decay constant κ_- for all positive q, so that this mode quite generally describes a surface magnetoplasmon whose high-field frequency varies like ω_c^{-1}. In contrast, the decay constant κ_+ for Eq. (14a) vanishes when $sq\omega_c=\Omega_p^2/2$; for larger values of $sq\omega_c$ this branch of the magnetoplasma spectrum ceases to represent a surface mode, merging into the continuum of modes that propagate in the x-y plane.

The surface magnetoplasmons have the unusual feature that the $+$ and $-$ modes are *intrinsically* different. For definiteness, consider $q>0$ and $\omega_c>0$. The $+$ mode represents a wave moving in the positive direction with phase velocity ω_+/q, whereas the $-$ mode moves in the negative direction with a *distinct* phase velocity $|\omega_-|/q$. Consequently, no linear combination of these two modes can produce a standing wave in nonzero field, in contrast to the bulk modes in Eq. (4). This situation is unaffected by the inclusion of waves with $q<0$; for general q, Eqs. (7)—(14) remain correct if q and ω_c are replaced by $|q|$ and $\omega_c\mathrm{sgn}q$. With this substitution, the $+$ and $-$ modes always propagate in the positive and negative directions with different phase velocities for any finite magnetic field. This same behavior will be seen to occur in the 2D geometry considered below.

III. BULK AND EDGE MAGNETOPLASMONS IN TWO DIMENSIONS

The basic problem of interest is the self-consistent oscillation of a charge-compensated 2D electron gas placed in a perpendicular magnetic field $B\hat{\mathbf{z}}$. If n and $\mathbf{v}$ are interpreted as the perturbation in the 2D number density and the 2D velocity vector, Eqs. (1) and (2) continue to apply, with $-\nabla\Phi$ as the in-plane component of the electric field, evaluated at the plane ($z=0$, say). In contrast, Poisson's equation involves the potential throughout all space, with a 3D Laplacian and the full 3D charge density on the right-hand side.

As an introduction to the difficult problem of edge magnetoplasma modes on a half-plane, consider first the simpler case of an unbounded 2D electron gas in a vacuum, where Eq. (3) is replaced by

$$\nabla^2\Phi=4\pi en\,\delta(z)\ . \tag{15}$$

Equations (1), (2), and (15) have plane-wave solutions with the density n and velocity $\mathbf{v}$ proportional to $e^{i(\mathbf{q}\cdot\mathbf{r}-\omega t)}$; the potential has the same form, but the amplitude $\Phi_{\mathbf{q}}(z)$ depends on the distance from the plane as well as on the wave vector $\mathbf{q}$. Substitution into Eq. (15) yields the equation for this amplitude,

$$[(d/dz)^2-q^2]\Phi_q(z)=4\pi e n_q \delta(z)\ , \tag{16}$$

where n_q is the corresponding amplitude for the density perturbation. In the absence of distant boundaries, the only allowed solution is proportional to $\exp(-q\,|z\,|)$, and the coefficient is easily found by integrating across the layer

$$\Phi_q(z)=-2\pi e q^{-1} n_q \exp(-q\,|z\,|)\ . \tag{17}$$

For $z=0$ this relation determines the electrostatic potential in the plane of the charge, and a combination with Eqs. (1) and (2) immediately gives a dispersion relation of the form in Eq. (4a), but with the bulk 3D plasma frequency replaced by the bulk 2D value[8-10]

$$\Omega_q=(2\pi n_0 e^2 q/m)^{1/2}\ , \tag{18a}$$

which depends explicitly on the wave vector $\mathbf{q}$. In the collisionless limit, this solution becomes[12]

$$\omega^2=\Omega_q^2+\omega_c^2+s^2q^2\ . \tag{18b}$$

Typical experimental values[2] are $n_0=10^8$ cm^{-2}, $q=10$ cm^{-1}, $\Omega_q=1.3\times10^9$ rad/sec, $s=(k_BT/m)^{1/2}=3.9\times10^5$ cm/sec, and $\omega_c=1.8\times10^{10}$ rad/sec for $B=10^3$ G; thus the dispersion correction in Eq. (18b) is indeed negligible. As in the corresponding 3D situation, the squared frequency of all bulk 2D magnetoplasma modes increases linearly with ω_c^2. A straightforward analysis also shows that the electrons execute an elliptical trajectory, with the same characteristic features as in the discussion below Eq. (4).

The differences between 2D and 3D become significant for a bounded system. Consider a 2D electron fluid confined to the half-plane $x<0$, and $z=0$. Poisson's equation now becomes

$$\nabla^2\Phi=4\pi e\delta(z)\Theta(-x)\ , \tag{19a}$$

where Θ denotes the usual step function. Since the system is translationally invariant along the boundary, the solutions may be taken as plane waves of the form $e^{i(qy-\omega t)}$, but the amplitudes now depend on x (and z for the potential). It is again convenient to consider q positive, but to let ω_c have either sign. In contrast to the case of an unbounded 2D system, Poisson's equation is still a partial-differential equation

$$\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}-q^2\right]\Phi(x,z)=4\pi e n(x)\delta(z)\Theta(-x)\ , \tag{19b}$$

where the q dependence of the amplitudes has been suppressed. Since the charge density $-en(x)$ is everywhere finite, Φ and $\partial\Phi/\partial x$ are continuous, and a Fourier transform in x yields the ordinary differential equation

$$[(d/dz)^2-(k^2+q^2)]\overline{\Phi}(k,z)$$
$$=4\pi e\delta(z)\int_{-\infty}^{0}dx\,e^{-ikx}n(x)=4\pi e\delta(z)\overline{n}(k)\ , \tag{20}$$

where $\overline{n}(k)$ is the Fourier transform of $n(x)\Theta(-x)$. This equation has the same form as Eq. (16), and its solution in the plane reduces to

$$\overline{\Phi}(k,0)=-2\pi e\overline{n}(k)(k^2+q^2)^{-1/2}\ . \tag{21}$$

The inverse Fourier transform then gives a nonlocal integral relation[2] between the electrostatic potential $\Phi(x,z=0)$ in the plane and the corresponding charge density $-en(x)$,

$$\Phi(x,z=0)+4\pi e\int_{-\infty}^{0}dx'L(x-x')n(x')=0\ , \tag{22a}$$

where

$$L(x)=\tfrac{1}{2}\int_{-\infty}^{\infty}\frac{dk}{2\pi}\frac{e^{ikx}}{(k^2+q^2)^{1/2}}=(2\pi)^{-1}K_0(q\,|x\,|) \tag{22b}$$

is a particular Bessel function, obtained as the inverse transform of $\overline{L}(k)=\frac{1}{2}(k^2+q^2)^{-1/2}$.

It is instructive to compare this situation with that for the 3D half-space, where Poisson's equation was solved directly by considering separately the regions $x<0$ and $x>0$. If (perversely) a Fourier transform is used, it produces an integral relation of the form (22a), but with a different and simpler exponential kernel $L(x)=(2q)^{-1}\exp(-q\,|x\,|)$, the inverse transform of $\overline{L}(k)=(k^2+q^2)^{-1}$. This nonlocality in three dimensions is rather trivial, however, for application of the differential operator $(d/dx)^2-q^2$ immediately reproduces Eqs. (5d) and (6) because this new L is just the appropriate Green's function. In contrast, no simple operator seems capable of eliminating the nonlocality of Eq. (22), which presumably reflects the fringing electric fields in the surrounding vacuum.

Equation (22a) is a special form of integral equation, one with a displacement kernel that depends only on the difference of the variables and whose range is the half-space. In principle, such equations can always be solved with the Wiener-Hopf technique.[11] In three dimensions this approach reproduces the solution found in Sec. II because the appropriate kernel $\overline{L}(k)$ is a meromorphic function of k that is readily factored or decomposed with partial fractions. In contrast, the 2D kernel $\overline{L}(k)$ has branch points, which complicates the analysis. The Appendix contains a brief discussion of this exact approach, but a complete treatment (which would involve considerable numerical work) has not been attempted.

In problems of this sort it has often proved valuable to approximate the exact integral kernel $L(x)$ by another simpler one, L_0, that has the same integrated area and second moment[11] (typically L is even so that the first moment vanishes). Equivalently, the exact and approximate Fourier transforms must have the same first two terms in a power series about $k^2=0$. In addition, the approximate kernel $\overline{L}_0(k)$ should have a simpler analytic structure, and all these conditions are satisfied by choosing

$$\overline{L}_0(k)=q(k^2+2q^2)^{-1}\ , \tag{23a}$$

which has poles instead of branch points. In this way, the exact integral relation (22a) is replaced by one of the same form, but with the approximate kernel

$$L_0(x)=2^{-3/2}\exp(-\sqrt{2}q\,|x\,|)\ . \tag{23b}$$

As an added benefit, the solution of this approximate integral equation is now elementary, because L_0 is essentially the Green's function for the differential operator

$(d/dx)^2-2q^2$. Thus, *in this approximation,* Poisson's equation becomes effectively local, with the potential $\Phi(x)$ in the plane of the charge satisfying two equations of the same form as Eqs. (5d) and (6) for the 3D problem:

$$(\partial^2-2q^2)\Phi_<(x)=4\pi eqn(x) \quad (x<0)\ , \tag{24a}$$

$$(\partial^2-2q^2)\Phi_>(x)=0 \quad (x>0)\ . \tag{24b}$$

These equations must be combined with the hydrodynamic equations (5a)–(5c) and the boundary conditions on Φ and v_x. In this way, the coupled integro-differential equations are replaced by differential ones, and the same methods used in the 3D half-space configuration will now be seen to provide the *exact* solution to this approximate problem.

For $x>0$ the solution has the form [see Eq. (7)]

$$\Phi_>(x)=\Phi_0\exp(-\sqrt{2}qx)\ . \tag{25}$$

For $x<0$ the solutions are decaying exponentials $\exp(\kappa_1 x)$ and $\exp(\kappa_2 x)$, where κ_1^2 and κ_2^2 are the smaller and larger roots of the quadratic equation

$$\kappa^4-[3q^2+2\Omega_q^2 s^{-2}+(\omega_c^2-\omega^2)s^{-2}]\kappa^2 +2q^2[q^2+\Omega_q^2 s^{-2}+(\omega_c^2-\omega^2)s^{-2}]=0\ . \tag{26}$$

Here and henceforth, damping has been omitted ($\tilde{\omega}=\omega$), and Ω_q is the bulk 2D plasma frequency from Eq. (18a). If κ^2 is positive, then the corresponding magnetoplasma mode is the 2D analog of the 3D surface mode and can be called an edge mode (or, in Ref. 3, a perimeter wave). Since the principal experimental interest is in these bound states, the additional class of scattering states will not be considered.

The remaining steps in obtaining the dispersion relation for the edge magnetoplasma modes are the same as in 3D, and straightforward manipulations give the explicit expression

$$\begin{aligned}2q(\Omega_q^2+s^2q^2)(\sqrt{2}\omega-\omega_c) &= s^2[(\kappa_1^2+\kappa_1\kappa_2+\kappa_2^2)q\omega\sqrt{2}+(\kappa_1+\kappa_2)q^2\omega_c\sqrt{2}\\ &\quad +\kappa_1\kappa_2 q\omega_c+\kappa_1\kappa_2(\kappa_1+\kappa_2)\omega]\ .\end{aligned} \tag{27}$$

For fixed q (>0) and ω_c (of either sign), this equation must be solved for the allowed frequencies (which appear implicitly in the quantities κ_1 and κ_2, as well as explicitly). In general, this procedure requires numerical work, since no simple analytic solution [compare Eq. (14)] has been found. In the long-wavelength limit, however, the situation becomes much simpler, which will now be considered in detail.

For the 3D surface modes the condition $sq \ll \Omega_p$ defined the long-wavelength limit, and a similar condition $sq \ll \Omega_q$ holds for the 2D edge modes as well. Alternatively, the same condition can be considered to arise from omitting the dispersive corrections ($s=0$), but it is preferable not to take that limit too early in the calculation. Apart from corrections of relative order q^2, the two roots of Eq. (26) reduce to

$$\kappa_1\approx Cq+O(q^3)\ , \tag{28a}$$

$$\kappa_2\approx s^{-1}(2\Omega_q^2+\omega_c^2-\omega^2)^{1/2}+O(q^2)\ , \tag{28b}$$

where C is independent of q and given by

$$C^2=2\frac{\Omega_q^2+\omega_c^2-\omega^2}{2\Omega_q^2+\omega_c^2-\omega^2}\ . \tag{29}$$

To leading order in q, the dispersion relation (27) becomes

$$2(\sqrt{2}\omega-\omega_c)=\omega(C+\sqrt{2})[2+(\omega_c^2-\omega^2)/\Omega_q^2]\ , \tag{30}$$

which determines the allowed frequencies in terms of the plasma frequency Ω_q and the cyclotron frequency ω_c. Some manipulation (including squaring both sides) yields a quadratic equation

$$3\omega^2-2\sqrt{2}\omega\omega_c-2\Omega_q^2=0\ , \tag{31}$$

along with an additional set of roots $\omega^2=\omega_c^2$. It is clear by inspection that $\omega=-\omega_c$ indeed satisfies Eq. (30), but (see below) it is probably a spurious result of the approximation method. In contrast, Eq. (31) has precisely the structure of Eq. (11) for 3D surface magnetoplasmons and provides the approximate dispersion relation for the 2D edge magnetoplasma modes.

For zero magnetic field, the roots are $\pm(\frac{2}{3})^{1/2}\Omega_q$, which should be compared with the 3D values $\pm(2)^{-1/2}\Omega_p$. Evidently, the 2D modes lie closer to the continuum and are less tightly bound. For general fields, it is again convenient to label the modes as

$$\omega_+=\tfrac{1}{3}\sqrt{2}\,\mathrm{sgn}\omega_c[(3\Omega_q^2+\omega_c^2)^{1/2}+|\omega_c|]\ , \tag{32a}$$

$$\omega_-=-\tfrac{1}{3}\sqrt{2}\,\mathrm{sgn}\omega_c[(3\Omega_q^2+\omega_c^2)^{1/2}-|\omega_c|]\ . \tag{32b}$$

In contrast to all the other magnetoplasma modes of a 2D electron gas, $|\omega_-|$ decreases with increasing magnetic field and ultimately varies inversely with B in the large-field limit, when its coefficient is proportional to Ω_q^2 and hence to the unperturbed electron density.[1]

It is important to consider the physical quantities associated with each of these modes, particularly the two long-wavelength screening constants κ_1 and κ_2. For both modes, the larger screening constant κ_2 remains finite for all values of the ratio $(\omega_c/\Omega_q)^2$,

$$s^2\kappa_{2\pm}^2\approx\Omega_q^2+\tfrac{1}{9}[(3\Omega_q^2+\omega_c^2)^{1/2}\mp 2|\omega_c|]^2\ . \tag{33}$$

As in the corresponding 3D case, κ_{2-} increases with increasing field and ultimately produces a screening length proportional to B^{-1}. The quantity C associated with the smaller screening constant is more complicated. For the $-$ mode, it is proportional to the positive-definite quantity $(\omega_c^2+3\Omega_q^2)^{1/2}+2|\omega_c|$, so that the two independent long-wavelength solutions of Eq. (26) indeed represent decaying exponentials. Direct substitution confirms that the $-$ mode indeed satisfies Eq. (30) for all magnetic fields. For the $+$ mode, however, C_+^2 is proportional to

$$[(\omega_c^2+3\Omega_q^2)^{1/2}-2|\omega_c|]^2\ ,$$

which vanishes at $(\omega_c/\Omega_q)^2=1$; thus C_+ has a discontinuous first derivative at $\omega_c=\Omega_q$. For this field, the long-wavelength screening constant κ_{1+} vanishes, presumably because the mode merges with the continuum. For larger values of the magnetic field, it is not difficult to show that the + mode (32a) ceases to satisfy the original dispersion relation in Eq. (30), even though it is a root of Eq. (31). This latter behavior is similar to that in the 3D case for finite q [see the discussion below Eq. (14)]; a more accurate solution for the 2D edge mode will be needed to decide whether this behavior arises from the present approximations.

It is important to keep in mind that these solutions are not exact, even though they do provide a very satisfactory fit to the preliminary experimental data.[2] In this connection it is interesting to analyze the electron's trajectory for the edge modes [compare Eq. (13) for the 3D case]. For $q \approx 0$ the same general form remains valid, but there are additional factors of

$$(C\omega+\omega_c)(\omega_c^2-\omega^2)^{-1} .$$

In 3D the quantity C is identically 1, so that the corresponding ratio is finite for $\omega=-\omega_c$; in the present 2D case, however, this cancelation fails to occur, and the amplitudes of the motion become resonant. This behavior arises from the additional root of Eq. (30), and it most probably would not occur in an exact 2D solution. This question will require further work.

IV. SCREENING BY GROUNDED PLANES

In practical experimental studies of electrons on the surface of liquid He,[1–3,10] the charges are confined by electrostatic fields instead of a uniform rigid positive background. As a first step toward a more realistic description, this section considers the effect of the grounded conducting planes located above and below the electron fluid, but I still assume that the system is neutralized by a positive background. In this case the plasma frequency for an unbounded charged layer differs from that in Eq. (18) by a geometrical screening factor. For simplicity I assume that the 2D electron fluid is symmetrically placed a distance h from the top and bottom grounded planes, and the square of the bulk 2D plasma frequency is then given by[13,14]

$$\Omega_q^2=\frac{4\pi n_0 e^2 q}{m(1+\epsilon)}f(q) , \qquad (34a)$$

where ϵ is the dielectric constant of the liquid He, and $f(q)$ is the screening function

$$f(q)=\tanh(qh) . \qquad (34b)$$

Note that f lies between 0 and 1. If the planes are far away compared to the wavelength ($qh \gg 1$), then Ω_q reproduces that in Eq. (18a) apart from the trivial correction for the dielectric constant: in contrast, if the planes are nearby ($qh \ll 1$), the 2D plasma dispersion relation reduces to that for a compressional wave, with a propagation speed

$$[4\pi n_0 e^2 h/m(1+\epsilon)]^{1/2} .$$

This latter limit is that used by Glattli *et al.*,[3] where they essentially took $qh \to 0$. Although this limit is indeed appropriate for long-wavelength magnetoplasmons, it fails to treat the short-wavelength behavior correctly. This section provides a simple model for the edge magnetoplasmons that interpolates between the two limiting cases. As mentioned below, this same approach has been used by Wu *et al.*[15] in the slightly different case of edge magnetoplasmons in a layered electron fluid.

The only effect of the grounded planes is on Poisson's equation (15), which now satisfies the boundary conditions that the potential vanish at $z=\pm h$ (and an additional modification because of the dielectric constant of the liquid He). Equation (18b) remains correct for the magnetoplasmons in an unbounded 2D electron fluid if Eq. (34) is used for Ω_q. In the case of a half-plane, Eq. (22a) again holds, but the detailed form of the integral kernel is modified. In particular, the exact kernel has the Fourier transform

$$\bar{L}(k)=\frac{f((k^2+q^2)^{1/2})}{(k^2+q^2)^{1/2}(1+\epsilon)} , \qquad (35)$$

where the screening function has the form given in Eq. (34b). Evidently, an exact solution of this problem is no easier than before, except for $h \to 0$, when $\bar{L}(k)$ reduces to a constant, independent of k. In that special limit, treated from a different viewpoint in Ref. 3, the inverse transform $L(x)$ becomes a δ function.

To study the behavior for general values of qh, the approximation method used in Sec. III remains applicable[15] and will be seen to work with equal ease. Specifically, $\bar{L}(k)$ will be approximated by a function with simple poles in the k plane that reproduces the first two nonzero terms in the Taylor series about the origin:

$$\bar{L}_0(k)=\frac{2qf(q)}{(1+\epsilon)[2q^2+k^2 g(q)]} . \qquad (36)$$

Here, $g(q)$ is another screening function, given generally by

$$g(q)=1-(q/f)(df/dq) , \qquad (37a)$$

where $f(q)$ characterizes the screening correction for bulk plasmons [see Eq. (34)]. In the present case, an easy calculation shows that

$$g(q)=1-2qh/\sinh(2qh) , \qquad (37b)$$

so that g also lies between 0 and 1, with the limiting behavior

$$g(q)\approx\begin{cases}1, & qh \gg 1\\ \frac{2}{3}q^2h^2, & qh \ll 1 .\end{cases} \qquad (37c)$$

As in Sec. III, Eq. (36) is readily inverted to find the approximate kernel in position space,

$$L_0(x)=f(1+\epsilon)^{-1}(2g)^{-1/2}\exp[-q(2/g)^{1/2}|x|] , \qquad (38)$$

which generalizes Eq. (23b). Once again, this function is essentially a Green's function, and the problem can be reduced to a pair of ordinary differential equations for the potential in the two regions $x<0$ and $x>0$,

$$(\partial^2-2q^2/g)\Phi_<(x)=8\pi eq(1+\epsilon)^{-1}fg^{-1}n(x) \quad (x<0) , \qquad (39a)$$

$$(\partial^2-2q^2/g)\Phi_>(x)=0 \quad (x>0) \,. \tag{39b}$$

The remaining steps in the solution are identical with those in Sec. III. For $x<0$ there are again two decaying exponential solutions, obtained from the roots of the equation [compare Eq. (26)]

$$\kappa^4-[(2+g)g^{-1}q^2+2\Omega_q^2/s^2g+s^{-2}(\omega_c^2-\omega^2)]\kappa^2 +(2q^2/g)[q^2+s^{-2}(\Omega_q^2+\omega_c^2-\omega^2)]=0 \,, \tag{40}$$

where damping has been neglected and Ω_q is now the screened plasma frequency given in Eq. (34). With minor modifications, Eq. (27) again describes the approximate dispersion relation of the edge magnetoplasma modes for general wave vectors, and it is not difficult to extract the "long-wavelength" behavior ($qs \ll \Omega_q$) by taking the limit $s \to 0$. This procedure gives the desired dispersion relation

$$(2+g)\omega^2-2(2g)^{1/2}\omega\omega_c-2\Omega_q^2=0 \,, \tag{41}$$

which reproduces Eq. (31) in the unscreened limit ($g=1$).

Equation (41) has several interesting features. In the absence of a magnetic field, it predicts an edge plasmon with frequency

$$\omega=\pm\left[\frac{2}{2+g}\right]^{1/2}\Omega_q \tag{42}$$

that lies below the bulk value for any nonzero g. In the limit $g=0$, however, this edge plasmon merges with the bulk 2D plasmons, in agreement with the predictions of the model used in Ref. 3. For a given geometrical configuration, the edge plasmons with shorter wavelengths lie increasingly below the bulk continuum, approaching the unscreened limit of $\pm(\frac{2}{3})^{1/2}\Omega_q$ as $qh\to\infty$ and $g\to 1$. In the presence of a magnetic field, Eq. (41) has two distinct solutions,

$$\omega_+=\mathrm{sgn}\omega_c\sqrt{2}(2+g)^{-1}\{[(2+g)\Omega_q^2+g\omega_c^2]^{1/2} +g^{1/2}|\omega_c|\} \,, \tag{43a}$$

$$\omega_-=-\mathrm{sgn}\omega_c\sqrt{2}(2+g)^{-1}\{[(2+g)\Omega_q^2+g\omega_c^2]^{1/2} -g^{1/2}|\omega_c|\} \,, \tag{43b}$$

which clearly reduce to Eq. (32) in the unscreened limit. A detailed study shows that the $-$ mode is a true edge wave for all values of the magnetic field, with the amplitude decaying exponentially toward the interior. In contrast, the $+$ mode is an edge wave only for a limited range of low magnetic fields, and it merges with the bulk continuum modes at a critical field specified by the condition

$$|\omega_c|=\left[\frac{g}{2-g}\right]^{1/2}\Omega_q \,, \tag{44}$$

where the decay constant κ_{1+} vanishes. This critical value reproduces that found previously [see the discussion below Eq. (33)] in the unscreened limit ($g=1$), and it has the striking feature that it vanishes for $g\to 0$. This last behavior is consistent with the conclusion of Ref. 3 that only a single edge magnetoplasma mode occurs in the fully screened limit ($qh\approx 0$). Figure 1 illustrates these two modes in the particular cases of (a) $qh=0.5$ and (b) $qh=2$, which correspond, respectively, to $g=0.15$ and 0.85. Note the limited range of the upper ($+$) mode in these two cases.

This method of including the effect of the grounded planes is similar to that of Wu *et al.*,[15] who consider a layered electron fluid with separation a between adjacent layers, placed in a medium with dielectric constant ϵ. In an unbounded system, the bulk plasmon is again given by Eq. (34a), but with a different screening function given by[9,15]

$$f(q)=\frac{\sinh(qa)}{\cosh(qa)-\cos(q_za)} \,, \tag{45}$$

and with the factor $1+\epsilon$ replaced by 2ϵ. Here, q_z denotes the wave number for propagation perpendicular to the layers. The formalism for the edge magnetoplasmons remains valid, with a modified screening function g. In the particular case of a wave propagating strictly in the y direction ($q_z=0$), it has the simple form $g=1+qa/\sinh(qa)$, which now lies between 1 and 2. For $qa \gg 1$ (widely separated layers), the results reduce to those of Sec. III for a single layer; in the opposite limit ($qa \ll 1$), they reproduce the conclusions of Sec. II for the 3D surface magnetoplasmons,[15] where both the $+$ and $-$ modes were true bound surface waves in the long-wavelength limit [compare Eq. (44) as $g\to 2$]. Curve (c) of Fig. 1 illustrates this behavior, giving the edge magnetoplasmons for a layered electron gas with the typical values $qa=1$, $q_za=0$, and $g=1.85$.

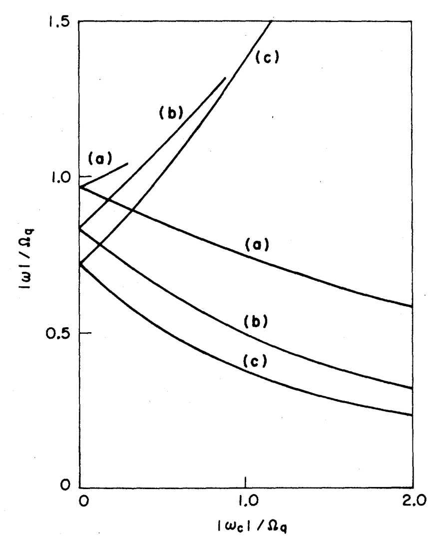

FIG. 1. Magnetic field dependence of $+$ (upper) and $-$ (lower) edge magnetoplasmons, including various screening corrections. Curves (a) and (b) refer to a single 2D charged layer with grounded planes a distance h away, with $qh=0.5$ and 2, respectively. In each case, the termination of the $+$ mode is apparent. Curve (c) describes a layered electron fluid, with $q_za=0$ and $qa=1$. The latter upper curve terminates at $|\omega_c|/\Omega_q\approx 3.51$.

V. DISCUSSION

The present theoretical approach raises several questions. First, is the basic approximation reasonable, and can it be improved? In principle, this problem can be solved exactly for a half-plane, as discussed in the Appendix. Alternatively, the question can be treated by expanding in the difference between the exact and approximate kernels; the first correction would give an estimate of the approximation's validity.

A second concern is the assumption of a rigid uniform positive background charge, when, in fact, the electrons are held in place by applied electrostatic fields. The authors of Ref. 3 have considered this effect, particularly the question of a nonuniform unperturbed electron density $n_0(\mathbf{r})$. Another relevant practical point is the role of a finite geometry, where charge conservation may be more important than in the present semi-infinite configuration.

At present there appear to be some differences between the models of Refs. 2 and 3; for example, I find a localized 2D edge plasmon even in the absence of a magnetic field (as in 3D), whereas Glattli *et al.*[3] apparently ascribe the localization to the presence of a field. The discussion in the preceding section indicates that a magnetic field is necessary for the localization only in the limit of perfect screening ($qh=0$), since otherwise the edge plasmon lies below the bulk value Ω_q even in zero field. In addition, the authors of Ref. 3 suggest an analogy between the edge magnetoplasmon and the Rayleigh wave on the surface of an elastic continuum. This latter bound surface wave involves both longitudinal and transverse displacements,[16] because neither alone can satisfy the free-surface boundary conditions. In contrast, the transverse component of the localized surface or edge plasmon varies with the external field and vanishes if $B=0$. Thus the detailed dynamical motions of the two waves must differ considerably.

Another generalization of interest is the possibility of edge modes on a 2D Wigner lattice of electrons.[3] This system has a nonzero shear modulus, which should affect the dynamics of the magnetoplasmons, particularly the transverse component of the motion.

In conclusion, this paper has presented an approximate solution for the magnetoplasma modes of a 2D electron fluid confined to a half-plane, stimulated by recent studies of electrons on the surface of liquid He.[1,2] In addition to the propagating waves that are analogous to the modes of an unbounded system, two new edge modes have been found whose amplitudes decay exponentially away from the boundary. These new modes are very similar to the surface magnetoplasmons found on a 3D electron fluid confined to a half-space. They provide a good qualitative fit to the anomalous modes observed in Refs. 1 and 2. The present approximation methods also suffice for considering the screening effects of grounded planes above and below the 2D electron fluid. The same approximations should prove useful in other contexts.

ACKNOWLEDGMENTS

This work was the result of many conversations with A. J. Dahm and D. B. Mast, and I am grateful for their valuable comments and suggestions. It has also benefited from helpful discussions with E. Y. Andrei. The research has been supported in part by the National Science Foundation through Grants No. DMR-81-18386 and No. DMR-84-18865.

APPENDIX

In this appendix I sketch the application of the Wiener-Hopf technique to the problem of surface and edge magnetoplasmons on a 3D or 2D half-space.[11] The basic observation is that if a function vanishes for $x>0$ [such as the electron density $n(x)$ or the components of its velocity], its Fourier transform

$$\bar{n}_+(k)=\int_{-\infty}^{0} dx\, e^{-ikx}n(x) \tag{A1}$$

is analytic in the upper half complex k plane. Since the potential $\Phi(x)$ can be decomposed into the sum of two parts that vanish separately for positive and negative x, the corresponding Fourier transforms $\overline{\Phi}_+(k)$ and $\overline{\Phi}_-(k)$ are analytic in the upper and lower half k planes, respectively.

For both the 2D and the 3D configurations, Eqs. (5a)—(5c) specify the dynamical equations of motion. In contrast to the usual case of an unbounded domain, however, the present semi-infinite geometry complicates the situation. For example, the Fourier transform of dn/dx is $ik\bar{n}_+(k)+n(0)$, where the last term is the electron density at the edge of the half-space ($x=0$). Thus the Fourier transforms of Eqs. (5a)—(5c) contain inhomogeneous terms proportional to $n(0)$ and $\Phi(0)$, as well as the transformed quantities $\bar{n}_+(k)$ and $\bar{\mathbf{v}}_+(k)$ [note that v_x vanishes at $x=0$ so that $v_x(0)$ does not appear]. Elimination of $\bar{\mathbf{v}}_+(k)$ yields the following inhomogeneous relation between $\bar{n}_+(k)$ and $\overline{\Phi}_+(k)$,

$$(k^2+\mu^2)\bar{n}_+(k)-(n_0e/ms^2)(k^2+q^2)\overline{\Phi}_+(k) = i(k+iq\omega_c/\tilde{\omega})[n(0)-(n_0e/ms^2)\Phi(0)]\ , \tag{A2}$$

where

$$\mu^2=q^2+(\omega/s^2\tilde{\omega})(\omega_c^2-\tilde{\omega}^2) \tag{A3}$$

is independent of the Fourier variable k.

The other basic equation is the Fourier transform of Eq. (22a), which becomes

$$\overline{\Phi}_+(k)+\overline{\Phi}_-(k)+4\pi e\bar{L}(k)\bar{n}_+(k)=0\ . \tag{A4}$$

Here, $\bar{L}(k)$ is the appropriate kernel function $[=\frac{1}{2}(k^2+q^2)^{-1/2}$ for the 2D case and $(k^2+q^2)^{-1}$ for the 3D case]. A combination of Eqs. (A2) and (A4) leads to the single equation

$$(k^2+\mu^2)\overline{\Phi}_-(k)+G(k)\overline{\Phi}_+(k)=(k+iq\omega_c/\tilde{\omega})A\bar{L}(k)\ , \tag{A5}$$

where A is a known constant, independent of k, and

$$G(k)=k^2+\mu^2+(k^2+q^2)\bar{L}(k)(4\pi n_0e^2/ms^2)\ . \tag{A6}$$

Although $G(k)$ does not, in general, have simple analytic properties, standard but intricate techniques[11] permit its factorization into the product $G_+(k)G_-(k)$, where the first (second) factor is analytic in the upper (lower) half k plane. Elementary manipulation then gives

$$\frac{k-i\mu}{G_-}\overline{\Phi}_- + \frac{G_+}{k+i\mu}\overline{\Phi}_+ = \frac{k+iq\omega_c/\widetilde{\omega}}{(k+i\mu)G_-}A\overline{L} \ , \quad \text{(A7)}$$

where the two terms on the left-hand side have simple and obvious analyticity. The same techniques allow the right-hand side to be decomposed additively in the form $P_+(k)+P_-(k)$, and a slight rearrangement gives

$$\frac{k-i\mu}{G_-}\overline{\Phi}_- - P_- = -\frac{G_+}{k+i\mu}\overline{\Phi}_+ + P_+ \ . \quad \text{(A8)}$$

The usual arguments of the Wiener-Hopf technique[11] imply that each side is an entire function, which is taken to be zero. Thus, each of the functions $\overline{\Phi}_+(k)$ and $\overline{\Phi}_-(k)$ is determined, with the proper analyticity. Substitution back into Eq. (A2) finally yields the following expression for $\overline{n}_+(k)$:

$$\begin{aligned}\overline{n}_+(k) = (k^2+\mu^2)^{-1}\{&(n_0e/ms^2)(k^2+q^2)\overline{\Phi}_+(k)\\ &+i(k+iq\omega_c/\widetilde{\omega})\\ &\times[n(0)-(n_0e/ms^2)\Phi(0)]\} \ .\end{aligned} \quad \text{(A9)}$$

This function is analytic in the upper half k plane and cannot be singular at $k=i\mu$. Thus the quantity in curly braces must vanish at that point, which determines the dispersion relation for the edge or surface magnetoplasmon:

$$\frac{n_0e}{ms^2}(q^2-\mu^2)\overline{\Phi}_+(i\mu) = \left[\mu+\frac{q\omega_c}{\widetilde{\omega}}\right]\left[n(0)-\frac{n_0e}{ms^2}\Phi(0)\right] \ . \quad \text{(A10)}$$

In three dimensions this procedure is readily implemented, since the function G is linear in k^2; straightforward algebra then reproduces Eq. (10) obtained previously by direct means. For two dimensions, in contrast, the corresponding $G(k)$ has branch points at $\pm iq$, and the subsequent decompositions are very complicated. Nevertheless, the problem is thus solved in principle. It would be interesting to carry out the numerical analysis in detail; comparison with the approximate solution obtained in Sec. III would clarify the replacement of the exact kernel by an approximate one with simpler analytic structure.

It is evident that this exact method can also treat the screening corrections of Sec. IV. In that case, the exact kernel function $\overline{L}(k)$ in Eq. (35) has an infinite set of simple poles at the points given by

$$k^2 = -q^2-(\pi/2h)^2(2n+1)^2$$

for $n=0,1,\ldots$. These merge to form branch points in the limit $qh\to\infty$, but the behavior for general qh would require an additional investigation.

[1]D. B. Mast and A. J. Dahm, Physica **126B+C**, 457 (1984).

[2]D. B. Mast, A. J. Dahm, and A. L. Fetter, Phys. Rev. Lett. **54**, 1706 (1985).

[3]D. C. Glattli, E. Y. Andrei, G. Deville, J. Poitrenaud, and F. I. B. Williams, Phys. Rev. Lett. **54**, 1710 (1985).

[4]See, for example, G. Barton, Rep. Prog. Phys. **42**, 65 (1979); A. D. Boardman, in *Electromagnetic Surface Modes,* edited by A. D. Boardman (Wiley, New York, 1982), pp. 1—76.

[5]K. N. Stepanov, Zh. Tekh. Fiz. **35**, 1349 (1965) [Sov. Phys.—Tech. Phys. **10**, 1048 (1966)]; K. W. Chiu and J. J. Quinn, Phys. Rev. B **5**, 4707 (1972); G. C. Aers, A. D. Boardman, and E. D. Isaac, IEEE Trans. Plasma Sci. **PS-5**, 123 (1977).

[6]The more general case of 3D surface magnetoplasma polaritons has been studied by G. C. Aers and A. D. Boardman [J. Phys. C **11**, 945 (1978)] and R. F. Wallis (in *Electromagnetic Surface Modes,* Ref. 4, pp. 575—631).

[7]J. J. Brion, R. F. Wallis, A. Hartstein, and E. Burstein, Surf. Sci. **34**, 73 (1973). The corresponding experiments are reported in A. Hartstein and E. Burstein, Solid State Commun. **14**, 1223 (1974).

[8]F. Stern, Phys. Rev. Lett. **18**, 546 (1967).

[9]A. L. Fetter, Ann. Phys. (N.Y.) **81**, 367 (1973); **88**, 1 (1974).

[10]C. C. Grimes and G. Adams, Phys. Rev. Lett. **36**, 145 (1976).

[11]G. F. Carrier, M. Krook, and C. E. Pearson, *Functions of a Complex Variable* (McGraw-Hill, New York, 1966), Secs. 8-1—8-4.

[12]N. J. M. Horing and M. Yildiz, Phys. Lett. **44A**, 386 (1973). See also Sec. VI C of T. Ando, A. B. Fowler, and F. Stern, Rev. Mod. Phys. **54**, 437 (1982). The more general case of a 2D magnetoplasmon with retardation is considered in M. Orman, N. J. M. Horing, and M. L. Glasser, in *Proceedings of the Seventh International Vacuum Congress,* edited by R. Dobrozemsky *et al.* (Berger, Vienna, 1977), p. 545.

[13]A. V. Chaplik, Zh. Eksp. Teor. Fiz. **62**, 746 (1972) [Sov. Phys.—JETP **35**, 395 (1972)].

[14]C. C. Grimes and G. Adams, Surf. Sci. **58**, 292 (1976).

[15]J.-W. Wu, P. Hawrylak, and J. J. Quinn, Phys. Rev. Lett. **55**, 879 (1985).

[16]L. D. Landau and E. M. Lifshitz, *Theory of Elasticity* (Pergamon, Oxford, 1959), Sec. 24.