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Measurement of the conductivity exponent in two-dimensional percolating networks: Square lattice versus random-void continuum

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Using an analog simulation technique, we have studied the conductivity transition in two-dimensional percolating networks. A computer-controlled x-y plotter scribes a percolating pattern on a sheet of aluminized plastic while the resistance of the sheet is continuously monitored. With this technique, we have measured the conductivity exponent t for two systems: site percolation on a square lattice and random-void continuum percolation. We find $t = 1.29 \pm 0.03$ for the lattice and $t = 1.34 \pm 0.07$ for the continuum, in agreement with a recent theoretical prediction that the conductivity exponents for these two systems are the same. We have also verified a theoretical estimate of the magnitude of conductance fluctuations due to finite-size effects.

Near the percolation threshold, the conductance G of a random metal-insulator composite obeys the power law, $G \propto (p - p_c)^t$, where p is the volume fraction of the metal component, p_c is the critical volume fraction, and t is the conductivity exponent. Until recently, the exponent t was believed to be universal in the sense that it depends only on the dimensionality of the system and not on its small-scale details. In particular, t was presumed to be the same for lattice and continuum systems. Experimental work,[1] as well as numerical simulation,[2] indicates that at least some two-dimensional (2D) continuum systems do indeed have the same exponent t as lattice systems. Experimental work[3-5] on 3D continuum systems, however, often results in values for t higher than those obtained from numerical simulation of lattice networks.[6-9]

Recently, Halperin, Feng, and Sen[10] (HFS) have estimated critical exponents for electrical conductivity, elastic constants, and fluid permeability in a particular class of continuum systems, the random-void models (also called the "Swiss-cheese" models) in which insulating holes are located randomly in a conducting background. They find exponents larger than those for lattice networks *except* for the case of electrical conductivity in 2D, where the lattice and continuum exponents are equal.

The value of the conductivity exponent for a random-void system is sensitive to the distribution of the conductances of the constrictions between voids. This distribution of bond strengths depends on the geometry of the constrictions and hence on the shape of the voids. If δ is the minimum width of the constriction between voids and g is the conductance of the constriction, then[10,11] $g \propto \delta^{1/2}$ for circular holes in 2D, $g \propto \delta$ for parallel-oriented square holes in 2D, and $g \propto \delta^{3/2}$ for spherical holes in 3D. More generally, the random-void models can be classified by the exponent m in the power law $g \propto \delta^m$. HFS and others[11-13] have argued that the random-void conductivity exponent t' and the corresponding lattice exponent t are related by $t' = t$ for $m < 1$ and $t' \approx t + m - 1$ for $m > 1$. The case $m = 1$, realized by square holes in 2D, is thus a borderline case. (In 2D there appears to be no void shape which corresponds to the interesting case $m > 1$.)

Using a simple new experimental technique, we have studied the dc electrical conductivity transition in two-dimensional percolating networks of metal and insulator. We report measurements of conductance versus composition for site percolation on a square lattice and for a random-void model of continuum percolation in which square holes with parallel edges are located randomly (see Fig. 1). We find that the conductivity exponents for these two systems are identical, as predicted by HFS. We have also verified an estimate, due to Straley,[14] of the magnitude of conductance fluctuations in the square-lattice system due to finite sample size.

Using a computer-controlled digital x-y plotter, we cut the percolating patterns in sheets of aluminized plastic. As a pattern is cut, a digital ohmmeter continuously monitors the resistance of the sheet, and the computer records the resistance as a function of the number of holes cut. The entire curve of conductance versus number of holes is obtained in the time it takes for the plotter to draw a percolating pattern (10 h for a 200×200 lattice). With this technique, different percolation problems can be studied with only a programming change. Because the patterns are computer generated, the samples are well characterized with none of the ambiguities sometimes inherent in experimental studies (poorly known volume fraction and particle size distribution, possible nonrandomness, etc.).

The sample sheets are 0.005 in. thick and are made of Mylar plastic covered with a 500-Å film of aluminum.[15] The sheet resistance of the samples is about 1.9 Ω/□ and varies by less than 0.3% over a 25×25 cm^2 sheet.

The pattern is cut with a little device consisting of a steel ball bearing embedded in the tip of an aluminum rod which contains a heating element. This hot tip is held by the penholder of the plotter, and when it touches the sheet it sears the plastic, breaking the aluminum film on top. A thin piece of cloth, placed between the sample sheet and the bed of the plotter, makes the surface of the sheet slightly flexible. We find that a smooth tip and a slightly springy surface are necessary in order for the hot tip to draw a smooth, continuous line without skipping or chattering. Drawing a square with the hot tip electrically disconnects the interior of the square from the rest of the sheet, effectively removing the entire square from the sample. Repeated tests of the reliability of complete disconnection show failure rates of less than 1 square in 10 000.

Two copper strips, clamped on opposite edges of the sheet, form the contacts for a two-probe resistance measure-

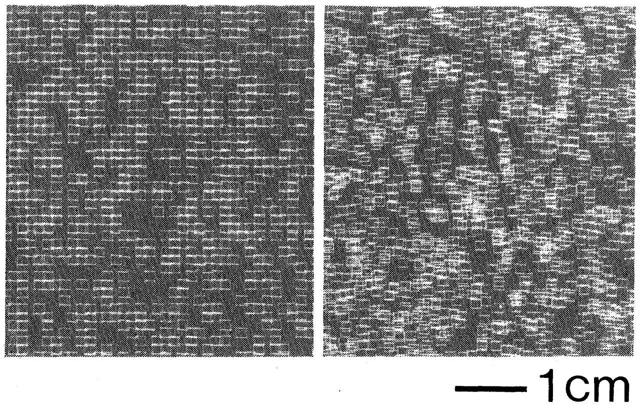

FIG. 1. Photographs of portions of sample sheets, showing the square-lattice pattern (on the left) and the random-void continuum pattern (right). The full sheets are 25×25 cm^2. Both patterns are shown at the critical hole concentration.

ment. After a correction for the lead resistance is made, the dominant source of error in the measurements is drift in the ambient temperature, resulting in errors not exceeding 0.1%.

In Fig. 1 are photographs of samples showing the two percolating patterns. The square sites of the square lattice are intentionally made slightly oversized to guarantee corner contact. The uncut sites thus form a percolating network with first-nearest-neighbor contact only. The program that generates the square-lattice pattern locates new sites at random but does not repeat previously drawn sites. In the continuum case, square sites are located at random with no underlying background lattice,[16] independently of previously drawn sites. The sites are thus freely interpenetrating.

Figure 2 shows conductance data for nine samples of a 200×200 square lattice. G is the normalized conductance ($G=1$ for the uncut sheet), and N is the number of square holes cut. In plotting our data, we have used the value of the critical hole number N_c known from Monte Carlo studies.[17] The critical hole fraction p_c established by these studies is 0.407 23, yielding $N_c=p_c\times200^2=16\,289$. Our measured critical hole fraction is 0.4069 ± 0.0025 (the uncertainty here is one standard deviation of the mean of the nine trials) and agrees with the known value.

Shown as the dashed vertical line in Fig. 2 is the hole fraction at which $\xi=L/2$, where ξ is the correlation length and L is the size of the system. In units of a lattice constant, L is 200 and ξ is taken[14] to be $[N_c/(N_c-N)]^{4/3}$. We find that the conductance obeys a power law over the range bounded by the conditions $\xi=L/2$ and $(N_c-N)/N_c=0.5$. At hole fractions closer to threshold, sample-to-sample fluctations due to finite sample size are considerable, while for $(N_c-N)/N_c>0.5$ a small deviation[18] from power-law behavior becomes measurable, though not evident in Fig. 2. A least-squares fit of the data in the power-law range to the expression $G\propto(N_c-N)^t$ yields $t=1.29\pm0.03$, the uncertainty being one standard deviation of the mean of the nine trials. N_c is not an adjustable parameter in this fit, but is fixed at 16 289. Our value of t agrees with the value established by numerical simulations,[19] $t=1.297\pm0.005$. The agreement between our results and those obtained by computer simulation gives us confidence that our technique contains no hidden systematic errors.

Straley[14] has estimated the magnitude of sample-to-sample fluctuations in the conductance G of a square-lattice system due to finite sample size. He finds that $\langle\delta G^2\rangle/\langle G\rangle^2=(\xi/L)^d$ where $\langle\delta G^2\rangle$ is the mean-square deviation of G from the mean and d is the dimensionality of the system. In Fig. 2 we have plotted $\langle G\rangle\pm\langle\delta G^2\rangle^{1/2}$ versus hole fraction using $d=2$, $L=200$, and $\xi=[N_c/(N_c-N)]^{4/3}$, and we find good agreement between our data and Straley's estimate.

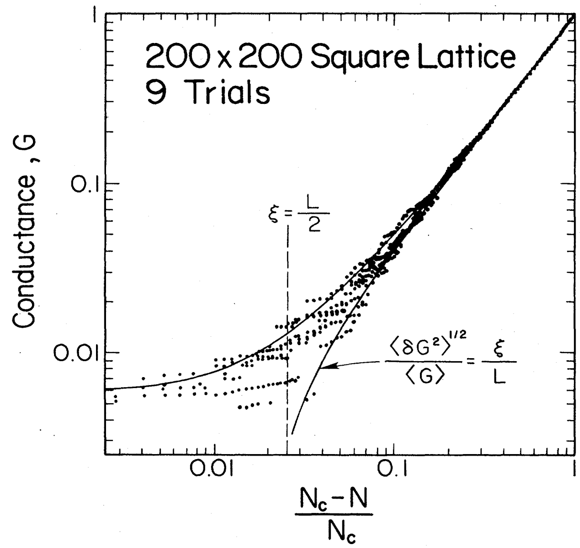

FIG. 2. Conductance vs hole-fraction data for nine trials of a 200×200 square lattice. N is the number of holes and N_c ($=16\,289$) is the critical hole number. The solid curves show Straley's estimate of the size of conductance fluctuations.

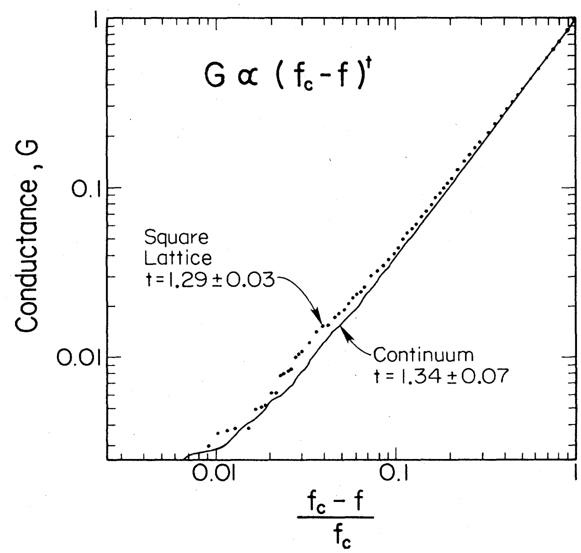

FIG. 3. Conductance data for the square-lattice system and the random-void continuum system. The data are the averages of nine trials. f is the hole fraction (fraction of sample area covered by holes), and f_c is the critical hole fraction.

Figure 3 compares conductance data for the random-void continuum and square-lattice systems. Each curve shown is the average of nine trials. For the continuum system, $L/l = 86 \pm 1$, where L is the sample length (25 cm) and l is the length of a single site; the mean critical hole number is $28\,440 \pm 230$. The data are plotted as a function of the hole fraction f defined as the fraction of the sample area covered by holes. For the square lattice, $f \propto N$, but for the continuum case, in which the squares are freely interpenetrating, f and N are related by[20]

$$f(N) = 1 - \exp(-Na/A) \ ,$$

where a is the area of each site and A is the area of the sample. For our continuum system, f_c, the critical hole fraction, is 0.613 ± 0.013, the uncertainty arising from the spread in the measured values of N_c, as well as from the uncertainty in the measured value of a/A.

A fit of the continuum data to $G \propto (f_c - f)^t$ yields $t = 1.34 \pm 0.07$. The uncertainty here is larger than in the lattice case because of the relatively large uncertainty in f_c. We see that, within experimental uncertainties, the lattice and the random-void continuum systems share the same exponent t.

We note that in both systems the critical regime over which power-law behavior is observed extends over nearly the whole curve. This wide critical regime has been seen before in other percolation systems,[1,2,4,21] but we know of no theoretical explanation for its existence.[22]

In summary, we have used an automated analog simulation technique to study the electrical conductivity transition in 2D percolating networks. We have verified the recent prediction of Halperin, Feng, and Sen that the square-lattice system and the 2D random-void continuum system share the same conductivity exponent, and we have verified Straley's expression for conductance fluctuations due to finite sample size.

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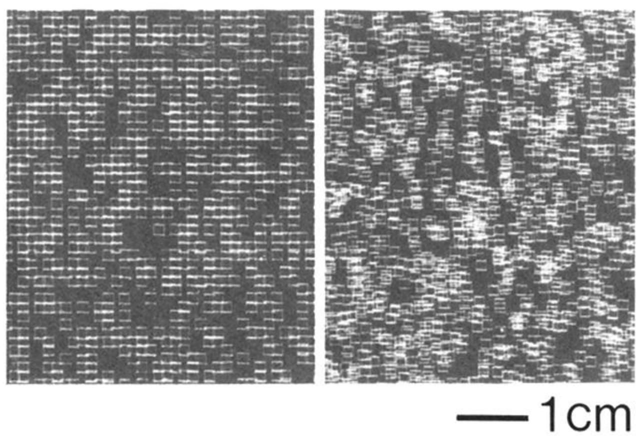

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