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Josephson phase transition

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A different theoretical approach gives the usual result for the Josephson current and its temperature dependence, and reveals that the coupling may be destroyed by a magnetic field. The calculated critical field agrees with new observations on cylindrical junctions. The theory predicts that this phase transition can be observed in narrow ($\sim 10\ \mu$m) flat junctions as well.

The Josephson current density is given by $J_0 \sin\gamma$. The phase difference between wave functions on the two sides, γ, varies spatially in the presence of magnetic fields and temporally in the presence of electric fields. These variations of γ account for all of the usual Josephson effects. Experiments on cylindrical junctions have shown that J_0 decreases with magnetic field so that the coupling is destroyed by a critical field B_s.[1,2] Measurements of the energy-gap parameters by quasiparticle tunneling showed that neither gap Δ_1 nor Δ_2 was measurably affected by B_s.[1] No theoretical explanation has been given for this phase transition in a magnetic field.

In this Rapid Communication, we present a simple theory of Josephson coupling, which yields the relation $J_0 \sin\gamma$, the temperature dependence of J_0, and the magnetic field induced phase transition observed in cylindrical junctions. This phase transition should also be observable in narrow ($\leq 10\ \mu$m), flat, singly connected junctions.

Let a planar superconductor lie in the y-z plane between $x=0$ and $x=d$. Let the space $x<0$ and $x>d$ be the insulator. Due to interaction with the insulator, the superconducting order parameter $\psi(x)$ is depressed at the interfaces. This depression is small, so $\psi(x)$ may be calculated from the linearized Ginsburg-Landau (GL) differential equation, $d^2\psi/dx^2=(2/\xi^2)\psi$, with solutions $\propto \exp(\pm\sqrt{2}x/\xi)$ and the de Gennes[3] boundary conditions, $(d\psi/dx)_0=(a/\xi_0^2)\psi_0$; $(d\psi/dx)_d=-(a/\xi_0^2)\psi_0$. Here ξ_0 is the BCS coherence length, ξ is the temperature-dependent GL coherence length, and a is the effective distance to which electrons penetrate the insulator. The result for $\psi(x)$ is

$$\psi(x)=\psi_0-\frac{\xi a}{\sqrt{2}\xi_0^2}\psi_0\frac{\cosh(d/\sqrt{2}\xi-\sqrt{2}x/\xi)}{\sinh(d/\sqrt{2}\xi)} \quad . \tag{1}$$

Let $\psi_0(\xi a/\sqrt{2}\xi_0^2)=\Delta\psi_0$.

Let us coherently couple this superconductor to another superconductor at $x=0$ to form a Josephson junction. We require that at $x=0$,

$$\psi(0)=\psi_0-\Delta\psi_0\coth(d/\sqrt{2}\xi)+C_0\psi_{20}\exp i\gamma \quad .$$

Here, C_0 is a coupling constant and γ is the phase difference between the order parameters of the two superconductors. The order parameter is now

$$\psi(x)=\psi_0-\Delta\psi_0\frac{\cosh(d/\sqrt{2}\xi-\sqrt{2}x/\xi)}{\sinh(d/\sqrt{2}\xi)}+C_0\psi_{20}\frac{\cosh(\sqrt{2}d/\xi-\sqrt{2}x/\xi)}{\cosh(\sqrt{2}d/\xi)}e^{i\gamma} \quad . \tag{2}$$

The Gibbs function $G(0)$ in zero magnetic field can be obtained by inserting $\psi(x)$ into the GL expression:

$$g(0)=\alpha|\psi|^2+(\beta/2)|\psi|^4+|\alpha|\xi^2|d\psi/dx|^2 \quad .$$

Here, $\alpha/\beta=-|\psi|^2$ and $\alpha^2/2\beta=B_c^2/2\mu_0$ and B_c is the bulk critical field of the superconductor. By dropping terms of order C_0^2 and $C_0(\Delta\psi_0)^2$ and higher we obtain the Gibbs function per unit area

$$\frac{G(0)}{A}=-\frac{B_c^2}{2\mu_0}d-\frac{\alpha\xi}{\sqrt{2}}(\Delta\psi_0)^2\frac{\sinh(\sqrt{2}d/\xi)}{\sinh^2(d/\sqrt{2}\xi)}+\sqrt{2}\alpha\xi C_0\psi_{20}\Delta\psi_0\left[\frac{\sinh(3d/\sqrt{2}\xi)+\sinh(d/\sqrt{2}\xi)}{\sinh(d/\sqrt{2}\xi)\cosh(2d/\sqrt{\xi})}\right]\cos\gamma \quad . \tag{3}$$

The first term is the condensation energy of the superconductor. The second, which is positive, is caused by the interaction with the insulator and the third, which is negative, is due to the coupling. This term plus a similar one from the other superconductor is the Josephson coupling energy. For a thick superconductor, $d \gg \xi$, the bracketed factor equals 2.

In this limit, with $\Delta\psi_0=\psi_0(a\xi/\sqrt{2}\xi_0^2)$ and $\xi^2=\phi_0^2/8\pi^2\lambda^2B_c^2$ (Ref. 4), we obtain the coupling energy between two thick superconductors:

$$\frac{G_c(0)}{A}=-\frac{\phi_0^2 aC_0}{4\pi^2\mu_0\lambda_1\lambda_2}\left[\frac{1}{\xi_{01}^2}+\frac{1}{\xi_{02}^2}\right]\cos\gamma \quad . \tag{4}$$

Some results of the BCS theory are already incorporated into Eq. (4) through the de Gennes boundary condition. The BCS results for $|\psi_0|^2$, ξ, and the effective penetration depth λ can be inserted by use of the relation[4]

$$\lambda^2(0)/\lambda^2(T)=[\Delta(T)/\Delta(0)]\tanh\Delta(T)/2k_BT \tag{5}$$

and $\lambda^2=m/2\mu_0|\psi|^2e^2$. Reexpressing $G_c(0)/A$ in this way and setting it equal to the usual expression $G_c(0)/A=-(J_0\phi_0/2\pi)\cos\gamma$, we have for J_0

$$J_0(T)=J_0(0)\left[\frac{\Delta_1(T)\Delta_2(T)}{\Delta_1(0)\Delta_2(0)}\tanh\left[\frac{\Delta_1(T)}{2k_BT}\right]\tanh\left[\frac{\Delta_2(T)}{2k_BT}\right]\right]^{1/2},$$
$$J_0(0)=\frac{a\phi_0C_0}{2\pi\mu_0\lambda_1(0)\lambda_2(0)}\left[\frac{1}{\xi_{01}^2}+\frac{1}{\xi_{02}^2}\right] \quad . \tag{6}$$

For like superconductors this temperature dependence is identical to the Ambegaokar-Baratoff result.[5,6] For unlike superconductors Eq. (6), when plotted, is indistinguishable from the plot of their infinite-series expression.

We wish now to consider the magnetic field contribution to the Gibbs function. Since the fields in question, though larger than those usually considered in Josephson effect, are well less than the critical fields of the superconductors we may ignore changes in $|\psi|^2$ caused by them. This is supported by the measurements as mentioned earlier. Therefore, $B(x)$ and the current density $J(x)$ may be calculated from the London theory using the BCS value for the effective penetration depth λ. Further, the magnetic contribution to the Gibbs function can be calculated by integrating $(\frac{1}{2})\mu_0\lambda^2 J^2 + (1/2\mu_0)(B-B_0)^2$ over the volume. For a superconductor in a field B_0 applied parallel to its surface, the result is $G(B)-G(0)=(B_0^2/2\mu_0)(V-\lambda A)$, where V is the volume and A the surface area of the superconductor.

Now the origin of the extra magnetic energy due to the coupling is a slight coupling-induced decrease in λ. From the last term of Eq. (2) and $\lambda^2 \propto 1/|\psi|^2$, this decrease is

$$d\lambda_1 = -C_0(\lambda_1^2/\lambda_2)\left(\frac{\cosh(\sqrt{2}d/\xi - \sqrt{2}x/\xi)}{\cosh(\sqrt{2}d/\xi)}\right)\cos\gamma \ . \quad (7)$$

Consider two identical flat bulk superconductors coupled at $x=0$ [Fig. 1(a)]. Then at $x=0$, $d\lambda = -C_0\lambda\cos\gamma$ in both. If $\xi >> \lambda$ then $d\lambda$ can be considered constant over the region of field penetration and the extra magnetic contribution to G is $[G_c(B)-G_c(0)]/A = 2(B_0^2/2\mu_0)C_0\lambda\cos\gamma$. From this expression and Eq. (4) we obtain the coupling energy in a magnetic field ($\xi_{01}=\xi_{02}=\xi_0$):

$$(G_c/A) = \frac{B_0^2\lambda}{\mu_0} - \frac{\phi_0^2}{2\pi^2\mu_0\lambda^2\xi_0^2}C_0\cos\gamma \ . \quad (8)$$

At the critical Josephson field B_s, $G_c=0$:

$$B_s = \left(\frac{a}{2}\right)^{1/2}\frac{\phi_0}{\pi\xi_0}\lambda^{-3/2} \ . \quad (9)$$

This approximate result should be valid for pure, thick superconductors for which $\xi_0 >> \lambda(0)$. Examples are tin and aluminum. If the superconductors have short mean free paths as in alloys or thin films the result will be substantially different. With $a=0.2$ nm, $\xi_0=230$ nm, $\lambda(0)=32$ nm, which are the values for pure tin, Eq. (9) gives $B_s(0)=4.7$ mT. To observe this effect the junction width W must be so small that the interference effects do not hide the critical-field effect. This may be expressed $B_1=(\phi_0/2\lambda W) \geq B_s$. For the parameters used this gives $W \leq 26$ μm at $T=0$. For alloys or films, W must be even smaller.

Next we consider a cylindrical junction like that shown in Fig. 1(b). The junction is formed between a bulk cylinder and a thin film of identical material coaxial with it. The Josephson currents are radial and the applied field is axial. The approximations used for the flat junction are not appropriate because of the film and because the field is applied to the outside of the sample and decays inward. The coupling induced change in penetration depth for the film is given by Eq. (7) with $\cosh[\sqrt{2}/\xi(d-x)]$ replaced by $\cosh[\sqrt{2}/\xi(d+x)]$, as can be seen from Fig. 1(c). In the bulk superconductor $d\lambda = -C_0\lambda\exp(-\sqrt{2}x/\xi)\cos\gamma$. In both $d\lambda$ decays with distance from the junction and has the value $d\lambda = -C_0\lambda\cos\gamma$ at the junction. We approximate this spatial dependence by taking $d\lambda$ to be $-C_0\lambda\cos\gamma$ within $\pm\xi/\sqrt{2}$ of the junction and zero elsewhere. A straightforward but tedious calculation yields the additional magnetic energy to first order in $d\lambda$:

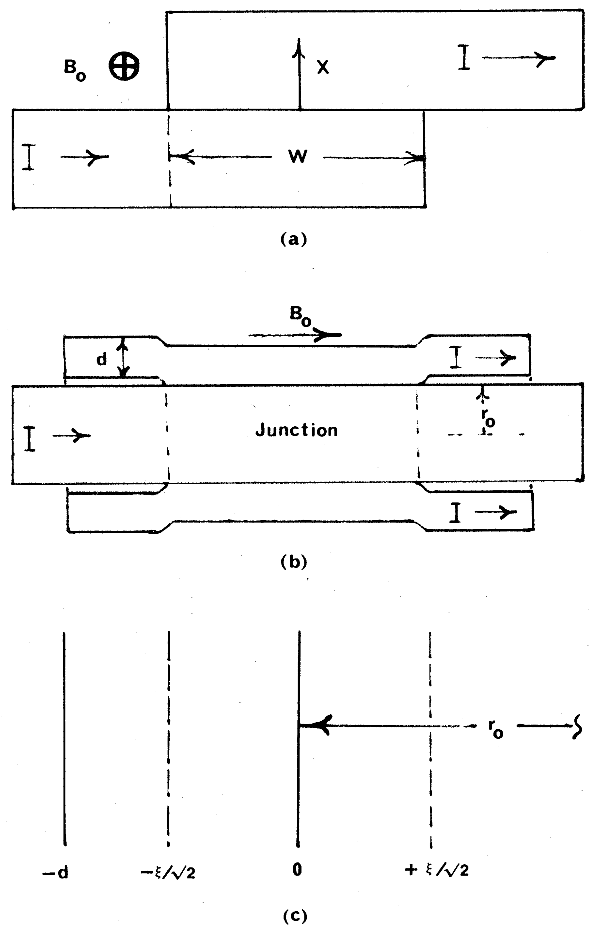

FIG. 1. Josephson junctions (a) flat, (b) cylindrical in a magnetic field. The approximation used to calculate $B(x)$ in the cylindrical junction is illustrated in (c).

$$[G_c(B)-G_c(0)]/A = \frac{B_0^2}{2\mu_0}e^{-2d/\lambda}f(\xi,d)C_0\cos\gamma \ ,$$
$$f(\xi,d) = \begin{cases}\sinh\sqrt{2}\xi/\lambda, & \xi/\sqrt{2} \geq d \ , \\ \exp 2d/\lambda - \exp(-\sqrt{2}\xi/\lambda), & \xi/\sqrt{2} \leq d \ . \end{cases} \quad (10)$$

The last term of Eq. (3) gives the film's contribution to the zero-field coupling energy. This same term with the bracketed expression replaced by 2 gives the contribution from the solid cylinder. When reexpressed as before and combined with Eq. (10) we have the coupling energy in an applied field B_0:

$$G_c/A = C_0\cos\gamma\left[-\frac{\phi_0^2 a}{8\pi^2\mu_0\lambda^2\xi_0^2}\left(\frac{\sinh(3d/\sqrt{2}\xi)+\sinh(d/\sqrt{2}\xi)}{\sinh(d/\sqrt{2}\xi)\cosh(d/\sqrt{2}\xi)}+2\right)+(B_0^2/2\mu_0)\exp(-2d/\lambda)f(d,\xi)\right] \ . \quad (11)$$

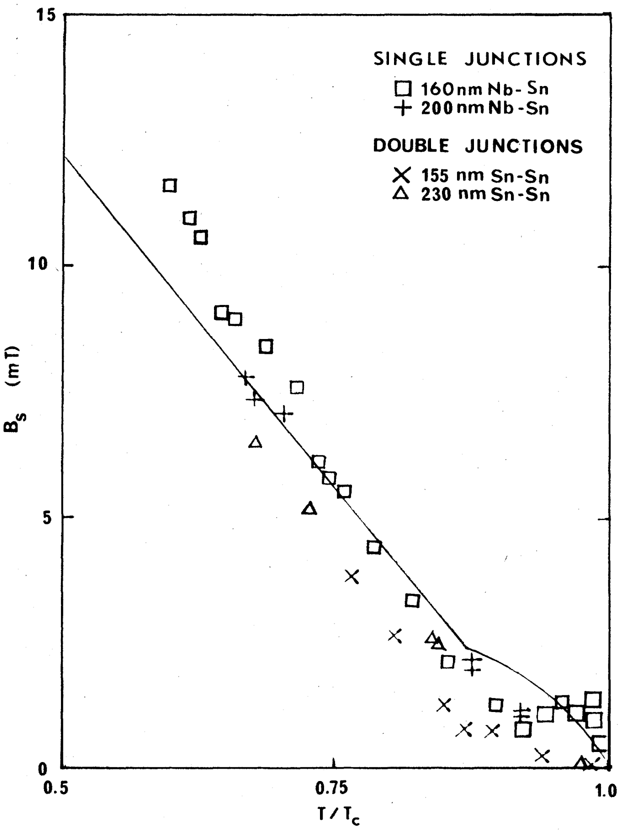

FIG. 2. The critical field B_s as a function of temperature. The solid curve is from Eq. (12). The points are data on junctions as described in the text. The film thicknesses are shown on the figure.

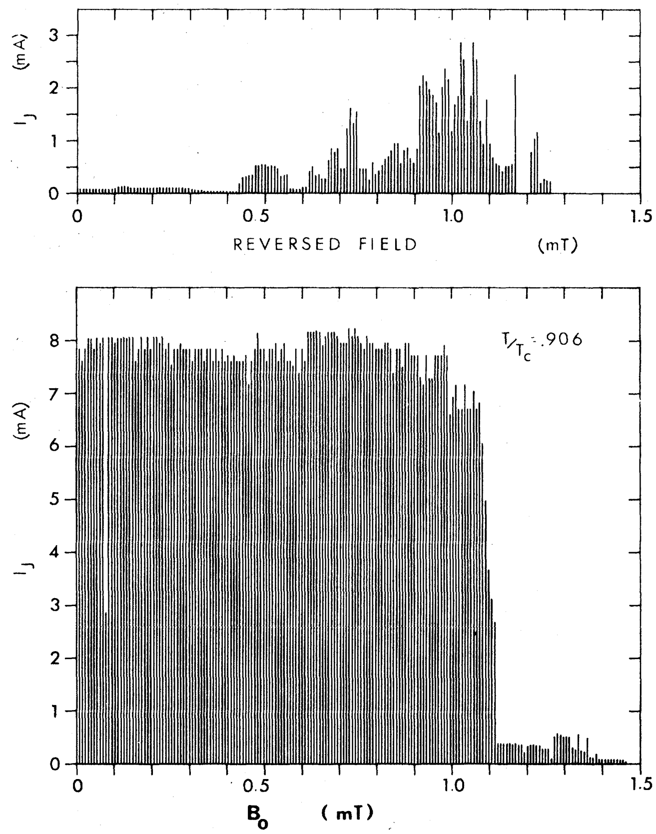

FIG. 3. The critical Josephson current as a function of axial field for a cylindrical junction between a 0.25-mm-diam Nb wire and a 160-nm tin film. Note the "steps" and the "flat top."

Again, G_c is zero when B_0 is B_s:

$$B_s=\frac{\phi_0}{2\pi\xi_0}\left[a\exp(2d/\lambda)\left(\frac{\sinh(3d/\sqrt{2}\xi)+\sinh(d/\sqrt{2}\xi)}{\sinh(d/\sqrt{2}\xi)\cosh(2d/\sqrt{2}\xi)}+2\right)\Big/2\lambda^3 f(d,\xi)\right]^{1/2} . \tag{12}$$

This result is plotted in Fig. 2 for $\xi(0)=\xi_0=101$ nm, $\lambda(0)=724$ nm, and $a=0.2$ nm. The discontinuity in slope at $(T/T_c)=t=0.86$ occurs when $\xi(T)/\sqrt{2}=d$ and $f(d,\xi)$ changes form. Below $t=0.5$ Eq. (12) predicts a decreasing slope for $B_s(T)$ with $B_s(0)=17$ mT and $(dB_s/dT)=0$ at $T=0$. There are no adjustable parameters in Eq. (12) but the value selected for a amounts to a guess. The data shown are for four types of junctions no one of which corresponds exactly to our model. The single junctions consist of a 0.125-mm-radius Nb wire/niobium oxide/Sn film. The double junctions consist of Nb wire/Nb oxide/Sn film/Sn oxide/Sn film. For the double junctions only the results for the outer, Sn/Sn oxide/Sn, junctions are shown.

The model gives a thermodynamic critical field B_s which depends upon material parameters and geometry but is independent of the coupling strength. This all agrees with our observations. A further prediction of the model is that the Josephson current density varies as $J_0(B)=J_0(0)[1-(B/B_s)^2]$. The junctions reported in Ref. 1 showed this dependence for $I_J(B)$. These junctions had Josephson currents of about 1 mA. The junctions reported here have larger critical currents and do not show this dependence except possibly very near T_c.

The dependence of Josephson current on axial field B_0 is shown in Fig. 3 for a single junction, Nb wire/Sn film. The film is 160 nm thick and its critical temperature is 3.85 K. In these measurements the current is increased until a voltage is sensed across the junction. The current is then returned to zero and the process is repeated. At the same time the magnetic field is slowly increased by an automatic current drive. The data were recorded on an x-y recorder. The sample was cooled in zero field and the data in the lower plot were recorded. The field was returned to zero and then increased in the opposite direction to obtain the upper plot. There are several interesting features of these data. Among these are the "flat top" and the "steps."[7] Note that the junction switches repeatedly at several values of the current.

Finally the sine-Gordon equation for a junction[8] must be modified by replacing λ_J^2 by $\lambda_J^2/[1-(B/B_s)^2]$. Here, λ_J is the Josephson penetration depth.

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