Comparison of the elastic moduli and the conductivity observed in a two-dimensional percolating system

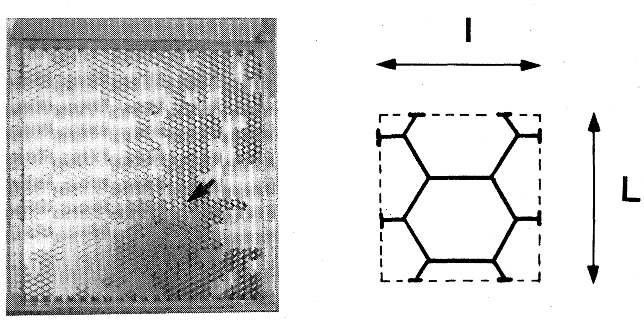
C. Allain, J. C. Charmet, M. Clement, and L. Limat
Laboratoire d'Hydrodynamique et Mécanique Physique, Ecole Supérieure de Physique et de Chimie Industrielles de la Ville de Paris (ESPCI), 10 rue Vauquelin, 75231 Paris Cédex 05, France
(Received 4 June 1985)

Measurements of the shear and Young moduli were performed on a thick grid whose links are progressively cut. The comparisons of the moduli and the conductivity have shown that these three quantities do not have the same critical behavior. However, the poor statistics (one actual experiment of 20×20 sites) does not allow us to calculate reliable values of the exponents.

Recently, there has been a growing interest in the problem of mechanical behavior of mixtures of hard and soft components near the percolation threshold. On one hand, different numerical simulations[1-3] and theoretical approaches[4-6] have been made. On the other, only few experimental studies on mixture of hard materials and voids[7,8] have been performed. More experimental results concerning the elasticity of gels[9,10] are available. However, even if the critical behavior of the elasticity of gels has first initiated the interest in this field, it does not rely on the same category of problems. Indeed, as was pointed out by de Gennes[11] and Alexander,[12] due to the presence of the solvent, the compressibility has no singularity at the gel point and large internal stresses are involved.

In the present paper, we report an experiment performed on a thick bidimensionnal grid, whose links are progressively cut. The shear and the Young moduli are measured as a function of the number of removed sites. Due to the nature of the grid used, the conductivity cannot be measured simultaneously. Hence, using a method developed by Mitescu,[13] it was computed for each configuration.

Figure 1 represents a picture of the grid partially cut. For each site, we removed simultaneously eight links as shown. The choice of this elementary cell is made in order to ensure that, even near the threshold, the rigidity of the sample is large enough to avoid irreversible deformations. As a



(a) (b)

FIG. 1. (a) Picture of the grid obtained after 138 sites have been removed. The dimensions of an elementary cell (b) are $L=8.5\times 10^{-3}$ m and $l=8.5\times 10^{-3}$ m. The third dimension (perpendicular to the plane of the picture) is 1.5×10^{-2} m.

consequence of this decimation recipe, the initial hexagonal geometry is transformed into a square site lattice. Finally, the resulting lattice on which the experiments are performed, is a 20×20 sites sample.

Experimentally, large variations of the moduli are expected, so we have decided to impose given displacements and measure the resulting forces; this prevents the structure from being irreversibly distorted. The displacements are smaller than 3×10^{-3} m (note that the samples are 0.2 m high); they are imposed with a precision of 1×10^{-5} m. The resulting forces ranging typically from 10 to 2×10^{-1} N, are determined with a precision of 10^{-3} N. For each measurement, the values of the moduli are deduced from the slopes of the displacement-force curves. Finally, including the reproducibility of the loading and unloading operations, the precision on the moduli is 1%.

The variations of the shear and Young moduli with the number of removed sites are plotted in Fig. 2. We also plotted the value of the conductivity for the same realization. Let us first outline the presence of small discontinuities on the curves. These drops which are larger than the precision of measurements, are related to the topological position of the removed sites. Indeed, if a site belongs to a finite-size cluster or to a dangling arm of the backbone, its removal affects neither the mechanical behavior nor the electrical one. On the contrary, if the removed site has a strategic position leading for instance to the appearance of a red site[14] or to the disappearance of a loop, a break in the decrease of moduli and conductivity values would appear. Using this criterion, we were able to anticipate *a priori* strategic sites for the realization corresponding to Fig. 2; then we measured the elastic moduli just before and after these points. In nearly all the cases, we observed a jump on the measured values. For instance, the suppression of the site underlined on Fig. 1 leads to the drop indicated by an arrow in Fig. 2.

It would be also interesting to calculate the exponent of the Young modulus as was done by Benguigui.[7] However, we consider that the data obtained from a single sample of size 20×20, have a statistical value which is too poor to deduce a valid result. One of the difficulties is entailed in the determination of the threshold p_c. For instance, for our realization p_c corresponds to removal of 151 sites, whereas the expected value computed for an infinite lattice would be 165.[15]

In order to compare the shear and Young moduli be-

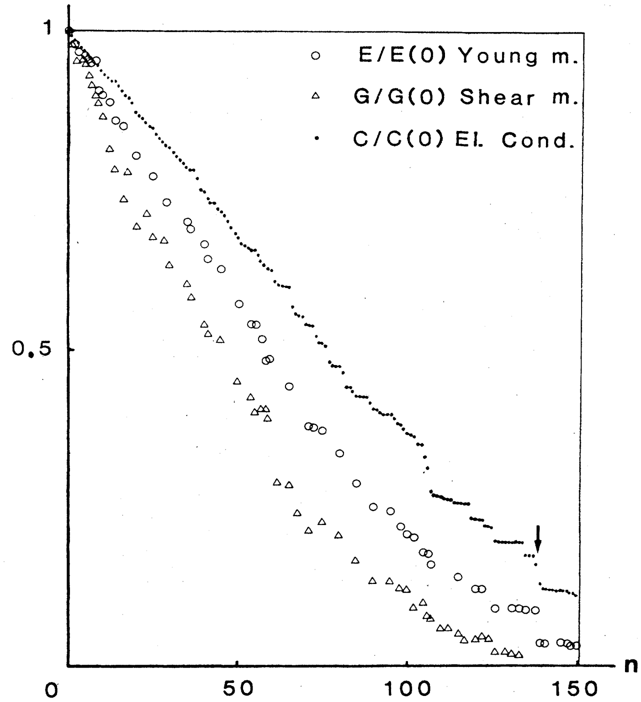


FIG. 2. Variations of the conductivity (·), Young modulus (○), and shear modulus (Δ) vs the number of removed sites. The drop indicated by an arrow corresponds to the suppression of the site underlined in Fig. 1.

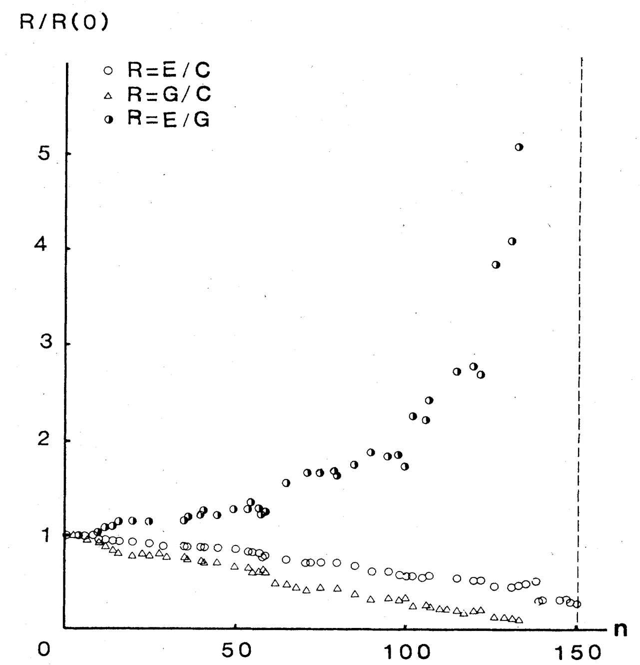


FIG. 3. Variation of the Young modulus/shear modulus ratio (◑) vs the number of removed sites. The variations of the shear modulus/conductivity (Δ) and Young modulus/conductivity (○) ratios are also plotted.

haviors, we studied the variation of their ratio R as a function of the number of removed sites. In absence of any cut link, the value of R taken as R_0, is equal to 1.88. As the number of removed sites is increased, R/R_0 increases; near the threshold, the slope becomes larger and larger (cf., Fig. 3). This behavior corresponds to the fact that the partially cut grid is more sensitive to shear stress than to compressional ones; the difference becoming more significant near the threshold. In terms of critical behavior, this result implies that the exponent of the shear modulus is larger than the exponent of the Young modulus. However, let us note that two effects may influence the variation of R. First, the elementary cell that is chosen is not completely symmetrical. However, as the number of links of the grid between two adjacent sites is the same (equal to two) regardless of the relative position of these sites, this effect is probably negligible. Secondly, one could expect that an average over a large number of realizations would lead to a value of R/R_0 independent of p. Nevertheless, given the sharp increase of R/R_0 in Fig. 2, this hypothesis would imply the existence of fairly large fluctuations. Hence in spite of these limitations, our results indicate a difference of the percolation behaviors of the shear and Young moduli. Let us now compare the mechanical and the electrical results. The two ratios of each of the elastic moduli with the conductivity are plotted in Fig. 3; the curves have been normalized by the values in absence of any cut link. For these quantities, we also observed a departure from a constant value which becomes particularly large near the threshold. It is noteworthy that the amplitudes of these variations are different. The largest effect is observed for the ratio, shear modulus/conductivity while the ratio, Young modulus/conductivity exhibits only a smooth decrease. In terms of critical properties, these results show clearly that the exponent of the shear modulus is larger than that of the conductivity while the Young modulus exhibits an intermediate behavior. Again, unfortunately, the statistic when we consider a sample of 20×20 sites is too poor to deduce reliable values of these exponents. So, it would be of great interest to increase the statistic but this is not very easy in experimental investigations. In effect, in our study it is difficult to perform experiments on a large number of samples or on a sample with many sites. Because of this, experiments on a mixture of hard materials and void or soft materials may seem attractive. However, since the elastic moduli tends to zero near the threshold, preparation of these samples in the critical region is to be made difficult by their collapse under their own weight.

In summary, we observe that the elastic moduli and the conductivity do not have the same behaviors near the percolation threshold, especially the shear modulus which decreases more quickly than the Young modulus. Further studies on this comparison between mechanical and electrical behaviors are now in progress.

We are indebted to C. Mitescu who first inspired us to percolation computations. We also want to thank I. Petitbon, F. Minière, and J. M. Mériaux for their help and patience in cutting the grids.

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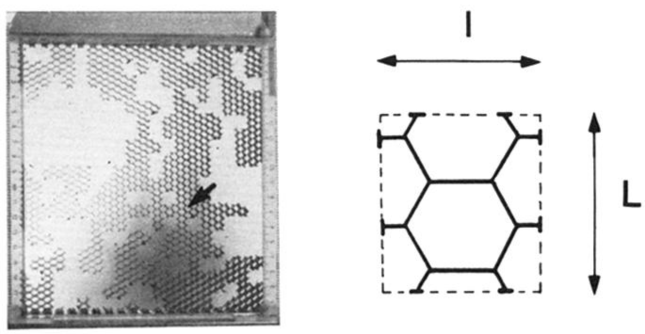

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