Dynamics of a particle in an external potential interacting with a dissipative environment

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The study of the recently introduced model of a particle in a periodic potential interacting with an external environment has been pursued, and two main new results are reported: (i) The Hamiltonian is exactly soluble for a particular value of the friction coefficient, $\eta=\hbar\pi/a^2$, a being the lattice constant. Earlier results based on the scaling laws obeyed by the model are confirmed. (ii) The influence of imperfections in the potential on the phase diagram of the system is analyzed. It is shown that near impurities, or at a surface, a new localized phase appears.

I. INTRODUCTION

The problem of a quantum system interacting with a dissipative environment has recently attracted a great deal of attention.[1] An interesting case is that of a particle moving in a periodic potential and experiencing a frictional force.[2] It has been shown to have a nontrivial transition to a self-localized state if the friction coefficient is large enough.[3]

The analysis of the phase diagram presented in Ref. 3 (hereafter called I) was based on the renormalization flow of the interactions of the Hamiltonian, completed with a duality transformation[2] which maps the strong friction and large-corrugation regime onto its opposite. The scaling laws obeyed by the model are similar to the Kosterlitz-Thouless type of flow deduced for the closely related two-level system interacting with a dissipative environment.[4-6] Hence, when the friction coefficient lies above a given value, the "fugacity" of the jumps of the particle between successive sites (instantons) increases as the cutoff is reduced, and, at zero temperature, the particle is localized in one of the wells defined by the periodic potential. Below this critical friction coefficient, the Hamiltonian scales towards a strong-coupling regime, away from the initial dilute instanton gas case, and this analysis cannot be continued. To study further the phase diagram, an interesting duality transformation[2] was used, which allows us to apply the same scaling laws to the situation opposite to the one discussed earlier, that of a particle weakly perturbed by the external potential. Then, the flow calculated in one of the limits can be smoothly extended to the other, if, as argued in I, no extra relevant interactions appear in the intermediate-coupling regime. The analysis was completed by an exact calculation of the order parameter, the mobility, along the transition line separating the two phases.

The purpose of the present work is twofold: first, we will show that the model is exactly soluble for a particular value of the friction coefficient, which lies in the region where the Hamiltonian scales towards its strong-coupling regime. The calculations confirm the predictions made in I. We think that this solution may also be relevant to studies of the discrete Gaussian model with r^{-2} interactions, onto which our system can be mapped.[7,8] Second, we will analyze deviations from perfect periodicity in the potential and their influence on the phase diagram of the system. Although the main conclusions of I remain unchanged, it will be shown that the renormalization flow of the system is altered, changing the positions of the critical lines, and, within the localized phase, a new regime may appear, in which the particle can tunnel between sites close to the impurity (or surface), although it is confined to a finite region in its vicinity.

II. THE MODEL

The model introduced in Ref. 2 describes a particle moving in a periodic potential with an extra term in its effective action, in such a way that its motion is dissipative in the classical limit. The effective action can be replaced by a coupling term to a heat bath of appropriately chosen harmonic oscillators.[9] It is this latter situation which we will consider. We will also study only the case in which the barriers between adjacent potential wells are so large that only the lowest-lying state of each well has to be taken into account. This is the limit in which a semiclassical approximation to the tunneling processes is appropriate (the instanton approach in Ref. 2), and, as extensively discussed in the same reference, is also equivalent to the opposite case, that of a nearly free particle. Thus, its analysis suffices to understand the complete phase diagram. Following the arguments given in Ref. 9, we will describe the environment by a set of harmonic oscillators whose frequencies range from 0 to a maximum ω_c. Finally, we choose a linear coupling between the oscillators and the particle, in order to describe the dissipative mechanism. The complete Hamiltonian then reads

$$H=\Delta\sum_n c_n^\dagger c_{n+1}+\text{H.c.}+\sum_k k b_k^\dagger b_k + \lambda\left[\sum_n n c_n^\dagger c_n\right]\left[\sum_k \sqrt{k}\,(b_k^\dagger+b_k)\right] + \lambda^2\left[\sum_n n c_n^\dagger c_n\right]^2\sum_k 1\,, \tag{1}$$

where Δ is the hopping probability between successive wells, the possible values of k lie between 0 and ω_c, and

$\sum_n nc_n^\dagger c_n$ is the position operator for the particle in our tight-binding basis. The last term in (1) is required to avoid an unphysical renormalization of the potential from the coupling to the environment.

Following the definition of the friction coefficient given in Ref. 2, we can relate it to the coupling coefficient λ by

$$\eta=\lambda^2\hbar/2a^2 , \tag{2}$$

where a is the distance between successive wells. In the following it will be convenient to express the magnitude of the coupling to the environment in terms of the dimensionless constant:

$$\alpha=\frac{\lambda^2}{4\pi}=\frac{\eta a^2}{2\pi\hbar} . \tag{3}$$

Following I, we define a canonical transformation such that

$$\begin{aligned} U&=\exp\left[\lambda\left[\sum_n nc_n^\dagger c_n\right]\left[\sum_k\frac{b_k^\dagger-b_k}{\sqrt{k}}\right]\right] , \\ \tilde{H}&=U^\dagger HU=\sum_k kb_k^\dagger b_k \\ &\quad+\Delta\sum_n c_n^\dagger c_{n+1}\exp\left[\lambda\sum_k\frac{b_k^\dagger-b_k}{\sqrt{k}}\right]+\text{H.c.} , \end{aligned} \tag{4}$$

where the translational symmetry of (1) is made explicit. Writing

$$c_q^\dagger=\frac{1}{\sqrt{N}}\sum_n e^{iqn}c_n^\dagger \tag{5}$$

we have

$$\tilde{H}=\sum_k kb_k^\dagger b_k+\Delta\sum_q c_q^\dagger c_q\exp\left[-iq+\lambda\sum_k\frac{b_k^\dagger-b_k}{\sqrt{k}}\right]+\text{H.c.} \tag{6}$$

Now H is diagonal in the particle operators. Thus, once the state q occupied by the particle is specified, we are left with an effective Hamiltonian for the oscillators

$$H_q=\sum_k kc_k^\dagger c_k+\Delta\exp\left[-iq+\lambda\sum_k\frac{b_k^\dagger-b_k}{\sqrt{k}}\right]+\text{H.c.} \tag{7}$$

Hamiltonians for different q's are related by canonical transformations

$$\begin{aligned} H_{q'}&=U_{q-q'}^\dagger H_q U_{q-q'} , \\ U_{q-q'}&=\exp\left[\frac{i(q-q')}{2\lambda}\lim_{k\to0}\sqrt{k}\,(b_k^\dagger+b_k)\right] . \end{aligned} \tag{8}$$

The physics behind this transformation become more transparent if we consider the operators $b_k^\dagger$ as defining a one-dimensional field

$$\begin{aligned} \Pi(x)&=\sum_k\sqrt{k/2}(b_k^\dagger e^{ikx}+b_k e^{-ikx}) , \\ \phi(x)&=i\sum_k\frac{1}{\sqrt{2k}}(b_k^\dagger e^{ikx}-b_k e^{-ikx}) , \\ H_q&=\tfrac{1}{2}\int\left[[\Pi(x)]^2+\left[\frac{\partial\phi}{\partial x}\right]^2\right]dx \\ &\quad+2\Delta\cos[q+\sqrt{2}\lambda\phi(0)] . \end{aligned} \tag{9}$$

Then, it can be seen that, by a global displacement of the values of ϕ, Hamiltonians with different values of q can be made to coincide. The generator of the transformation is the total momentum operator.

A convenient parameter to define the dynamics of the particle is the mobility,[2] given by

$$\mu=\lim_{\omega\to0}\eta\omega\langle x,x\rangle_\omega , \tag{10}$$

where $\langle x,x\rangle_\omega$ is the Fourier transform of the position correlation function for the particle at different times. In our tight-binding basis this operator is

$$x=a\sum_n nc_n^\dagger c_n . \tag{11}$$

A convenient way of expressing the mobility in terms of oscillator operators is to replace x by

$$X=\lim_{L\to\infty}\frac{L}{i}\left[\exp\left[\frac{ix}{L}\right]-1\right] . \tag{12}$$

This operator shifts the lattice momentum of the particle by $q=1/N$, $N=L/a$, so that

$$\langle X(t)X(0)\rangle=a^2\lim_{N\to\infty}N^2\langle0|(e^{iH_q t}-1)|0\rangle \tag{13}$$

and, making use of Eq. (8)

$$\begin{aligned} &\lim_{N\to\infty}N^2\langle0|(e^{iH_q t}-1)|0\rangle \\ &\quad=\lim_{N^2\to\infty}N^2\langle0|U_q^\dagger(e^{iH_0 t}-1)U_q|0\rangle \\ &\quad=\frac{1}{4\lambda^2}[\langle0|P(t)P(0)|0\rangle-\langle0|P^2|0\rangle] , \\ &P=\lim_{k\to0}\sqrt{k}\,(b_k^\dagger+b_k) , \end{aligned} \tag{14}$$

so that

$$\mu=\frac{\pi\alpha}{2\lambda^2}\lim_{\omega\to0}\omega\langle P,P\rangle_\omega \tag{15}$$

and the expectation value in the equation above is to be taken over the ground state of H_0, Eq. (7).

The equation of motion for P reads

$$i\frac{dP}{dt}=4i\lambda\Delta\sin[\sqrt{2}\lambda\phi(0)] \tag{16}$$

and, substituting in Eq. (15)

$$\mu = 8\alpha\Delta^2\pi \lim_{\omega\to 0} \frac{\langle S,S\rangle_\omega}{\omega} ,$$
$$S = \sin[\sqrt{2}\lambda\phi(0)] . \tag{17}$$

Thus we have integrated out the particle's variables, and we have now to calculate a correlation function which involves only the bosonic Hamiltonian H_0, given by Eq. (7) setting $q=0$. The last term in this Hamiltonian is reminiscent of the nonlinear term of the sine-Gordon model, although restricted to a single point in space. This feature, however, prevents the application of the methods used to solve the latter system.[10] It is also to be stressed that we are interested in a dynamical correlation function, which is usually much harder to compute.

In I it was shown that a perturbation expansion of H_0 in terms of Δ is formally equivalent to the partition function of a one-dimensional gas of charges equal to $\pm\sqrt{2\alpha}$ interacting with a logarithmic potential. Contrary to the similar analysis for the Kondo and related Hamiltonians, the signs of these charges need not be in a definite order, which lead to changes in the phase diagram. When $\alpha>1$, these charges are bound in neutral pairs, irrespective of the value of Δ. For $\alpha<1$, their fugacity increases when the cutoff is lowered, and eventually the scaling trajectories move away from the region of validity of the approximations needed to define the renormalization equations. As mentioned earlier, in I the fact was exploited that the scaling equations can also be known in the nearly-free-electron case, when the periodic potential is a weak perturbation acting on the particle. Thus, a smooth interpolation is feasible provided that no relevant variables appear in the intermediate coupling region. Finally, when $\alpha=1$, there is not renormalization flow. In I it was possible to calculate exactly the mobility along this line, using the equivalence between bosons and fermions in one dimension.[11,12] Its value increases as a function of Δ, in agreement with the identification of this line with a line of fixed points, and μ with a critical exponent.

III. EXACT SOLUTION WHEN $\alpha=\frac{1}{2}$

In order to analyze the problem when $\alpha=\frac{1}{2}$, we will also replace H_0 [Eq. (7)] by a fermionic Hamiltonian. It is well known that the free part of H_0 is equivalent to a free Fermi field. As we are considering a single bosonic branch, $k>0$, the spectrum of these fermions will have a single branch as well. We can also define

$$\psi^\dagger(x) = \sqrt{\omega_c}\exp\left[\sqrt{2\pi}\sum_k \frac{b_k^\dagger e^{ikx} - b_k e^{-ikx}}{\sqrt{k}}\right] , \tag{18}$$

where ω_c is our high-energy cutoff. It can be proved[10,11] that these operators have the anticommuting properties of a Fermi field, and also the right low-energy propagators in terms of the free-field Hamiltonian. Thus, the fermionic equivalent of H_0 is

$$H_f = \sum_k k c_k^\dagger c_k + \Delta\sum_k (c_k^\dagger + c_k) \tag{19}$$

and, to compute the mobility we also need the fermionic representation of S [Eq. (17)],

$$S_f = -\frac{i}{2}\sum_k (c_k^\dagger - c_k) . \tag{20}$$

The fact that the second part of H_f does not commute with the first one prevents a straightforward solution by a factorization of the eigenfunctions into separate k components.

To transform H_f into a Hamiltonian *quadratic* in the fermion operators, we will choose a particular regularization by assuming that the fermionic operators are only defined on the sites of a semi-infinite lattice,

$$H_f = \tfrac{1}{2}\sum_{n=0}^{\infty} c_n^\dagger c_{n+1} + \text{H.c.} + \Delta(\tfrac{1}{2}\pi)^{1/2}(c_0^\dagger + c_0) ,$$
$$S = -\frac{i}{2}(\tfrac{1}{2}\pi)^{1/2}(c_0^\dagger - c_0) . \tag{21}$$

The free part of H_f in the previous equation is easily diagonalizable, and it can be seen that Eq. (19) is just the continuum limit of (21).

We now use the Wigner-Jordan transformation and express H_f in terms of Pauli matrices, representing a set of coupled two-level systems,

$$c_n^\dagger = \prod_{i(<n)} \sigma_i^z \sigma_n^+ ,$$
$$c_n = \prod_{i(<n)} \sigma_i^z \sigma_n^- ,$$
$$H_s = -\tfrac{1}{4}\sum_{n=0}^{\infty} (\sigma_n^x\sigma_{n+1}^x + \sigma_n^y\sigma_{n+1}^y) + \Delta(\tfrac{1}{2}\pi)^{1/2}\sigma_0^x , \tag{22}$$
$$S_s = \tfrac{1}{2}(\tfrac{1}{2}\pi)^{1/2}\sigma_0^y ,$$

which is the semi-infinite X-Y model with a magnetic field at the boundary. We can apply a canonical transformation well known in the study of duality in one-dimensional spin systems,[13]

$$\sigma_n^x = \prod_{i=0}^{n} \bar\sigma_i^z ,$$
$$\sigma_n^z = \bar\sigma_n^x \bar\sigma_{n+1}^x ,$$
$$\bar H_s = -\tfrac{1}{4}\sum_{n=0}^{\infty} (\bar\sigma_{n+1}^z - \bar\sigma_n^x \bar\sigma_{n+1}^z \bar\sigma_{n+2}^x) + \Delta(\tfrac{1}{2}\pi)^{1/2}\bar\sigma_0^z , \tag{23}$$
$$\bar S_s = \frac{i}{2}(\tfrac{1}{2}\pi)^{1/2}\bar\sigma_0^x \bar\sigma_0^z \bar\sigma_1^x$$

and now we can go back to fermionic operators

$$\bar c_n^\dagger = \prod_{i(<n)} \bar\sigma_i^z \bar\sigma_n^+ ,$$
$$\bar c_n = \prod_{i(<n)} \bar\sigma_i^z \bar\sigma_n^- ,$$
$$\bar H_f = -\tfrac{1}{4}\sum_n [(2\bar c_{n+1}^\dagger \bar c_{n+1} - 1) + (\bar c_n^\dagger - \bar c_n)(\bar c_{n+2}^\dagger + c_{n+2})] + \Delta(\tfrac{1}{2}\pi)^{1/2}(2\bar c_0^\dagger \bar c_0 - 1) , \tag{24}$$
$$\bar S_f = \frac{i}{2}(\tfrac{1}{2}\pi)^{1/2}(\bar c_0^\dagger + \bar c_0)(\bar c_1^\dagger + \bar c_1) .$$

Note that $\overline{H}_f$ as defined above can be split into two decoupled parts, one acting only on odd and the other on even sites. The operator $\overline{S}_f$ contains operators acting on the two subspaces, and the correlation required to calculate the mobility [Eq. (17)] can be factorized into two terms, when computed in real times.

The Hamiltonian $\overline{H}_f$ is now quadratic, and can be solved by a transformation to a new set of Fermi fields.[14,15] The calculation is straightforward, although tedious; the details are given in the Appendix. The main result is that, in terms of the operators $c_k^\dagger$ and $c_k'^\dagger$ which diagonalize, respectively, the even and odd parts of $\overline{H}_f$, we can write

$$\begin{aligned}
&\bar{c}_0^\dagger+\bar{c}_0=\sum_k \alpha_k(c_k^\dagger+c_k)\ ,\\
&\bar{c}_1^\dagger+\bar{c}_1=\sum_k \alpha_k'(c_k'^\dagger+c_k')\ ,\\
&\lim_{k\to 0}\alpha_k=\frac{1}{\pi\sqrt{2\Delta}}\ ,\\
&\lim_{k\to 0}\alpha_k'=\frac{2}{\sqrt{\pi}}\ ,
\end{aligned} \tag{25}$$

where $0<k<\pi/2$, and we also know that

$$\lim_{k\to 0}\langle c_k(t)c_k^\dagger(0)\rangle=\lim_{k\to 0}\langle c_k'(t)c_k'^\dagger(0)\rangle=e^{ikt}\ , \tag{26}$$

so that

$$\lim_{t\to\infty}\langle S_f(t)S_f(0)\rangle=\frac{1}{4\pi^2\Delta^2t^2} \tag{27}$$

and, finally,

$$\begin{aligned}
&\lim_{\omega\to 0}\langle S_f,S_f\rangle=\frac{\omega}{4\pi\Delta^2}\ ,\\
&\mu=1\ ,
\end{aligned} \tag{28}$$

as predicted in I. Using the duality transformation from Ref. 2, this result also means that the particle is always localized ($\mu=0$) when $\alpha=2$, for any finite strength of the periodic potential.

IV. INFLUENCE OF IMPERFECTIONS ON THE PHASE DIAGRAM

In the following, we will analyze the scaling equations satisfied by the Hamiltonian:

$$\begin{aligned}
H=&\sum_{m,n}\Delta_{m,n}c_m^\dagger c_n+\text{H.c.}+\sum_k kb_k^\dagger b_k\\
&+\left[\sum_n\lambda_n nc_n^\dagger c_n\right]\left[\sum_k\sqrt{k}\,(b_k^\dagger+b_k)\right]\\
&+\left[\sum_n\lambda_n nc_n^\dagger c_n\right]^2\sum_k 1\ ,
\end{aligned} \tag{29}$$

which is a simple generalization of (1), but allowing for an arbitrary distribution of hopping terms and a position-dependent friction coefficient proportional to λ_n^2. As particular cases, we will consider a local imperfection characterized by an altered barrier, and that of a surface, i.e., a semi-infinite chain of an otherwise perfect sequence of potential wells; extension to higher dimensions is straightforward. On the other hand, we can no longer use the duality transformation[2] which allowed us to extend the results to the nearly-free-particle case. We will restrict ourselves to the tight-binding case [Eq. (29)]. As before, we can perform a canonical transformation

$$\begin{aligned}
&U=\exp\left[\left[\sum_n\lambda_n nc_n^\dagger c_n\right]\left[\sum_k\frac{b_k^\dagger-b_k}{\sqrt{k}}\right]\right]\ ,\\
&\tilde{H}=U^\dagger HU\\
&\quad=\sum_k kb_k^\dagger b_k+\sum_{m,n}\Delta_{m,n}c_m^\dagger c_n\exp\left[(-\lambda_m m+\lambda_n n)\right.\\
&\qquad\left.\times\left[\sum_k\frac{b_k^\dagger-b_k}{\sqrt{k}}\right]\right]+\text{H.c.}
\end{aligned} \tag{30}$$

We can treat the second term in $\tilde{H}$ as a perturbation, and expand all correlation functions in powers of the $\Delta_{m,n}$'s; the integrations over the oscillators' operators can be performed exactly. Following the same steps as in I and Ref. 6, we can map this expansion onto the partition function of a one-dimensional gas of charges interacting via a logarithmic potential. Each jump of the particle between sites m and n can be seen as a charge $q_{m,n}$ with fugacity $\tilde{\Delta}_{m,n}$, whose values are

$$\begin{aligned}
&\tilde{\Delta}_{m,n}=\frac{\Delta_{m,n}}{\omega_c}\ ,\\
&q_{m,n}=\frac{\lambda_m m-\lambda_n n}{\sqrt{2\pi}}\ .
\end{aligned} \tag{31}$$

The charge associated with the inverse hop has the same value, with the sign changed. ω_c is the high-energy cutoff in the oscillators' energies.

As the cutoff is lowered, these parameters have to be replaced by scaled quantities. The ensuring equations can be analyzed with the method used in Ref. 2. A general procedure valid up to second order, is given in Ref. 16. The main new result with respect to the well-known case of a two-level system (TLS) is the appearance of new charges, arising from the composition of two simpler ones. The scaling equations are

$$\begin{aligned}
&\frac{d\tilde{\Delta}_{m,n}}{dl}=\left[1-\frac{q_{m,n}^2}{2}\right]\tilde{\Delta}_{m,n}+\sum_i\tilde{\Delta}_{m,i}\tilde{\Delta}_{i,n}\ ,\\
&\frac{dq_{m,n}}{dl}=\sum_i q_{i,m}\tilde{\Delta}_{i,m}^2+\sum_j q_{n,j}\tilde{\Delta}_{n,j}^2\ ,
\end{aligned} \tag{32}$$

where $l=\ln\omega_c$.

We will now assume that the initial Hamiltonian has all the λ_n's equal to a constant value λ and the hopping is only possible between nearest-neighbor sites. Then, we

consider the two cases mentioned above.

(i) A single impurity characterized by a hopping Δ', in a chain with an otherwise constant value of the hopping Δ. Numbering the sites from the impurity, we have the following equations:

$$\begin{aligned}
\frac{d\tilde{\Delta}_0}{dl} &\simeq \left[1-\frac{q_0^2}{2}\right]\tilde{\Delta}_0 , \\
\frac{dq_0}{dl} &= -2q_0\tilde{\Delta}_0^2+2q_1\tilde{\Delta}_1^2 , \\
&\vdots \\
\frac{d\tilde{\Delta}_n}{dl} &= \left[1-\frac{q_n^2}{2}\right]\tilde{\Delta}_n , \\
\frac{dq_n}{dl} &= -2q_n\tilde{\Delta}_n^2+q_{n-1}\tilde{\Delta}_{n-1}^2+q_{n+1}\tilde{\Delta}_{n+1}^2 ,
\end{aligned} \tag{33}$$

where $\tilde{\Delta}_{-n}=\tilde{\Delta}_n$, $q_{-n}=q_n$, and the initial value of $\tilde{\Delta}_0$ is Δ'/ω_0. All initial q's are equal. In Eq. (33) we have also neglected the extra charges which correspond to hops between sites which are not nearest neighbors, and which appear in the scaling process. The value of these extra charges is, initially, twice that of the original ones. Hence, if we are close to the critical region, $q^2\sim 2$, their fugacity will increase with decreasing cutoff and they will become irrelevant.

Far from the critical region, the renormalization of the charges is also irrelevant (it is a second-order process), and the equations for the fugacities give the vertical flow in the $(\tilde{\Delta},q)$ plane which characterize the perfect model. Let us now examine the critical region, $q^2\sim 2$, assuming that $\Delta'<\Delta$, i.e., the impurity decreases the hopping probability in its vicinity. Then, the charge associated with this hop will increase with decreasing cutoff (because the nearest hops give a kind of antiscreening effect), while the charge related to the nearest hop will move in the opposite direction. As the scaling proceeds, these deviations from the behavior of the perfect system around the impurity will be transmitted to hops further from it, because the changes in the values of the charges influence the fugacities, which, in turn, alter the equations for the neighboring charges. In general, even charges will tend to increase, while odd ones will decrease. This process is self-amplifying: a charge $q^2<2$ will make its corresponding fugacity grow which will enhance its antiscreening effect on the neighboring charges. Thus, between the two phases of the perfect model, the localized and the diffusive one, we may have an extra one, in which some of the hops will be suppressed, while the others not. Of course, the particle will be localized, because it cannot move through the entire lattice, but it will not be in a single well of the external potential as before. The analysis when $\Delta'>\Delta$ proceeds along the same lines and leads to similar results.

(ii) A semi-infinite chain. In this case the equations are

$$\begin{aligned}
\frac{d\tilde{\Delta}_0}{dl} &= \left[1-\frac{q_0^2}{2}\right]\tilde{\Delta}_0 , \\
\frac{dq_0}{dl} &= -2q_0\tilde{\Delta}_0^2+q_1\tilde{\Delta}_1^2 , \\
&\vdots \\
\frac{d\tilde{\Delta}_n}{dl} &= \left[1-\frac{\tilde{q}_n^2}{2}\right]\tilde{\Delta}_n , \\
\frac{dq_n}{dl} &= -2q_n\tilde{\Delta}_n^2+q_{n-1}\tilde{\Delta}_{n-1}^2+q_{n+1}\tilde{\Delta}_{n+1}^2 ,
\end{aligned} \tag{34}$$

where we start to label sites from the surface. All initial charges and fugacities are equal. The analysis of these equations resembles closely the one presented before. Near the critical region, the hopping processes near the surface do not experience the antiscreening effects typical of the bulk and, hence, they are more easily deconfined. The phase diagram for these charges is similar to that of the TLS. On the other hand, once the fugacity of these charges grows, the charges further inside the system tend to increase, leading to a reduction in the corresponding hopping processes. Again, we can have an extra phase in which the particle is localized, but not anywhere in the system; it will tend to be close to the surface.

V. CONCLUSIONS

We have continued the analysis of the dynamics of a particle in a periodic potential, interacting with a dissipative environment.[2,3] We have shown that the model is exactly soluble for a particular value of the friction coefficient. This solution contributes to clarify the behavior of the system in the intermediate coupling regime, a region of the phase diagram not directly accessible by a renormalization analysis[3] or a duality transformation.[2] We think that the method proposed here may also be relevant to the study of other systems which can be mapped onto the discrete Gaussian model,[7] and to Hamiltonians with commuting and anticommuting terms.

We have also studied the changes in the phase diagram induced by imperfections in the periodic potential. It has been shown that the critical region lying between the dissipative and localized phase is altered. The flow in the scaling equations changes in this region, and, for certain values of the initial parameters, a new localized regime appears, in which some of the hops of the particle between potential wells are not suppressed.

APPENDIX

Here we will calculate the excitation spectrum of the Hamiltonians:

$$\begin{aligned}
H_1 &= -\tfrac{1}{4}\sum_{n,\text{even}}[(2\bar{c}_{n+2}^{\dagger}\bar{c}_{n+2}-1)+(\bar{c}_n^{\dagger}-\bar{c}_n)(\bar{c}_n^{\dagger}+\bar{c}_{n+2})] \\
&\quad +\Delta(\tfrac{1}{2}\pi)^{1/2}(2\bar{c}_0^{\dagger}\bar{c}_0-1) , \\
H_2 &= -\tfrac{1}{4}\sum_{n,\text{odd}}[(2\bar{c}_n^{\dagger}\bar{c}_n-1)+(\bar{c}_n^{\dagger}-\bar{c}_n)(\bar{c}_{n+2}^{\dagger}+\bar{c}_{n+2})] .
\end{aligned} \tag{A1}$$

H_1 and H_2 are the even and odd sites parts of $\overline{H}_f$ [Eq. (24)].

We will first consider H_1. We look for new Fermi operators, $c_k^\dagger$, such that

$$c_k^\dagger = \sum_k \alpha_k^n(\bar{c}_n^\dagger + \bar{c}_n) + \sum_k \beta_k^n(\bar{c}_n^\dagger - \bar{c}_n) ,$$
$$[H_1 c_k^\dagger] = \epsilon_k c_k^\dagger . \tag{A2}$$

The coefficients α_k^n and β_k^n satisfy

$$\left.\begin{aligned} \epsilon_k \alpha_k^n &= -\tfrac{1}{2}\beta_k^n - \tfrac{1}{2}\beta_k^{n+2} , \\ \epsilon_k \beta_k^n &= -\tfrac{1}{2}\alpha_k^n - \tfrac{1}{2}\beta_k^{n-2} , \end{aligned}\right\} n \neq 0$$
$$\epsilon_k \alpha_k^0 = 2\Delta(\tfrac{1}{2}\pi)^{1/2}\beta_k^0 - \tfrac{1}{2}\beta_k^2 , \tag{A3}$$
$$\epsilon_k \beta_k^0 = 2\Delta(\tfrac{1}{2}\pi)^{1/2}\alpha_k^0 .$$

The two first equations admit the solutions

$$\alpha_k^n = (i)^n e^{\pm ikn} ,$$
$$\beta_k^n = \pm(i)^{n-1} e^{\pm ik(n-1)} , \tag{A4}$$
$$\epsilon_k = \sin k , \quad 0 < k < \pi/2 .$$

Once the "bulk" solutions have been obtained, we can satisfy the boundary conditions given by the two last equations in (A3), by choosing an appropriate combination of these solutions:

$$\beta_k^0 = \frac{\sqrt{2\pi}\Delta \cos k}{\frac{\epsilon_k e^{ik}}{2} - 2\pi i \Delta^2} ,$$

$$\left.\begin{aligned} \beta_k^n &= (i)^{n-1} e^{ik(n-1)} - e^{i\phi_k}(i)^{n-1} e^{-ik(n-1)} , \\ \alpha_k^n &= (i)^n e^{ikn} + (i)^n e^{i\phi_k} e^{-ikn} , \end{aligned}\right\} n \neq 0 \tag{A5}$$

$$e^{i\phi_k} = \frac{\epsilon_k/2e^{-ik} + 2\pi i \Delta^2}{\epsilon_k/2e^{ik} - 2\pi i \Delta^2} .$$

So far we have neglected a normalization factor, required to obtain the right anticommutation relations for the operators $c_k^\dagger$. This factor is $1/\sqrt{8N}$, where N is the number of sites. The diagonalization of H_2 can be achieved following the same steps presented above. In fact, the eigenenergies, and the bulk part of the equations are the same as for H_1. The only difference lies in the boundary conditions, which are actually simpler.

To complete the solution, we need the inverse transformation to (A2), in order to calculate the correlation function for $\overline{S}_f$

$$\bar{c}_n^\dagger = \sum_k \bar{\alpha}_k^n(c_k^\dagger + c_k) + \sum_k \bar{\beta}_k^n(c_k^\dagger - c_k) . \tag{A6}$$

When n is even, we obtain the following equations for the coefficients:

$$\left.\begin{aligned} \epsilon_k \bar{\alpha}_k^n &= -\tfrac{1}{2}\bar{\beta}_k^n - \tfrac{1}{2}\bar{\beta}_k^{n-2} , \\ \epsilon_k \bar{\beta}_k^n &= -\tfrac{1}{2}\bar{\alpha}_k^n - \tfrac{1}{2}\bar{\alpha}_k^{n+2} , \end{aligned}\right\} n \neq 0$$
$$\epsilon_k \bar{\alpha}_k^0 = 2\Delta(\tfrac{1}{2}\pi)^{1/2}\bar{\beta}_k^0 , \tag{A7}$$
$$\epsilon_k \bar{\beta}_k^0 = 2\Delta(\tfrac{1}{2}\pi)^{1/2}\bar{\alpha}_k^0 - \tfrac{1}{2}\bar{\alpha}_k^2 ,$$

which are formally equivalent to (A3). The only difference is the normalization factor which now turns out to be $\frac{1}{2}\sqrt{\pi}$.

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