Generation of charge-density-wave conduction noise by moving phase vortices

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(Received 7 November 1984; revised manuscript received 20 May 1985)

A moving phase vortex at the end of a charge-density-wave conductor plays an essential role in the conversion of condensed carriers to free carriers. Under sliding conditions the periodic nucleation and destruction of these vortices generates the voltage oscillations (narrow-band noise) observed in these conductors. The role of free carriers in the noise generation is emphasized and the calculated ac voltage amplitude is compared with experiment. The exponential decay of the ac electric field away from the contact is determined by the "phase-slip" length which also determines the "rigidity" of the sliding current. A general discussion of the role of phase slips and dislocations in inhomogeneous geometries is given. Results from sample-length-dependence studies and thermal-gradient experiments are discussed. Competing theories for the noise based on interaction with random impurities are surveyed and their predictions compared with known experiments.

I. INTRODUCTION

A growing number of inorganic quasi-one-dimensional compounds has been shown to possess very interesting transport properties related to the depinning of the charge-density wave (CDW) by an applied dc electric field E.[1] One of these properties is the appearance of well-defined spontaneous voltage oscillations ("narrow-band noise") when E exceeds the threshold E_T for depinning of the CDW. Fleming and Grimes[2] first observed the oscillations in $NbSe_3$. Aside from this compound narrow-band noise has been reported[3] in TaS_3 (both orthorhombic and monoclinic polytypes), in $(TaSe_4)_2I$, and $(NbSe_4)_{3.33}I$, and in blue bronze, $K_{0.3}MoO_3$.

Early models[4,5] of the noise regarded the oscillations as arising from periodic modulation of the bulk CDW velocity by its interaction with the impurities. (A survey of these models is given in Sec. VI.) Some recent experiments, however, have shown that the origin of the noise is localized at the ends of the sample rather than in the bulk. In particular, Verma and Ong[6] (VO) showed that in a thermal gradient the fundamental frequency in the noise spectrum splits into two sets which change in frequency in accordance with the local temperature at each end. In an earlier experiment it was also shown[7] that the silver paint contacts which normally envelop a large portion of the sample divide the uncovered portions into independently oscillating segments. In the region under the paint the CDW is pinned because of the reduction of the field strength along the chain direction. At the interface between the pinned and depinned CDW segments phase slippage accompanied by conversion of condensed to free carriers occurs. In this paper we propose[8] a mechanism for this conversion process and discuss how voltage oscillations arise naturally from such phase-slip processes. To relieve the conflict of different phase-winding rates on the two sides of this interface we propose that a train of vortices moves perpendicular to the CDW current. (The vortices are equivalent to edge dislocations in the superlattice. The motion of the dislocations along the CDW wave front corresponds to "climb".) We emphasize the role of the moving vortices in converting condensed electrons to free carriers, and describe how the nucleation or annihilation of these vortices at the sides of the sample induces abrupt changes in the ratio of CDW current to free carrier current. The changes in the free carrier current in turn cause jumps in the electric field near the contacts which are observed as voltage oscillations. Our model is a particular realization of Anderson's[9] general considerations on phase slippage in condensed systems when different phase-winding rates of the order parameter $\dot{\Phi}_1$ and $\dot{\Phi}_2$ are imposed in two regions 1 and 2 of a sample. To relieve the phase conflict a train of vortices crosses a line joining 1 and 2 at a rate $\dot{N}_v$ given by $2\pi\dot{N}_v=\dot{\Phi}_1-\dot{\Phi}_2$. A related model on CDW noise generation by phase slippage introduced by Gor'kov[10] will be compared with our model in Sec. VI. Recently, Fisher[11] has discussed length scales in the CDW conduction problem. Our discussion which focuses on the role of moving dislocations and phase slippage in noise generation may be viewed as complementary to Fisher's description, which neglects the effect of dislocations or other mobile defects in the superlattice.

II. PHASE SLIPPAGE AND CURRENT CONVERSION

First, we assume that the sample extends infinitely in the x-y plane and is of thickness b along z. Variations along z will be suppressed. The highly conducting axis of the sample is oriented along x so that the CDW condensate when depinned moves only in the $\pm x$ direction regardless of the orientation of the local electric field E. For simplicity we consider the antisymmetric case in which CDW current I_s is approaching the y-z plane (S) at the origin from both the $x>0$ and $x<0$ directions (Fig. 1, upper figure). Since the total current is zero in both $x>0$ and $x<0$ regions a net current of free carriers I_n must flow away from S along both x directions. Thus S is a sink for condensed charge and a source for free carriers. In the vortex model, charge conversion proceeds by

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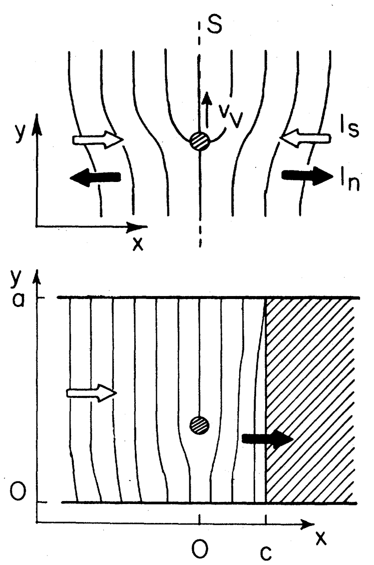

FIG. 1. (Upper figure) A phase vortex (cross-hatched circle) moving with velocity v_V represented as an edge dislocation in the superlattice. Solid lines are the phase contours of the charge-density-wave (CDW) order parameter. In this hypothetical situation CDW current (open arrows) flows towards the surface S while free carriers flow (solid arrows) away from S. (Lower figure) The idealized geometry used in the calculation. The lines $y=0$, $y=a$ represent the sample sides. The sample is covered with conductive paint for $x>c$. In the region $x>0$ ($x<0$) the CDW is pinned (moves to the right.) At S the CDW current is converted to free carriers which are swept into the covered region $x>c$.

the motion of a vortex (or an edge dislocation in the superlattice) moving perpendicular to the direction of I_s. For definiteness we describe the sole vortex (which is oriented along z and moving with velocity v_V along y) as the phase singularity

$$\Phi_v = \tan^{-1}[(v_V t - y)/x] \ . \tag{1}$$

Using the relation for the condensed charge density $\rho_s = (n_s e/Q)\partial\Phi_v/\partial x$ and the CDW current density $J_s = -(n_c e/Q)\partial\Phi_v/\partial t$ (where $n_s e$ is the condensed charge density, $n_c e$ the uniform condensate density, and Q is the CDW wave vector) we calculate the charge and current distribution around the vortex

$$\rho_s = \frac{n_s e}{Q}\frac{y - v_V t}{x^2 + (y - v_V t)^2} \ , \tag{2}$$

$$J_s = -\frac{n_c e}{Q}\frac{v_V x}{x^2 + (y - v_V t)^2} \ . \tag{3}$$

The interpretation of Eqs. (2) and (3) is straightforward. As the vortex moves along the y axis it piles up condensed charge ahead of it, while leaving a region of depleted charge behind so that the charge distribution is dipolar. Condensed charge flows towards S to fill in the channel dug out by the passage of the vortex in such a way that the continuity equation is satisfied. For the infinite geometry J_s is symmetric (antisymmetric) about the x (y) axis. Integrating Eq. (3) over all y and z we obtain the total CDW current

$$I_s = b \int dy \, J_s = (n_c e/Q) b v_V \pi \ . \tag{4}$$

In a time t the vortex moves a distance $v_V t$ and converts an amount of condensed charge equal to $v_V t b \lambda n_c e$ (where λ is the CDW wavelength). It is readily verified that this equals the amount of condensed charge transported towards S in time t by the currents I_s in the $x>0$ and $x<0$ regions. At steady state the normal current I_n which removes the free carriers from S to equal and opposite in sign to Eq. (4). [Because of the extreme anisotropy of the CDW conductance tensor the two quantities J_s and J_n (the free current density) cannot be equal. Near S J_s is sharply peaked at the vortex position whereas the J_n and ρ_n (the free carrier density) are rather more uniform because of the finite transverse conductivity for the free carriers.]

Returning to a more realistic geometry we now let the sample boundaries be at $y=0$ and $y=a$ (Fig. 1, lower figure). The paint contact covers the region $x>c$. [For convenience we keep S (defined as the surface $x=0$) distinct from the paint boundary $x=c$. This distinction is not essential to the argument.] Due to the shunting effect of the conductive paint E in the region $x>c$ is much reduced from its value in the uncovered regions. The CDW is pinned in the region $x>0$ while in the region $x<0$ it slides towards S with a drift velocity v_D. Therefore S again serves as a sink for condensed charge and a source for free carriers. In contrast to the previous symmetric case all the free carriers are swept into the contact by the applied field. Since in all systems studied as yet the CDW current in the bulk is accompanied by a current of free carriers (either thermally activated as in TaS_3 and $K_{0.3}MoO_3$, or arising from unnested portions of the Fermi surface as in $NbSe_3$) we assume there is a finite bulk free carrier current I_{nb}. Then, by current conservation we have $I_{nc} = I_{nb} + I_s = I_{tot}$ where I_{nc} is the current under the contacts and I_{tot} is the total current in the sample, i.e., at S the free carrier current undergoes a jump in magnitude to compensate for the vanishing of I_s [Fig. 2(a)]. For small I_s a single vortex moving along $x=0$ converts the condensed charge into free carriers at a rate determined by the vortex velocity v_V. Although at steady state the total current flowing into S matches the outflow, there exists a charge buildup q_T at S which is determined by balancing the conversion rate against the ohmic conductance near S. (We shall call the charge buildup a conversion bottleneck.) As E is increased, condensed charge is carried to S at a higher rate which can be accommodated only to a limited extent by an increase in the vortex velocity v_V. Eventually the system finds it more favorable to spontaneously nucleate a new vortex at $y=0$. The (nonlinear) I-V curve with N vortices inside the sample is clearly different from that with $N+1$ vortices because of the different conversion rates. The different I-V curves are shown as parallel branches in Fig. 2(b). As a vortex is injected or removed from the sample the system jumps from one intersection of the "load line" with the family of I-V curves to another. [If the total current is held constant the load line is horizontal as shown in Fig. 2(b).] The *steady-state* voltage undergoes discrete jumps of magnitude proportional to

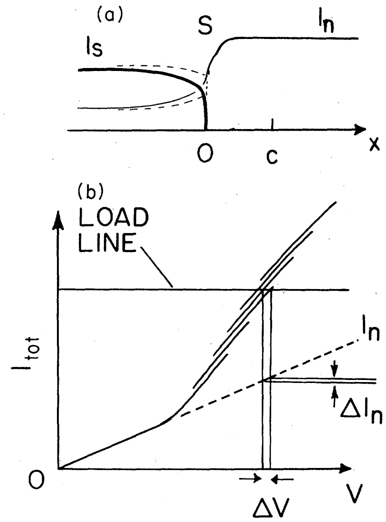

FIG. 2. (a) Schematic plots of the magnitude of the (steady-state) charge-density-wave current I_s (bold line) and the free carrier current I_n (thin line) versus distance along the sample axis x when the number of moving vortices in the sample equals N. At S, I_s vanishes and I_n increases to keep the total current constant. The dashed lines are the values of I_s and I_n when N increases by 1. (b) The family of nonlinear I-V (current-voltage) curves of the CDW, with each branch corresponding to a different number of vortices in the sample. A background free carrier current is also present. The intersection of the load line (which is horizontal when the total current is constant) with the I-V curves determines the observed V. At high field the intersection point jumps back and forth between two adjacent branches.

the normal resistivity.

The simple expressions in Eqs. (1)—(4) are no longer valid due to the distortions at the boundaries $y=0$ and $y=a$. However, the charge conservation arguments still apply and one may conclude quite generally that Eq. (4) holds in the region $x<0$ with an extra factor of 2 (due to the fact that the charge flow previously carried by I_s no longer exists in the region $x>0$). Thus,

$$I_s=2(n_c e/Q)bv_V\pi=n_c e\lambda bv_V \tag{5}$$

and in general the change in the CDW current caused by the nucleation of one vortex is

$$\Delta I_s=2(n_c e/Q)bv_V\,\Delta\Phi_v\ , \tag{6}$$

where in Eq. (6) we have introduced the phase jump

$$\Delta\Phi_v=\Phi_v(y=a)-\Phi_v(y=0)\ . \tag{7}$$

Equation (6) may also be obtained by integrating Eq. (3) over y. It implies that when a vortex nucleates at the site $(x,y)=(0,0)$ at time $t=0$ the phase difference jumps by π, inducing a corresponding surge in the CDW current given by Eq. (6) (Fig. 3). After a time a/v_V the exit of this vortex at $(0,a)$ causes a corresponding drop in I_s. The nucleation of a vortex generates free carriers at the rate given by Eq. (5) until it leaves the sample. This free charge is immediately swept into the contacts by an enhanced free current at $x>0$. However, at steady state a

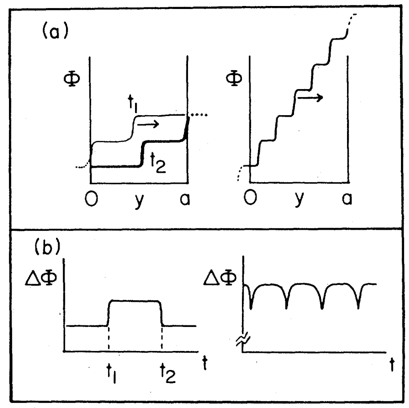

FIG. 3. (a) Schematic variation of the order-parameter phase Φ vs y along $x=0$. The stepwise increase in Φ represents a vortex moving along y. In the figure on the left-hand side, snapshots of Φ vs y at succeeding times t_1 and t_2 are drawn for applied fields E close to the threshold. At t_1 (t_2), a vortex enters (leaves) the sample at $y=0$ ($y=a$). In the figure on the right-hand side (for large E), the high phase-winding rate in the bulk forces the vortices to be densely packed in the sample. (b) Variation of $\Delta\Phi=\Phi(a)-\Phi(0)$ with respect to time for the two situations in panel (a). In the figure on the left-hand side, the entry (exit) of a vortex at time t_1 (t_2) causes $\Delta\Phi$ to increase (decrease) abruptly. The sample voltage is proportional to $\Delta\Phi$. If the density of vortices is high (right-hand-side figure) the jumps in $\Delta\Phi$ become spikes which point up (down) if the entry of a vortex at $y=0$ immediately precedes (follows) the exit of another at $y=a$.

free charge accumulation exists at S due to the conversion bottleneck.

[The process of integrating over y to derive Eqs. (5) and (6) implies that the strong variation in J_s relative to y is unimportant. This needs some justification. In the experiment $\mathbf{J}_s$ is strictly one-dimensional ($\mathbf{J}_s\cdot\mathbf{y}=0$) whereas the free carrier conductivity has a finite anisotropy ($\sim$20 in $NbSe_3$, 100 in TaS_3). Therefore the flow of free carriers along y is not prohibited. Furthermore, the presence of the equipotential plane at $x=c$ due to the high isotropic conductivity of the silver paint tends to force the equipotential lines to lie parallel to the y axis. (We assume that the paint boundary is flat. If it is curved the argument is the same with S parallel to the paint boundary.) Therefore the y variations in the *free carrier* current density J_n and E are strongly suppressed; despite the strong y variation of J_s in the vicinity of a vortex the y variation of J_n is rather more uniform. It is J_n which generates the observed voltage oscillations.]

III. SCREENING OF VORTEX ARRAY AND THE PHASE-SLIP LENGTH

In applying the above model to physical systems we should introduce a length scale which describes the screening of the vortex array from the bulk of the CDW. This length scale arises quite naturally when one considers the "rigidity" of the CDW current. As we emphasized above, the drift velocity of the CDW must vanish near a contact (or some other strong pinning center). The extent to which information on this blockage propagates into the bulk depends primarily on the ease with which local changes in the drift velocity v_D can be accommodated in the moving condensate. We assume that in the bulk of the drifting condensate *there already exists* a finite density N_Φ of mobile dislocations. Further creation of dislocations as necessitated by phase slippage at the contacts will lead to dramatic changes in v_D near the contacts on a time scale of the period of the voltage oscillations. However, these changes will be screened from the bulk by local motion of the preexisting dislocations. Thus, the nucleation of a new vortex near the interface S will cause v_D to increase near S. The drift velocity is sustained at this value as long as the vortex stays in the sample. However, collective motion of dislocations already present in the bulk will tend to compensate for this transitory increase, thereby screening out the perturbation in v_D from the interior of the condensate. On the average fluctuations in v_D decay exponentially into the bulk in a distance which we call (in the static limit $\omega \to 0$) the phase-slip length l_Φ. In the geometry of Sec. II we have

$$\langle \delta v_D(x) \rangle_t = \langle \delta v_D(0) \rangle_t \exp(-|x|/l_\Phi) , \tag{8}$$

where $\langle \ \rangle_t$ denotes time-averaged and δv_D is the fluctuation in v_D.

It may be useful to compare l_Φ with other length scales in the CDW problem. In the pinned state the CDW breaks up into Lee-Rice[12] domains roughly of size ξ_0. Thus, the static phase-phase correlation decays with the Lee-Rice length ξ_0 as

$$\langle \Phi(x)\Phi(0) \rangle \sim \exp(-x/\xi_0) . \tag{9}$$

From perturbation-calculations and scaling arguments Fisher[11] has proposed that in the sliding state the local velocities are semicoherent in regions of size ξ which may greatly exceed ξ_0. Near threshold the velocity coherence may extend over very large regions of the sample with ξ diverging as

$$\xi(E) = \xi_0(E - E_T)^{-\nu} , \tag{10}$$

where $\nu = \frac{1}{2}$ in mean-field theory.

Confining our attention to the sliding state far from threshold we examine the question of the "rigidity" of the CDW current. Clearly, in the absence of moving dislocations (or other means of carrier conversion) charge conservation requires very long-range correlation in the drift velocities v_D *along* the chain direction x, i.e., changes in the drift velocity in one region of the sample will be transmitted to a distant region unless the condensate is allowed to convert to free carriers somewhere in the middle. This long-range correlation is especially sensitive to phase slippage in general. We adopt the point of view that the generation of phase vortices is in fact the *most important mechanism* for disrupting the phase coherence and for adjusting the drift velocity in inhomogeneous geometries. The time averaged (dc) velocity-velocity correlation is thus given by

$$\langle v_D(\mathbf{x}) v_D(0) \rangle \sim \exp(-|\mathbf{x}|/l_\Phi) , \tag{11}$$

where l_Φ is the phase-slip length and $\mathbf{x}$ is parallel to the drift velocity. In the absence of inhomogeneous fields or conversion processes such as the phase slippage discussed above the dc velocity correlation length may become comparable to the sample length L. However, when these processes are allowed Eq. (11) restricts the dc velocity coherence to the shorter length l_Φ. These considerations apply in the dc limit. When vortices are created at the contacts at a finite frequency ω the effective screening length is further reduced to the coherence length $\xi(\omega)$ of the ac component of the drift velocity, given by

$$\langle \delta v_D(\mathbf{x},\omega) \delta v_D(0,\omega) \rangle \sim \exp[-|\mathbf{x}|/\xi(\omega)] , \tag{11a}$$

where $\delta v_D(\mathbf{x},\omega)$ is the Fourier component of the drift velocity at $\mathbf{x}$ and $\xi(\omega)$ is the magnitude of the length ξ in Eq. (10) when the spontaneous voltage oscillation frequency is ω. Allowing ω to approach 0 we must have the relationship

$$\lim_{\omega \to 0} \xi(\omega) = l_\Phi .$$

In Fisher's model, ξ diverges without limit as $\omega \to 0$ [Eq. (10)]. If our assumption that vortices are freely created is valid then this divergence is cut off at the length l_Φ. In general we have $l_\Phi \geqq \xi(\omega)$. According to Fisher $\xi(\omega)$ must also exceed ξ_0.

In the usual experiment the total current is held constant while the noise is monitored as voltage fluctuations. When a single vortex enters the sample it induces a finite jump in the local winding rate. The corresponding jump in I_s given in Eq. (6) is opposed by an equal and opposite free carrier current fluctuation $\Delta I_n = -\Delta I_s$. The local electric field $E(x)$ which is rigorously equal to $J_n(x)\rho$ (where ρ is the normal resistivity) will thus display a jump ΔE of magnitude $\Delta I_n \rho/(ab)$ at S, as shown in Fig. 2(b). From the foregoing discussion these changes in J_s and E are screened in a distance l_Φ (in the static limit) so that the amplitude of the ac electric field decays into the bulk as

$$\Delta E(x) = n_c e (\lambda v_V/a)(\Delta\Phi_v/\pi)\rho \exp(-|x|/l_\Phi) , \tag{12}$$

where Eq. (6) has been used. Thus, the observed voltage fluctuation is given by

$$\Delta V = n_c e (\lambda v_V/a) \rho l_\Phi (\Delta\Phi_v/\pi), \quad \omega \to 0 . \tag{13}$$

Equation (13) gives the voltage jump magnitude expected from the nucleation or annihilation of a vortex when the time scale is of the order of milliseconds. Note that it is proportional to n_c, ρ, and v_V and inversely proportional to a. The factor $(\lambda v_V/a) = v_D$ may be seen to be the drift velocity of the CDW in the bulk far from S. The phase jump $\Delta\Phi_v$ is $+\pi$ if a vortex enters at (0,0) and $-\pi$ if it exits at $(0,a)$ [see Eq. (7) and Fig. 3]. If in a time t, N vortices traverse the sample, the total phase change equals $2\pi N$.

Typical numbers for the voltage amplitude are 10 to 50 μV in $NbSe_3$ and 100 μV to 1 mV for TaS_3 above 90 K. The noise amplitude for a typical sample is affected by factors which can be experimentally controlled as well as by factors which cannot. Thus, tests of various models based on amplitude measurements alone are difficult and

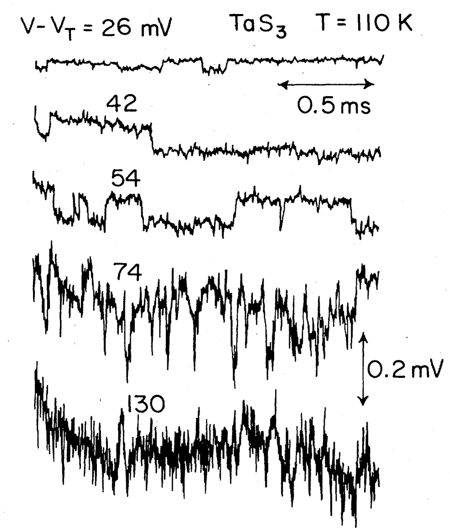

FIG. 4. Time variation of the voltage across a sample of orthorhombic TaS_3 for several values of the electric field. Near threshold (upper traces) voltage jumps occur stochastically. The jumps evolve into sharp spikes which are observed as quasiperiodic noise at higher field values. The data are from Ref. 13.

conflicting results exist in the literature. (See below.)

Equation (13) gives an expression for the amplitude which can be compared with experiment rather directly. In recent experiments Ong, Kalem, and Eckert[13] (OKE) report that near threshold in TaS_3 the periodic voltage spikes constituting the observed narrow-band noise degenerate into stochastic voltage jumps (in both directions) quite suggestive of the vortex creation and destruction events discussed above (Fig. 4). In the range of $E-E_T$ where individual steps are seen in isolation OKE show that the step heights are quantized in units which are E dependent. Using the data $a=10\ \mu$m, $b=3\ \mu$m, L (total sample length)$=50\ \mu$m, $l_\Phi=30\ \mu$m, $\lambda=1.2$ nm, $n_c=10^{21}$ cm^{-3}, $\rho=1.4\ \Omega$ cm (at 110 K), $\Delta V=100\ \mu$V, we calculate from Eq. (13) that $v_V=1.2$ cm/s. Therefore, the vortex takes 0.8 ms to traverse the sample width, in reasonable agreement with the data of OKE in which voltage jumps occur approximately 0.5 to 1 ms apart when the jump magnitude is 100 μV. One may argue in another way, using the phase-slip frequency f. A value of f of 1 kHz (approximately when periodicity is lost) implies that v_D (CDW drift velocity) equals 1.2 μm/s. This implies that v_V $(=a/v_D\lambda)$ is of the order of 1 cm/s, consistent with the value estimated from the observed ΔV.

IV. EXPERIMENTAL CHECKS

Other qualitative predictions of Eq. (13) are noteworthy. The magnitude of ΔV is expected to scale with ρ, n_c, and to be independent of L (when $L>l_\Phi$). In both TaS_3 and $NbSe_3$ ΔV increases approximately linearly with I_s very near threshold before saturating to a field independent value. The saturated ΔV decreases with the order parameter[14] as $[(T_c-T)]^{1/2}$ near the CDW transition in both systems in accordance with the factor n_c in Eq. (13). (However, this appears to be true of all models of the noise.) The dependence on ρ is also consistent with experiment. (For example, in impurity driven models the free carriers play no essential role in the ac generation.) In $NbSe_3$ there is some evidence that ΔV at saturation varies with T approximately as ρ over the temperature range 90 to 140 K. In orthorhombic TaS_3 ΔV initially increases as T decreases below 210 K, suggesting a parallel trend between ΔV and ρ. The maximum ratio of ΔV in TaS_3 to that in $NbSe_3$ (approximately 100) is somewhat smaller than calculated from the resistivity ratio alone (1000). Nonetheless there is some evidence that ΔV changes qualitatively as ρ in the two systems. More systematic comparisons of ΔV with ρ are desirable. In particular, high-field magnetoresistance can be used to tune ρ in $NbSe_3$ while keeping all other parameters fixed. However, since any model that involves the free carriers will satisfy this test it is not discriminating.

Next, we consider the sample length dependence of the ac voltage amplitude. Since experimental studies are usually performed on the Fourier components of the ac voltage at frequencies 1 to 30 MHz we replace l_Φ by $\xi(\omega)$ in Eq. (12) and write for the ac voltage amplitude

$$\Delta V(L)=n_cE(\lambda v_V/a)(\Delta\Phi_v/\pi)\rho\xi(\omega)\{1-\exp[-L/\xi(\omega)]\}. \tag{14}$$

Recalling that $\Delta\Phi_v/\pi$ is the number of vortices inside the sample and a/v_V is the time for each vortex to traverse the sample width we may write Eq. (14) in the more transparent form

$$\Delta V(L)=n_cE(\lambda\dot{n}_\Phi)\rho\xi(\omega)\{1-\exp[-L/\xi(\omega)]\}, \tag{15}$$

where $\dot{n}_\Phi$ is the number of vortices that cross a line parallel to the CDW drift velocity per second. When $L\gg\xi(\omega)$ as is usually the case, ΔV is independent of L as found by Verma *et al.*[15] For very short lengths ΔV varies linearly with L. In that case the phase-slip mechanisms at the two ends will be strongly synchronized. (See Sec. VII.) The CDW current will be phase coherent throughout the length of the sample, even in a thermal gradient, and one expects ΔV to scale linearly with the sample resistance or L.

The sample size dependence of ΔV has been studied by two groups who get different results. Mozurkewich and Grüner[16] (MG) find that ΔV scales as $\sqrt{L}$ in $NbSe_3$ and as 1/(cross section)$^{1/2}$ in both $NbSe_3$ and $(TaSe_4)_2I$ whereas Verma, Ong, and Eckert[15] (VOE) find ΔV to be independent of L over a factor of 60 in length variation in $NbSe_3$. VOE discuss the importance of thermal stability, large L variation and signal averaging in such measurements. There appear to be several other factors that strongly affect the magnitude and quality of the ac voltage although they are not amenable to experimental control. VOE found in length-dependent studies[16] that the spectral linewidths can be sharp (10 kHz) for an initial set of contacts. On warming up to room temperature and attaching new contacts the linewidths can broaden to 200 kHz. Finally when a third set of contacts are used the linewidths can sharpen up again to 10 kHz. Grimes[17] has found that in a pure sample of $NbSe_3$ on which several high quality gold film contacts had been evaporated no noise was observed in all but one contact (i.e., no ac signal is observed unless one of the preamplifier leads touches the noise generating contact). Fleming and Schneemeyer[18] have searched for voltage oscillations in $K_{0.3}MoO_3$ and found that only 1 out of 20 crystals display narrow-band noise. Recently Fisher[19] reported that voltage oscillations in TaS_3 could only be observed in the time domain only after the samples had been subjected to a 300 V/cm conditioning pulse. Fisher also concluded that contacts play a dominant role in generating the oscillations. All these observations suggest that the quality of the contacts (not easily defined) affect the noise amplitude.

An important consideration in the analysis of voltage noise in a conductor involves the screening of current transients inside the conducting medium. As derived by Schockley,[20] and more recently by Landauer,[21] the electrons in the electrodes effectively screen out transients in charge patterns. The motion of a charge q over a distance d inside the sample appears as a much reduced charge $q'=q(d/L)$ flowing in the external wires, where L is the sample length. A naive scheme in which periodic shot noise is generated by succeeding charged waves (or solitons, etc.) impinging on an electrode would certainly not work because of the foregoing objection. In our model the appearance of the vortices causes changes in the voltage drop along the sample. Insofar as these jumps are between steady-state solutions of the nonlinear circuit equations [see Fig. 2(b)] they represent changes in the *conductive* voltage (as opposed to transient electrostatic voltages). Therefore the arguments of Schockley and Landauer although correct are not pertinent to our model.

V. EFFECT OF A THERMAL GRADIENT

A different and experimentally more clear cut test of the phase-slip model is provided by thermal gradient experiments.[6,22] For as yet unknown reasons the threshold E_T varies dramatically with T in all CDW conductors with the exception of $K_{0.3}MoO_3$ below 60 K. In $NbSe_3$, E_T attains a minimum near 50 K. By holding the total current constant and *uniformly* heating the sample from 30 K to above the transition T_c at 59 K one observes that the phase-slip frequency f increases to a maximum value near 52 K before decreasing to zero a few degrees below T_c [Fig. 5(a)]. If different temperatures T_A and T_B are maintained at the two ends (A and B, respectively) of the sample different phase-slip rates occur at the two ends and one should expect to see the fundamental frequency split into two peaks (f_A and f_B) of equal amplitude in a spectrum analyzer. Such behavior was observed by Verma and Ong,[6] who further verified that if T_A is held constant while T_B is varied f_A stays fixed while f_B follows T_B. In particular f_B vanishes when T_B approaches T_c. Zhang and Ong[22] (ZO) have performed an experiment in which both ends of a $NbSe_3$ sample are heated while the middle is kept at 40 K. As the end temperatures approach T_c all frequency components in the spectrum vanish. ZO also observe that in very short samples (less than 0.8 mm) frequency locking between f_A and f_B obtains over a temperature difference of 10 K. These experiments provide unambiguous evidence that the source of the oscillations is located at the sample ends.

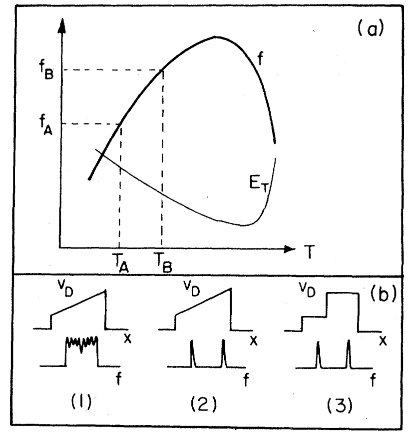

FIG. 5. (a) The variation of the narrow-band frequency f (bold line, at fixed current) and the threshold electric field (thin line) with respect to temperature in $NbSe_3$. (b) Velocity profile (v_D vs x) of the sliding charge density wave and the predicted frequency spectrum in three models. The sample is maintained in a thermal gradient with end A (B) at T_A (T_B). [See panel (a).] In all of the models f is proportional to v_D. In the incoherent impurity model (1) a continuous band is predicted in a gradient while the phase-slip model (2) predicts two lines of equal intensity, which is in fact observed. The observed spectrum can be fitted, *a posteriori*, to the coherent impurity model (3), but only by assuming a step-function profile.

The velocity profile (v_D versus x) of the sliding CDW in a thermal gradient assumes a trapezoidal shape [Fig. 5(b)]. At end A (B) the sliding velocity increases abruptly from zero and assumes the value appropriate to the local temperature T_A (T_B) while the velocity in the bulk varies linearly between the two extreme values. At steady-state dislocations will have to be created throughout the bulk of the sample to accommodate the continuously varying phase winding rates along the sample length. However, because the rate of creation of these bulk dislocations is rather low (i.e., not proportional to I_s) they play no role in the generation of the ac signal at the phase-slip frequency f (although their role in the broadband $1/f$ noise may be important). At the sample ends the relatively sharp change of v_D (from a finite value to zero over a distance l_Φ) leads to vortex creation at a rate directly proportional to the CDW current density existing at that end. It is this well-defined oscillation that is observed in spectrum analyzers.

VI. OTHER MODELS

In contrast to the phase-slip model discussed here (and the similar model by Gor'kov) there exists a class of models which ascribe the voltage oscillations to interactions between the sliding CDW and random impurities in the bulk. In the simplest models[4,5] the impurities are assumed to present a coherent periodic potential to the moving condensate which is regarded as rigid. The narrow-band noise is generated by modulations of the CDW velocity by the periodic impurity potential. However, the problem of how randomly distributed impurities can present a coherent periodic potential to a rigidly moving periodic structure has not been satisfactorily addressed.

Numerical simulations[23] of such classical models with the additional ingredient of deformability show that the oscillations are phase incoherent from one impurity to the next, and vanish for large samples. Sneddon, Cross, and Fisher,[23,11] and others,[24] have emphasized that the *observed* voltage oscillations cannot be a true bulk phenomenon. Sneddon[23] has shown this to be valid to all orders in perturbation theory. The phase of the oscillating CDW current (or ac velocity v_ω) due to periodic interactions with impurities will be coherent only in a region of size ξ_0 (or at best ξ). Because of the uncorrelated phase the oscillations add with random phase and in the large-volume limit the amplitude is expected to decrease as $(1/\text{volume})^{1/2}$. Klemm and Schrieffer[25] have also suggested a similar result for the ac amplitude. MG[16] have in fact interpreted their data along these lines. Barnes and Zawadowski[26] have suggested a quantum-mechanical mechanism for noise generation based on $2k_F$ scattering of electron pairs by impurities, in analogy with the Josephson effect. This quantum mechanical model has not been generalized to a system with many domains.

In these impurity models (suitably embellished with domains) we should distinguish the question of frequency coherence from that of phase coherence. The phase coherence of the ac component of the CDW current is restricted to a distance of the order of the correlation length ξ. The frequency coherence on the other hand is determined by the dc velocity correlation length which may extend over macroscopic distances, in the absence of conversion or phase-slip processes (as we discussed in Sec. III). Allowing the existence of phase vortices the frequency coherence distance is cut off at l_Φ. The experiments which study the noise amplitude ΔV vs L dependence are relevant to the quantity ξ whereas studies of the noise spectrum with the sample in a thermal gradient are more relevant to the frequency coherence length. Since no experiment shows ΔV scaling linearly with L we clearly have $\xi \ll L$ for L as short as 200 μm. The surprisingly narrow linewidth of the oscillation spectrum implies that (if the noise originates from the bulk of the sample) the frequency coherence length l_Φ is comparable to L. Consequently, viable theories for the noise based on interaction with the impurities must have $l_\Phi \sim L$ while $\xi \ll L$.

However, *no satisfactory mechanism* has been proposed whereby frequency coherence can be maintained over such macroscopic distances. Nominally, in the absence of any locking mechanism the washboard frequency ω is equal to $Qn_c eE/\Gamma$ where Γ is a relaxation rate for the phase of the CDW. In a uniform situation it is plausible that ω is uniform throughout the length of the sample. However, in the presence of "detuning" fields such as a thermal gradient the quantities n_c and E become dependent on the position x along the sample. A successful bulk model would have to incorporate a frequency-locking mechanism which effectively counters the divergent influence of the gradient, even for gradients as large as 150 K/cm. For example, with one end of a sample of $NbSe_3$ held above the critical temperature 59 K and the other at 50 K both n_c and E (which is proportional to the ohmic resistivity) decrease rapidly from one sample end to the other; yet the noise spectrum is observed to consist of one *sharp* fundamental frequency line close to the 50-K value. To be consistent with experiment the frequency-locking mechanism should lock the frequency of oscillation throughout the sample length to a value close to that of the cold end, thus compelling the sliding velocity v_D everywhere to maintain the cold-end value despite the rapid decrease in the force exerted by the field as we move towards the hot end. Such a mechanism is highly nonlocal indeed. It should also be noted that frequency coherence in this situation further requires the CDW current density $J_s \sim n_c v_D$ to decrease rapidly with x.

Assuming that the nonlocal mechanism discussed above exists, one can in principle force the observation of two lines f_A and f_B in a gradient to be consistent with the bulk impurity model by adopting the *ad hoc* assumption that the v_D vs x profile assumes a step function. One-half of the sample oscillates at the frequency f_A while the other half oscillates at f_B [Fig. 5(b), model (3)]. The difficulties inherent in this picture are discussed at length by ZO,[22] who find that the values of f_A and f_B are those that would be observed if the sample was *uniformly* at T_A or T_B, respectively (as opposed to some average value). (See Fig. 4.) ZO argue that the rigid requirement of a step function v_D vs x profile would lead to unphysical results.

The role of phase slippage at the ends of the sample has been studied by Gor'kov.[10] In his (one-dimensional) model the phase winds rapidly in the bulk while it is pinned at one end of the sample. To achieve phase slippage the system drives the order parameter to zero in a narrow region a distance x_0 from the pinning end. The value of x_0 $(=T_c/E)$ which computes out to be 0.5 cm in $NbSe_3$ when $E \sim 10$ mV/cm is rather too large compared to the sample lengths normally used ($L=2$ mm). Gor'kov's model may be more applicable to samples with transverse dimensions close to the BCS coherence length (10 nm). It is likely that wider samples will choose to generate dislocations to relieve the phase conflict (as described here) before resorting to driving the order parameter to zero over its entire cross section.

The phenomenon of phase slippage is more familiar in the context of superconducting whiskers and weak links. In superconducting whiskers very near the transition temperature the occurrence of a finite voltage drop along the sample length leads to a difference in winding rates of the Josephson phase at the two ends. The difference in winding rates is relieved by the spontaneous occurrence of phase-slip events along the sample length.[27] Each event corresponds to the order parameter being driven to zero at a localized spot so that an extra rotation of the phase costs little energy. In weak links or large area Josephson Junctions phase slippage is accomplished by the periodic nucleation of flux quanta at the junction.[28] Gor'kov's model for the voltage oscillations in CDW systems may be considered analogous to the one-dimensional superconducting whisker whereas the vortex model described here is the analog of the large Josephson junction.

VII. DISCUSSION

Just as mobile dislocations play an essential role in the response of ordinary crystals to strain we expect that phase vortices will be equally important in the dynamics of the CDW superlattice. We have focussed here on their role in generating the voltage oscillations, but it is interesting to speculate on the role they may play in bulk phenomena such as depinning of the CDW and the occurrence of hysteretic I-V curves when the CDW is repeatedly pinned and depinned. In this paper no attempt has been made to study how the nucleation of the vortices influences the threshold field value E_T. A better understanding of the vortex creation energy may elucidate some of the questions surrounding the depinning process.

The phenomenological model described here clearly can be improved in several directions. An important missing ingredient is a mechanism for coherence of the oscillations over the cross section of the sample. In pure $NbSe_3$ and TaS_3 the linewidth of the fundamental can be quite sharp (10 kHz). Upon recycling the sample to room temperature and back the single fundamental frequency often splits into a finite number (usually 3 or 4) of equally narrow lines rather than a single broad line. This suggests a domain structure in the *transverse* direction whereby the oscillating centers (vortex arrays) at the sample ends are mode locked within each domain to accommodate the disorder caused by strains in the contacts. Such mode locking is already seen in the studies[22] of ZO which demonstrate that in short samples (<0.8 mm) the phase-slip frequency at one end may be locked to that at the other despite a thermal gradient of 150 K/cm (provided the order parameter everywhere is fully developed, i.e., both T_A and T_B are below 50 K). Since the mode-locking mechanism extends lengthwise over 0.8 mm we expect it to extend over the sample cross section as well, thereby explaining the single sharp line seen in some unstrained samples. The source of the nonlinear field which generates the mode locking is not obvious. ZO suggest that the CDW strain field may be operative. A calculation incorporating the effects of superlattice strain into the vortex model is desirable.

The observation[13] by OKE (Fig. 4) of a smooth transition from stochastic voltage jumps to a periodic behavior (as $E-E_T$ is increased) in orthorhombic TaS_3 is also pertinent to this discussion. When $E-E_T$ is small the nucleation of vortices is stochastic and large fluctuations of the CDW current about the time averaged mean is observed. In OKE's data individual events (voltage jumps spaced 0.1 to 1 ms apart) which we interpret as the nucleation or annihilation of vortices at the sample sides are clearly observed in this regime. As E increases the events are more closely spaced until the interaction between vortices in the array enforces a periodicity which one observes as the narrow-band noise. (The mode-locking mechanism in the previous paragraph enforces the same frequency over macroscopic fractions of the cross section.) In the crossover from stochastic to periodic behavior one often sees intermittent behavior with random jumps interspersed between bursts of periodic spikes (3 to 10 periods long).

ACKNOWLEDGMENTS

Many aspects of this model have evolved in response to suggestions (and criticisms) from numerous colleagues. We thank Per Bak, Sue Coppersmith, Daniel Fisher, George Grüner, C. C. Grimes, Andrew Janossy, Rolf Landauer, Peter Littlewood, George Mozurkewich, and Gautam Verma for stimulating discussions. In particular, we are grateful to Daniel Fisher for emphasizing to us the role of free carriers at the contacts. This research is supported by the National Science Foundation (Grants No. DMR-81-09971 and No. DMR-82-14525). One of us (N.P.O.) wishes to thank the Alfred P. Sloan Foundation for partial support.

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