# Current distributions in the quantum Hall effect

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(Received 21 January 1985)

The spatial distribution of the current and electron density is calculated for an ideal two-dimensional Hartree quantum Hall device in which the electron interactions are treated in the Hartree approximation. The calculations are based on an equilibrium-thermodynamics approach in which a constant current is imposed as a constraint on the system. As a consequence of the two-dimensional nature of the Coulomb interactions, there are large inhomogeneities in the electron-density and current distributions. The net current is found to be due principally to a redistribution of charges at the edges of the sample, while currents flowing in the bulk of the sample play only a minor role. The fact that a few inhomogeneously distributed states carry large currents gives a qualitative explanation for the spatial inhomogeneity and the low total current observed at the breakdown of the dissipationless quantum Hall effect.

## I. INTRODUCTION

An unresolved question in the theory of the quantum Hall effect (QHE) concerns the nature of the current distribution in the Hall device. In the usual geometry, the flow of current in the $y$ direction in a two-dimensional (2D) electron system gives rise to a voltage in the $x$ direction when a uniform magnetic field is applied in the $z$ direction, as depicted in Fig 1. In his theory, Laughlin[1] uses gauge invariance to argue that the QHE current arises as a response by the edge states of the system. Středa and Smrčka[2] argue that the QHE results as the response of edge diamagnetic currents to the applied electric field. Their theory is based on a thermodynamic derivation by Widom[3] and has been connected to Laughlin's theory by MacDonald and Středa.[4] Some work has focused on trying to clarify the role of the edge currents and the current and voltage distributions near the edges of the system.[5–8] However, it remains unclear exactly how much current is carried in the bulk and how much by the edge states for an interacting system.

This question has relevance to the prediction of the breakdown of the dissipationless QHE, which occurs when the current in a particular device exceeds some critical value.[9–11] One proposed explanation of the breakdown phenomenon assumes a dramatic onset in phonon emission by the electrons as their drift velocity exceeds a threshold value. There are, however, still some difficulties with this theory. On the one hand, one can consider phonon-mediated electron transitions within the highest occupied Landau level. It is then necessary to assume that a slowly varying electrostatic potential causes variations in the electron occupation numbers, so that the electron system associated with this Landau level breaks up into occupied and unoccupied regions. Transitions can then occur between these regions.[12] On the other hand, one can consider phonon-mediated electron transitions between the highest occupied Landau level and the lowest unoccupied one. For a pure, noninteracting system one must then assume that a confining potential causes the Landau levels to bend up sharply at the edges. This will then allow the breakdown to occur at the edges.[13] The magnitude of the breakdown current density and of the resistivity will then depend on the specific form of the confining potential. If transitions in the bulk of the system are considered, the predicted current density for breakdown to occur is higher than the experimentally observed value[14] by a factor of about 20.

In this paper, we examine a 2D Hartree jellium system in a strong magnetic field. Because there is no dissipation for small enough total current, we adopt the principles of equilibrium statistical mechanics, and minimize the free energy of the system, subject to the constraint that the total current be constant.[6] This allows us to calculate explicitly the distributions of charge and current in the sample in the presence of 2D Coulomb interactions. Because of the nature of the 2D Coulomb interactions, there are large and very inhomogeneous diamagnetic edge currents and strong variations in the electron occupancies near the edges. These inhomogeneities and variations are further enhanced when a net current is forced through the system.

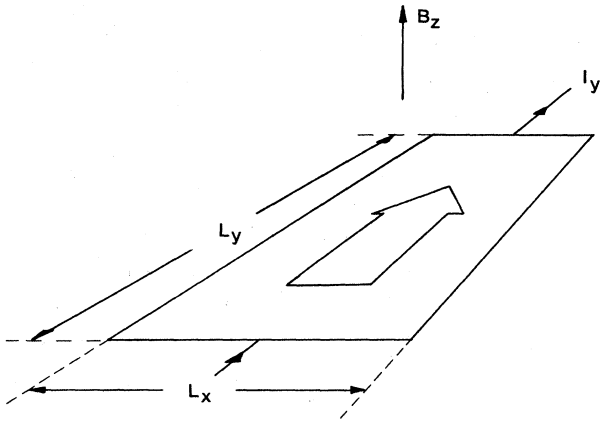


FIG. 1. In the typical QHE geometry an applied constant magnetic field $B\hat{z}$ and a current $I_y$ in the $y$ direction give rise to a voltage across the device in the $x$ direction.

 

The results of these calculations are compared with those of previous work, and the implications are discussed for the theory of the breakdown of the dissipationless QHE.

It is not unreasonable to expect that the charge distribution in an infinitely long 2D conductor of finite width will respond differently to an applied, constant electric field across the conductor from the response to be expected for a three-dimensional (3D) conductor (of infinite height). In the 3D conductor, the equipotential surfaces are planes of constant charge density, perpendicular to the applied electric field, so that the potential resulting from the redistribution of free charges is linear in the transverse direction (the direction of the electric field). In the 2D conductor, however, the equipotential points in the plane of the conductor are straight lines of constant charge density. The electrostatic potential due to a line of charge at the edge of the sample is thus logarithmic in the transverse direction. One then concludes that the charge distribution required to give rise to a field exactly canceling the applied field will be radically different from that for the 3D conductor.

A 2D conductor will also behave differently from an infinitely high 3D conductor at the edges. A simple model of the conductor-vacuum boundary for the 3D conductor would be a dipole *sheet,* since the Fermi pressure will cause the electron density to spill over the edge, thus leaving a deficiency in negative charge density just inside the conductor and a net negative charge density just outside. The constant electrostatic potential in the bulk will then be separated by a potential step from its constant value $V_0$ in the vacuum. The work function $\Phi_F$ is then well defined as the difference between $eV_0$ and the Fermi energy $\epsilon_F$ in the bulk.

For the 2D conductor, however, the conductor-vacuum boundary in the plane of the conductor will be a dipole *line,* and the electrostatic potential will vary as $|x-L_x/2|^{-1}$, where $L_x/2$ is the position of the boundary. One consequence of this is that the work function is not as easily defined as for the 3D conductor. We will return to this point in Sec. III.

So far, the only attempt to describe the charge density and the resulting current and voltage distributions for a QHE system has been made by MacDonald *et al.*[7] They used a Hartree scheme in which the electrostatic potential was approximated locally by a linear function, thus avoiding the mixing of Landau levels. In their paper they showed the resulting charge, current, and voltage distributions to be weighted towards the sample edges with a decay length that depends on the sample size and the strength of the magnetic field.

The validity of their results rests on the assumption that the curvature of the electrostatic potential energy, $V''(x)$, is much less than $m^*\omega_c^2$, with $m^*$ the effective electron mass and $\omega_c=eB/m^*c$. However, the Coulomb potential energy $V(x)$ is of the order of $e^2/\kappa l_B$, where $\kappa$ is the static dielectric constant and $l_B^2=\hbar c/eB$. The length scale of variation of $V(x)$ is $l_B$, and so $V''(x)\sim e^2/(\kappa l_B^3)$. For $V''(x)$ to be much less than $m^*\omega_c^2$, we must have $e^2/(\kappa l_B^3)\ll m^*\omega_c^2$, from which $e\alpha/(\kappa\lambda_e^*)^2\ll B$, where $\lambda_e^*$ is the Compton wavelength for an electron of effective mass $m^*$ and $\alpha$ is the fine structure constant. Substituting the values for GaAs ($\kappa=13$, $m^*=0.07m_e$) we find $e\alpha/(\kappa\lambda_e^*)^2=6$T, which is a typical magnetic field strength for experimental studies. The assumptions made by MacDonald *et al.*[7] are thus of questionable validity, and suggest the need for a more complete study of these interesting questions.

## II. ELECTROSTATICS AND THERMODYNAMICS

Our system is a 2D jellium slab in the $xy$ plane of dimensions $L_x$ and $L_y$. A current $I$ flows in the $y$ direction and a constant, uniform magnetic field $\mathbf{B}=B\hat{z}$ is applied; we use the Landau gauge and write $\mathbf{A}=(0,Bx,0)$. For $L_y$ very large, the Hartree electrostatic potential energy $V(x)$ is a function of $x$ only. Explicitly

$$V(x)=2e^2\int_{-\infty}^{+\infty}[n_+(x')-n_-(x')]\times\ln|2x'/L_x-2x/L_x|\,dx'+C\ , \quad (1)$$

where $n_-(x)$ is the electron density and $n_+(x)$ is the jellium charge density given by

$$n_+(x)=\begin{cases}N/L_xL_y, & |x|\le L_x/2\\ 0, & |x|>L_x/2\end{cases} \quad (2)$$

with $N$ the number of electrons; $C$ is a constant such that $V(0)=0$. With periodic boundary conditions in the $y$ direction the system is then translationally invariant in that direction. This means that the $y$ component of momentum, which we denote by $k$, is a good quantum number with $k=2\pi l/L_y$, where $l=0,\pm1,\pm2,\ldots$, and we can thus write the wave functions as $\psi_k(x,y)=e^{iky}\phi_k(x)$. In the absence of electrostatic fields, the Schrödinger equation then becomes

$$\left[-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}+\tfrac{1}{2}m\omega_c^2(x-x_k)^2\right]\phi_k(x)=\epsilon_k\phi_k(x)\ , \quad (3)$$

where $x_k=kl_B^2$. Equation (3) then describes a harmonic oscillator centered at $x=x_k$. One can easily show[15,6] that the current $i_k$ carried by the state $k$ is given by

$$i_k=\frac{e}{m\omega_cL_y}\frac{\partial\epsilon_k}{\partial x_k}=\frac{c}{BL_y}\frac{\partial\epsilon_k}{\partial x_k}\ . \quad (4)$$

The question then arises of how to solve Eq. (3) when an external Hall field $E_H$ is applied. One cannot simply write the applied potential $V(x)$ as $V(x)=eE_Hx$, since then the electrons would be unbounded in a potential that would tend to $-\infty$ as $x\to-\infty$. A better approach is suggested by the manner in which the experiments are actually performed. In the laboratory, the current $I$ is usually arranged to be maintained at a certain constant value. The response to this current is noted by measuring the resulting Hall voltage $V_H$ and also the longitudinal voltage drop $V_y$ occurring in the direction of the current. In view of the fact that there is no dissipation in the system (for small enough current), one can then use equilibrium thermodynamics with the constant current included as a constraint on the system. We minimize the free energy $F$ given by

$$F=U+k_BT\sum_\alpha[f_\alpha\ln(f_\alpha)+(1-f_\alpha)\ln(1-f_\alpha)] \quad (5)$$

subject to the constraints

$$\sum_\alpha f_\alpha = N \ , \tag{6a}$$

$$\sum_\alpha i_\alpha f_\alpha = I \ . \tag{6b}$$

Here $U$ is the internal energy (in the Hartree approximation), $k_B$ is Boltzmann's constant, and $f_\alpha$ is the average occupancy of the quantum state $|\alpha\rangle$, which can be labeled by its wave number $k$ and the Landau level $n$ to which it belongs. The minimization gives

$$f_\alpha = \{\exp[\beta(\epsilon_\alpha - \mu - \xi i_\alpha)] + 1\}^{-1} \ , \tag{7}$$

where the constants $\mu$ and $\xi$ are fixed by the constraints (6a) and (6b). The term $\mu + \xi i_\alpha = \mu + (\xi c / BL_y)\partial\epsilon_\alpha / \partial x_\alpha$ then acts as an electrochemical potential with the interesting property that whereas a high energy $\epsilon_\alpha$ disfavors occupation of a state, a high current, i.e., large value of the derivative of the energy with respect to $x_\alpha$, will favor occupation of the state. The competition between these two effects will then cause spatial fluctuations, as we will see later.

## III. RESULTS

Equations (1)—(3) and (7) were numerically solved self-consistently. Each wave function was expanded in its five lowest Fourier-Hermite components, and from the starting point of an assumed initial configuration of electrons, iterations led to convergence. Let us first look at some results from calculations for which the filling factor $\nu$ equals unity. The filling factor is the relevant dimensionless density of the problem and describes how many electrons are in the system relative to how many states are contained in the lowest Landau level; it is defined as $\nu \equiv 2\pi N l_B^2 / L_x L_y$. We take $L_x = 7.2$, $L_y = 50\pi$, $\kappa = 1$, $l_B = 1.0$, $\hbar\omega_c = 1.0$, and $k_B T = 5 \times 10^{-3}$ in units where $\hbar = e = m^* = 1$. Here $\kappa$ is the static dielectric constant. Energies and distances are given in units of $\hbar\omega_c$ and $l_B$, respectively. For these values of magnetic field, effective electron mass and dielectric constant the ratio of the Coulomb energy $e^2/\kappa l_B$ to the magnetic energy $\hbar\omega_c$ is unity, which is fairly close to the value found in typical QHE experiments. Figure 2 shows the electron energies of the two lowest Landau levels, for which $n = 0$ and $n = 1$.

A remarkable feature of these results is the structure in the energies around the edges of the system ($|x| = 3.6$). To verify that these structures did not arise as an artifact of a system with $L_x / l_B$ so small that the electrons at one edge extended to the other, the calculations were repeated for a system with $L_x = 14.4$. Another plausible objection could be that the structure was a Gibbs phenomenon, since the Fourier-Hermite expansion of the electron wave functions was truncated at five terms. To check that this was not the case, we also repeated the calculations for a system with $L_x = 7.2$ but with each wave function expanded in its ten lowest Fourier-Hermite components. In each case the electron energies around the edges were found to have exactly the same values as before to within the numerical accuracy of the calculations.

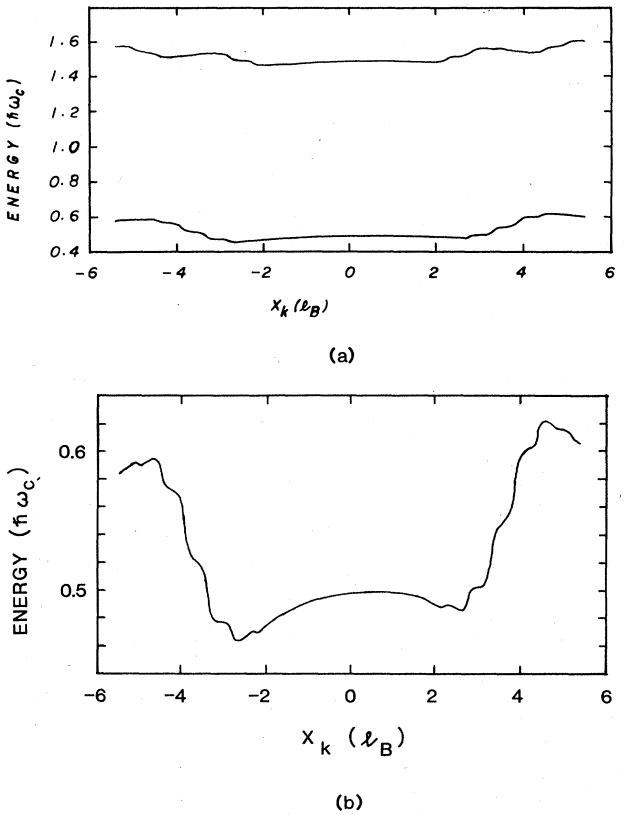


FIG. 2. (a) Electron energies in units of $\hbar\omega_c$ are shown for the two lowest Landau levels at unit filling factor as a function of position of the state. (b) Detail from (a) for the lowest Landau level.

The explanation for these structures is seen in the plot of the self-consistent electrostatic potential energy (Fig. 3). The natural unit of length characterizing variations in the potential energy in our calculations agrees fairly well with the decay length discussed by MacDonald *et al.*,[7] namely $W = (L_x e^2 i / \hbar\omega_c)^{1/2}$, where $i$ is the integer closest to $\nu$. Our results resemble those of MacDonald and co-workers[7] in that both show peaks of this width near the edges. There are, however, important differences between their results and our own. Whereas their calculations show a

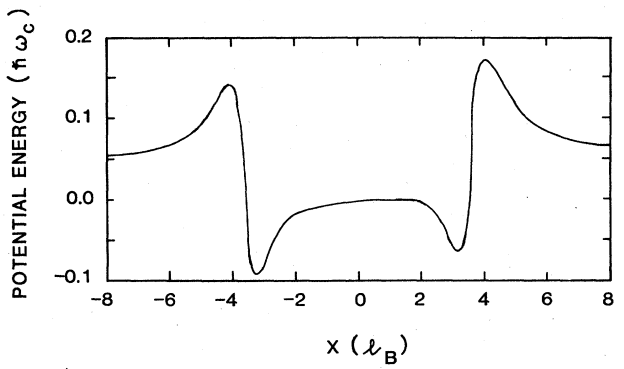


FIG. 3. Self-consistent electrostatic potential energy in units of $\hbar\omega_c$ for unit filling factor.

peak in the electrostatic potential energy at the edge, from which the potential energy decays monotonically both into the bulk and the vacuum, the self-consistent electrostatic potential energy obtained from our solutions has a different appearance. As one would expect, it has an inflexion point exactly at the boundary (since the charge density changes sign there), and a variation as $|x-L_x|^{-1}$ further away from the boundary, as a consequence of the line-dipole charge distribution. Furthermore, the potential energy has a local maximum *and* a local minimum near the boundary, at which points the modulus of the curvature of the potential energy has maxima.

The differences are fundamental, since a close look reveals that all the structure in the energy curves corresponds to points where the potential energy or its curvature have maxima or minima, or to inflexion points of the potential energy. That is, the structure is an effect of the combination of the potential energy and the kinetic energy associated with it, and stems from the 2D nature of the interaction. It is important to emphasize that it is *not* an exchange or a correlation effect, since neither of these interactions were included in the theory, nor is it a manifestation of the electrons redistributing themselves self-consistently in response to the interactions. The fact that it a consequence *only* of the kinetic energy was further demonstrated by the form of the electron energies for the case where all states in the lowest Landau level within the sample were occupied ($f_k=1$ for $|x_k| \leq L_x/2$) but in which the electron distribution was not allowed to relax. The electron energies then showed exactly the same kind of structure as in the case when redistribution was permitted. Also, when a self-consistent solution was generated within the space of the lowest ($n=0$) Landau-level wave functions (so that the kinetic energies were all degenerate), the structures were absent.

An important consequence of these results stems from the fact that the current carried by each state is proportional to the derivative of the energy; the currents that each state can carry will thus show large peaks in the regions around the edges. The electron occupancies will then tend to show wide variations around the edges for the following reasons. On the one hand, the energies increase at the edges, and this has the effect of lowering the occupancies. On the other hand, there is a wide range of currents carried by the various states, and this favors an increased occupancy for the state if the current carried is positive, and strongly disfavors occupancy if the current carried is negative. In Fig. 4 we show the electron occupancies of the lowest Landau level. At the left edge of the system there is a dip in the occupancies at $x=3.35$, corresponding to the large negative derivative of the energies at this point, which causes the electrons there to carry a large negative current. On the other hand, there is a broad peak in the occupancies for $-3.7<x<-3.5$, since in that interval, $\partial\epsilon/\partial x$ is very small, so that even though the energies have increased somewhat from $x=-3.35$, these states carry such small negative currents that they can be more easily occupied. Similarly, at the right edge there is a dip in the electron occupancies at $x=3.6$ since those states carry relatively small currents, but at $x=3.9$ the electrons carry large currents so that these states are easily occupied.

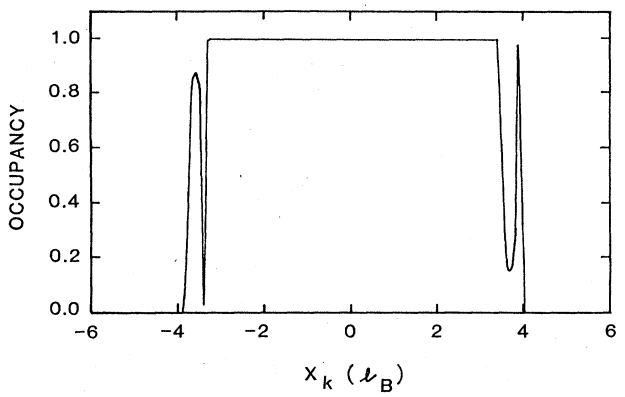


FIG. 4. Electron occupancies of the lowest Landau level for $\nu=1$ in the presence of a small current.

The effect of these peaks in the current distribution and occupancies is that there are a few states at both edges that carry very large currents. Figure 5 shows the current carried per state, the total current in this case being 0.01 in the system of units used. While the current density is negligible in the interior of the sample, there are large, inhomogeneously distributed currents circulating at the edges. The net total current constrained to flow in the system occurs in two ways — through modification of the current carried by states already occupied and by changes in occupancy. That is, the current occurs through changes in both the terms $i_\alpha$ and $f_\alpha$ that enter Eq. (6b). Changes in $f$ occur as electrons are displaced from the left side, where the current carried by the states is predominantly negative, to the right side, where the electrons carry positive current.

We are now in a position to identify the location of the major contribution to the total current. We subtract the currents that circulate in equilibrium, which are the normal diamagnetic response to the applied magnetic field, from the current density in the case where a net current is

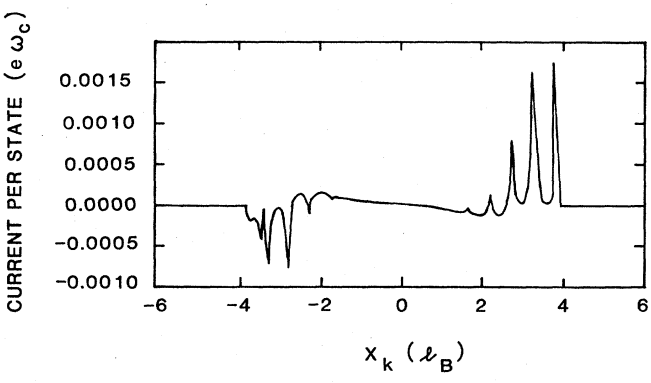


FIG. 5. Net current carried per state in the lowest Landau level for $\nu=1$ and $I=0.01\ e\omega_c$.

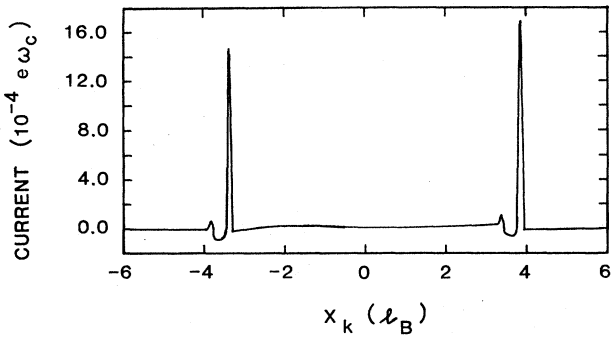


FIG. 6. Change in current carried per state across the sample due to an applied current of strength $I=0.01\ e\omega_c$ for $\nu=1$.

constrained to flow. The result is shown in Fig. 6 for the same set of parameters as before. The filling factor $\nu$ remains equal to unity. The large peaks in this function illustrate very clearly that the majority of the externally imposed current is carried at the edges of the sample.

The sharp spatial structure in electron occupancy found in these calculations is not restricted to the case of unit filling factor. It is, in fact, even more pronounced at fractional filling factors. The case of $\nu=\frac{1}{3}$ is shown in Fig. 7, where the occupancies for zero current and a current of strength $I=0.005$ are plotted as function of position.

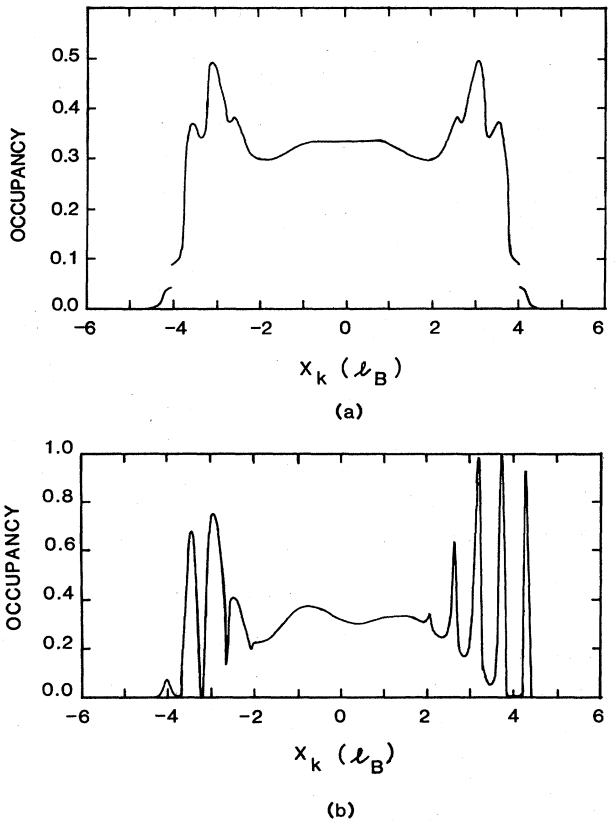


FIG. 7. (a) Electron occupancies as a function of position of the states for $\nu=\frac{1}{3}$ and $I=0$. For this system, $L_x=8.0l_B$. (b) Electron occupancies for $\nu=\frac{1}{3}$, $I=5\times10^{-3}e\omega_c$ and $L_x=8.0l_B$.

The fact that there are large, inhomogeneously distributed edge currents is very interesting. With a state $k$ that carried a current $i_k$ we can associate a drift velocity $v_d=i_kL_y/e$. We can estimate the maximum drift velocity by the following argument. At the edges, the charge density changes in a distance of order $l_B$. Thus the difference between the positions of the maximum and the minimum of potential energy is of order $l_B$. The order of magnitude of the peak-to-peak difference in potential energy is of order $e^2/\kappa l_B$, so it follows that $dV/dx\sim e^2/\kappa l_B^2$. Since it is largely the change in potential energy that will contribute to $\partial\epsilon/\partial x$, we have $\partial\epsilon/\partial x\approx\partial V/\partial x\sim e^2/\kappa l_B^2$ and thus from Eq. (4)

$$i_k\sim\frac{c}{BL_y}\frac{e^2}{\kappa_{lB}^2}=\frac{e}{L_y}\frac{e^2}{\kappa\hbar}\ , \tag{8}$$

which gives $v_d\sim e^2/\kappa\hbar\sim10^6\ \mathrm{m\,s^{-1}}$. This qualitative estimate is in accord with our detailed calculations, in which we find the peak current to be about $1.1\times10^{-5}$ A. This corresponds to a drift velocity of $6.0\times10^6\ \mathrm{m\,s^{-1}}$ which agrees well with the above argument. For GaAs, for which $\kappa=13$, the same qualitative ideas predict a maximum drift velocity of $v_d\sim1\times10^5\ \mathrm{m\,s^{-1}}$, while the actual computations give a maximum drift velocity of the order of $3\times10^4\ \mathrm{m\,s^{-1}}$.

We have not, within the model presented here, attempted to calculate the onset of dissipation in the system. However, the preceding argument does lead to some implications for the breakdown. In a real system, there will be spatial variations in the donor charge density,[16] which in GaAs is located above the electron gas. These spatial variations will induce similar variations in the electron charge density, but since the length scale of these variations is much larger than $l_B$, we can still locally describe the electron states as line charges with logarithmic interactions. It then follows that around these spatial variations there will be structures in the electron energies, similar to the structures found near the edges in our calculations. Theses will then cause $f_k$ and $i_k$ to have strong variations on a length scale of order $l_B$. Thus there will be a few states that carry very large currents. The breakdown of the dissipationless QHE must occur at these states at an average drift velocity $\langle v_d\rangle=IL_y/eN$ much lower than the critical velocity obtained in Ref. 14. Also, it follows that the breakdown will be spatially inhomogeneous, as is observed experimentally.[9]

It is of interest to compare the net current in the bulk to the magnitude of the edge currents as the filling factor increases. We have performed calculations for filling factors ranging from 1.0 to 2.0 ($\hbar\omega_c=1$, $l_B=1$, $L_x=8.0$, $L_y=50\pi$) with an applied constant current $I=0.01$ and have calculated both bulk currents and edge currents. We have also calculated the Hall conductivity $\sigma_H$ in order to compare it with results from a previous publication,[6] where we presented a theory for the plateaus in $\sigma_H$ that also was based on the equilibrium thermodynamic approach. In that publication we calculated $\sigma_H$ for a layered system with an effective one-dimensional Hartree interaction. One question is whether the Hartree interactions in a truly 2D system would change the shape of $\sigma_H$ versus $\nu$, since the current carried in the bulk is deter-

mined by the slope of the energy eigenvalues, which will depend on the potential energy in the bulk.

For a 3D system, where the Coulomb potential is one dimensional, the electrostatic potential energy will be steplike when passing through the conductor-vacuum boundary and then approach a constant value far away from the boundary. In a 2D system, the potential will decay as $|x|^{-1}$ away from the boundary (in the plane of the system). As $\nu$ increases from unity, electrons will initially enter states at the edges of the system, since those states will still have energies much less than $\frac{3}{2}\hbar\omega_c$. In the 3D layered system, this increases the work function and thus the Fermi energy until it is energetically favorable to put electrons in the next Landau level. For the 2D system, however, as $\nu$ increases there are always states at $x=\pm\infty$ with lower energies than $\frac{3}{2}\hbar\omega_c$, so electrons will escape to $x=\pm\infty$. In other words, a 2D jellium system will not give rise to a work function. Because a more realistic calculation involving lattice structure and Bloch states for core and conduction electrons was beyond the scope of this work, a potential

$$u(x)=\begin{cases}0, & |x|<L_x/2\\ u_0(|x|-L_x/2)^2, & |x|>L_x/2\end{cases}$$

in addition to the Hartree interaction was applied as an artificial device having the sole purpose of confining the electrons within the sample. This will of course influence the specific nature of the edge states, but the Hartree interactions will still give rise to the interesting behavior of the electron energies at the edges. Moreover, the net edge current is insensitive to the explicit form of the potential at the edges and will depend only on the Fermi energy difference between bulk and edge states. Also, the net current carried in the bulk will depend only on the shape of the Hartree potential in the bulk, and not on the edges.

When the Hall voltage is measured in an experiment, an electron is injected at one edge of the system and another electron is removed from the other edge. The Hall voltage is then measured as the difference in energies at which the one electron is injected and the other one removed, divided by the electron charge. The Hall voltage was accordingly calculated as the difference in electrochemical potential between the two edges of the electron system divided by the electron charge. This was achieved by fitting the electron distribution at each edge as a function $f$ of energy to an expression of Fermi-Dirac form. The Hall voltage was then identified as the difference between the two values of $\epsilon$ at which $f=0.5$, divided by the electron charge.

In Fig. 8 we show the edge currents and bulk current as a function of the filling factor $\nu$ in the case where the parameter $u_0$ describing the confining potential is given the value $0.15m\omega_c^2$. At $\nu=1.0$, the magnitudes of the edge currents are already larger than the net current, and they increase rapidly as $\nu$ increases, raising the Fermi energies as more electrons enter the lower Landau level. It is interesting to note that the total bulk current is small, which for $\nu=1.2$ and $\nu=1.4$ results from $\epsilon_k(x)$ being almost symmetric around $x=0$, so that the bulk currents nearly cancel. Moreover, as the upper Landau level is populated

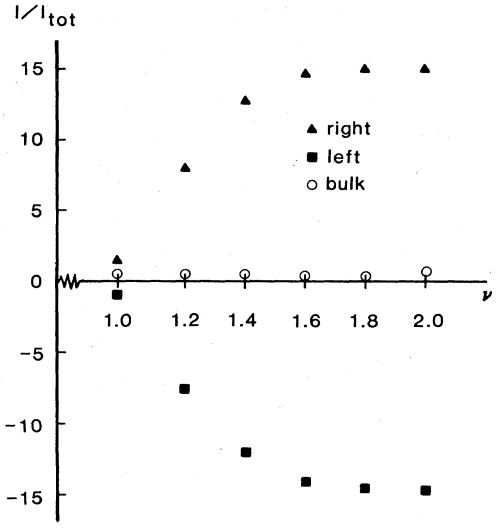


FIG. 8. The relative contributions of the three components of the total current are shown as a function of the filling factor $\nu$ for $u_0=0.15m\omega_c^2$. The two opposing edge currents are much larger in magnitude than the bulk contribution.

more and more, $\epsilon_k(x)$ becomes practically flat so that the states in the bulk carry little current until the upper Landau level is filled all the way to the edges.

What this implies for $\sigma_H$ versus $\nu$ is shown in Fig. 9. To within the numerical accuracy of our calculations, $\sigma_H$ remains equal to $e^2/h$ until the upper Landau level is practically full and the edge states are beginning to be populated, at which point the Hall conductivity increases by a factor of 2. The 2D system of interacting electrons thus exhibits an almost perfect step in the Hall conductivity at $\nu=2.0$; this result is thus closely similar to that previously derived for the case of a noninteracting system.[6] This seems to lead to the conclusion that Hartree interactions in the 2D electron gas give rise to no net bulk current. This does not imply that we do not accord an important role to impurity states in determining the final form of $\sigma_H(\nu)$; indeed, the presence of noncurrent-carrying states will be necessary to move the step between plateaus from $\nu=2$ to $\nu=1.5$. We do make the point, however, that localized states are not needed to explain the presence of the plateaus in $\sigma_H(\nu)$.

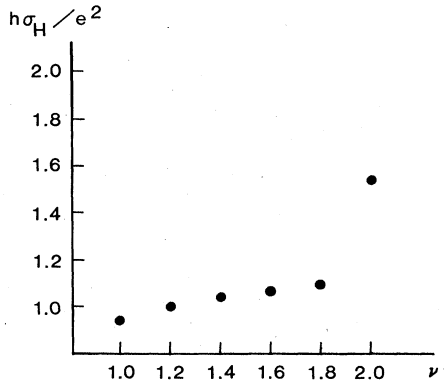


FIG. 9. The Hall conductivity is shown as a function of filling factor $\nu$ in units of its quantum, $e^2/h$.

## IV. CONCLUSIONS

We have presented calculations of current and electron distribution for a 2D Hartree interacting QHE system by using equilibrium thermodynamics with a constant-current constraint. Because of the specific nature of the 2D Coulomb interactions and the peculiar form of the electron distribution function, there are large inhomogeneities in electron and current distributions, and these inhomogeneities are further enhanced by driving a current through the system. The net current is principally due to a redistribution of charge at the edges, which adjusts the large edge currents to give the required net current. Very little net current appears to be carried by states within the bulk of the sample. The fact that a few inhomogeneously distributed states carry large currents gives a qualitative explanation for the spatial inhomogeneity and the rather low total current observed at the onset of breakdown of the dissipationless QHE.

## ACKNOWLEDGMENTS

O.H. acknowledges the hospitality of the Great Lakes Summer Workshop in Condensed Matter Research and P.L.T. that of the Aspen Center for Physics at which part of this work was done. The research was supported by the National Science Foundation through Grant No. DMR83-04244.

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