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Determination of the random-walk dimension of fractals by means of NMR

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It is proposed that NMR magnetic-field-gradient experiments may be used to determine the random-walk dimension of fractals.

In a recent Letter, Katz and Thompson[1] (K-T) have shown that the void spaces of many typical sandstones are volumetrically fractal objects within a significant range of length scales. Evidence that the surfaces of some rocks are fractal up to length scales of 200 Å had been published previously.[2] The novel claim is that the pore volume has statistically the same fractal structure as the surface over length scales of relevance to transport properties.

Here we remark that fractal pore volumes produce an unconventional behavior in magnetic-field-gradient NMR experiments: this technique can complement the K-T experiments to reveal the random-walk dimension[3] d_w, which allows the deduction of the Archie conductivity exponent discussed in their paper.

The key point is that diffusion in the presence of a gradient leads to randomization of the phase angle of the transverse magnetic polarization. Under the hypothesis that the probability distribution of phase angles is Gaussian (central limit theorem), it is well known[4] that the transverse magnetization decays in a normal bulk material as

$$m_{xy}(t) \propto \langle \cos\phi_D \rangle = m_0 \exp\left(-\frac{\gamma^2 G^2 D t^3}{12}\right) . \qquad (1)$$

Following Carr and Purcell, we may interpret the t^3 dependence in Eq. (1) heuristically as follows. The phase developed by a proton diffusing in a fluid relative to its phase in the absence of a gradient is

$$\phi_D(z,t) = \phi[z_D(t),t] - \phi(0,t) = \gamma G \int_0^t dt_1 z(t_1) , \qquad (2)$$

where γ is the proton gyromagnetic ratio and G is the field gradient.

The variance of the Gaussian probability distribution is then

$$\langle \phi_D^2 \rangle = \gamma^2 G^2 \int_0^t dt_1 \int_0^t dt_2 \langle z(t_1) z(t_2) \rangle_c . \qquad (3)$$

Let $g(a,b;t_1,t_2)$ denote the probability of a random walk from a at t_1 to b at t_2. Then, for $t_2 > t_1$,

$$\langle z(t_1) z(t_2) \rangle_c = \int \int dz\, dz' g(0,z;0,t_1) \times z g(z,z';t_1,t_2) z' . \qquad (4)$$

On the assumption that g depends only upon differences in its arguments, as in Ref. 5, the right-hand side of Eq. (4) can be rewritten as

$$\langle z(t_1)[z(t_1) + z'(t_2 - t_1)] \rangle = \langle z^2(t_1) \rangle .$$

This quantity behaves as t_1^{2/d_w}, where $d_w = 2$ on any Euclidean space. Inserting this into Eq. (3) leads to $\langle \phi_D^2 \rangle \propto t^{2+2/d_w}$ in the general case. This power of t replaces t^3 in Eq. (1). A computer simulation of the dephasing due to diffusion on a Sierpinski gasket (for which d_w is exactly known) has been carried out, and confirms the predicted time dependence.

In a pulsed gradient experiment,[6] t^3 in Eq. (1) would be replaced by $\delta^2 \Delta^{2/d_w}$, where δ is the pulse width and Δ is the time interval between the pulses, in the limit $\delta \ll \Delta$.

Thus, if the diffusion is occurring within a fractal volume, the decay will reveal d_w. This is an effect which is distinct from the very short-time decay described by de Gennes,[7] in which the fractal dimension of a fractal surface may be measured if, e.g., surface paramagnetic impurities dephase the magnetization.

There may, in fact, exist three possible regimes: a de Gennes regime since the surface is believed to be fractal, a fractal volume regime, and a long-time regime described by a reduced "bulk" diffusion constant in Eq. (1) which represents diffusional motion to length scales larger than those over which the porous material is fractal. There is obviously a better chance to observe the later regimes if the surface is free of impurities.

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