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# Size effects on electron-electron interactions in GaAs-$Al_xGa_{1-x}As$ heterostructures

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Magnetoresistance measurements are made on the two-dimensional (2D) electron gas in GaAs-$Al_xGa_{1-x}As$ heterostructures to study the electron-electron interaction as a function of the sample width $w$. For $w \gtrsim 20\ \mu m$, the data fit the 2D interaction theory. For $w < 20\ \mu m$, the magnetoresistance shows a large enhancement for $w \simeq 3\ \mu m$, confirming the 2D-to-1D crossover expected from the interaction theory, and a drastic decrease for $w \lesssim 2\ \mu m$, due to the increasing importance of boundary scattering.

There is a great interest in the quantum corrections, due to localization and electron-electron interactions, to the classical Drude conductivity of electronic systems.[1] In the case of the two-dimensional (2D) system realized in semiconductor inversion layers, 2D-to-1D crossover phenomena are expected when the width $w$ of the conducting channel is close to the length scales characteristic of the underlying physics. Several recent publications[2–6] have already reported new features in the transport through narrow channels. Indeed, localization correction is now sufficiently well understood that its 2D-to-1D crossover has been demonstrated to occur when $w$ approaches the electron inelastic scattering length.[4] The interaction correction, on the other hand, is not so well understood and the associated 2D-to-1D crossover has yet to be clearly demonstrated. Recently, Paalanen, Tsui, and Hwang[7] showed that in the classically strong magnetic field regime, localization is completely suppressed and the magnetoresistance directly measures the quantum correction due to electron-electron interactions. We have utilized this experimental fact and investigated the interaction correction as a function of $w$ of the 2D electron gas in GaAs-$Al_xGa_{1-x}As$ heterostructures. Our results agree with predictions from the 2D interaction theory for $w \gtrsim 20\ \mu m$. For $w \lesssim 20\ \mu m$, while agreement with the 1D interaction theory is observed for $w \gtrsim 3\ \mu m$, clear disagreement is apparent for $w < 2\ \mu m$. In this Rapid Communication, we present these results and demonstrate that they manifest the 2D-to-1D crossover in the interaction effect when $w \simeq (\hbar D/kT)^{1/2}$, as predicted by Altshuler, Khmel'nitzkii, Larkin, and Lee.[1] The deviation from the the 1D interaction theory for $w < 2\ \mu m$ is attributed to the increasing importance of boundary scattering, which is expected to decrease the orbital magnetoresistance effect.

From the interaction theory, the correction to the Drude conductivity of a 2D electron system is[8]

$$\delta\sigma = \frac{-1}{2\pi^2}\frac{e^2}{\hbar}g_{2D}\left[\psi\left(\frac{1}{2}+\frac{\hbar}{kT\tau}\right)-\psi\left(\frac{1}{2}\right)\right], \tag{1}$$

where $\psi$ is the digamma function and $\tau$ the elastic impurity scattering time. The interaction parameter $g_{2D}$ is given by[9,10]

$$g_{2D} = 4 - 3\frac{2+F}{F}\ln\left(1+\frac{F}{2}\right), \tag{2}$$

where $F$ is the direct-Coulomb-interaction parameter defined in Ref. 11. When $\hbar/kT\tau >> 1$, Eq. (1) is reduced to

$$\delta\sigma = -\frac{1}{2\pi^2}\frac{e^2}{\hbar}g_{2D}\ln\left(\frac{\hbar}{kT\tau}\right). \tag{3}$$

This correction is insensitive to magnetic field $B$, even in the classical high-field regime $\omega_c\tau > 1$ (Ref. 12) (where $\omega_c = eB/m^*$ is the cyclotron frequency). The Zeeman effect is important only when $B > B_c$, with $B_c = kT/g^*\mu_B$.[13–15] (Here, $\mu_B$ is the Bohr magneton and $g^*$ is the effective electron $g$ factor.) For GaAs, $B_c \approx 43$ kG at $T \approx 1.5$ K.[16] Since our experiments were performed with $B << B_c$, the correction is essentially independent of $B$. Concurrently, the interaction theory also predicts $\delta\sigma_{xy} = 0$. The correction to the resistivity, obtained by inverting the conductivity tensor, is[12]

$$\delta\rho(B) = -\frac{1-(\omega_c\tau)^2}{\sigma_0^2}\delta\sigma, \tag{4}$$

where $\sigma_0$ is the Drude conductivity. Equation (4) predicts an *orbital* magnetoresistance proportional to $B^2$, even though $\delta\sigma$ is independent of $B$. Indeed, the absence of the parallel-field magnetoresistance with $B << B_c$ from our measurements confirms the orbital nature of this magnetic field effect.

Altshuler *et al.*[1] first pointed out that the interaction effect will be 1D when $w$ is less than the thermal diffusion length defined by $L_T = \pi(\hbar D/kT)^{1/2}$, where $D$ is the electron diffusivity. In the 1D case, the theory gives

$$\delta\sigma = -\frac{e^2}{\pi\hbar}\frac{1}{w}g_{1D}\left(\frac{\hbar D}{2kT}\right)^{1/2}, \tag{5}$$

where $\delta\sigma$ is expressed in $(\Omega/\Box)^{-1}$. The interaction parameter is[14]

$$g_{1D} = \frac{4.91}{\pi}\left(1 - 12\frac{1+1/4F-(1+F/2)^{1/2}}{F}\right), \tag{6}$$

for $B << B_c$ and, as in the 2D case, $\delta\sigma_{xy} = 0$. This vanishing of $\delta\sigma_{xy}$ follows from the formulation that $M_y^L(\mathbf{q})$ and $M_x^R(\mathbf{q})$ in Eqs. (59) and (60) of Ref. 12 are odd functions of $q_x$ and $q_y$. When the summation is done on $q_y$ for a finite set of discrete values of $q_x$, $\delta\sigma_{xy} = 0$. Consequently, Eq. (4) also holds for 1D, with $\delta\sigma$ given by Eq. (5). In other words, regardless of the dimensionality of the electron sys-

TABLE I. Sample parameters.

| Device No. | $w$ ($\mu$m) | $N_s$ ($10^{15}$ m$^{-2}$) | $\tau$ ($10^{-12}$ s) | Theor. $T_c$ (K) |
|---|---|---|---|---|
| 1 | 156 | 5.03 | 11.4 | 0.0017 |
| 2 | 34.5 | 5.43 | 9.68 | 0.031 |
| 3 | 6.2 | 5.02 | 11.3 | 1.1 |
| 4 | 3.5 | 5.43 | 9.68 | 3.0 |
| 5 | 3.0 | 4.79 | 7.45 | 2.9 |
| 6 | 1.1 | 4.84 | 10.3 | 28 |

tem, $\delta\sigma$ can be obtained directly from measuring the magnetoresistance in the $B^2$ regime. More specifically, Eqs. (1), (4), and (5) can be combined to yield

$$\Delta\rho(B)=\delta\rho(B)-\delta\rho(0)$$
$$=-\frac{1}{n_s^2}\frac{1}{2\pi^2\hbar}g_{2D}\left[\psi\left(\frac{1}{2}+\frac{\hbar}{kT\tau}\right)-\psi\left(\frac{1}{2}\right)\right]B^2 \text{ for 2D },\tag{7}$$

and

$$\Delta\rho(B)=-\frac{1}{n_s^2}\frac{1}{\pi\hbar w}g_{1D}\left(\frac{\hbar D}{2kT}\right)^{1/2}B^2 \text{ for 1D },\tag{8}$$

where $n_s$ is the electron density. Although $\Delta\rho$ has the same $B^2$ dependence in both dimensions, its dependences on $w$, $F$, and $T$ are sufficiently different in the two cases to allow an unambiguous experimental test of the interaction theory in both dimensions.

Several devices were fabricated by using photolithographic techniques for four-terminal measurements. Their specifications, along with the 2D-to-1D transition temperatures $T_c$ calculated using $w=L_T$, are shown in Table I. The data are taken above 1.5 K with $B<8$ kG. The measurements are made using a lock-in amplifier operated at 145 Hz and a constant-current source of $10^{-8}$ A to avoid electron heating.[17]

Figure 1 shows the perpendicular-$B$ dependence of the channel resistance at 4.2 and 1.6 K, taken from four devices with different $w$. For the samples with $w=156$ and 34.5 $\mu$m, the parbolic magnetoresistance is observed for $B=200$ G up to $\sim4$ kG. In this range of $B$, the quantum corrections due to localization are completely suppressed,[1] but the Zeeman correction has not yet set in. For narrower channels, this parabolic behavior is observable when $B\gtrsim2$ kG, and a temperature-insensitive magnetoresistance is developed at lower $B$. The range of this $T$-independent magnetoresistance increases with decreasing $w$ and it extends to $B\simeq2.5$ kG for $w=1.1$ $\mu$m. At present there is no explanation for this phenomenon, and experiments are planned for further investigations. Here, we focus on the parabolic regime and extract the interaction parameter $g$ directly from its $T$ and $w$ dependences.

In Fig. 2 we show the experimental data, plotted as

$$|\Delta\rho(B)/B^2|n_s^2 2\pi^2\hbar \text{ vs } [\psi(\tfrac{1}{2}+\hbar/kT\tau)-\psi(\tfrac{1}{2})] ,$$

from four wider devices in the parabolic regime. [In the case where the Shubnikov–de Haas (SdH) effect is apparent, the midpoints of the oscillations are being taken.] Here, $\tau$ is deduced *directly* from the conductivity at $B=0$. For devices with $w=156$ and 34.5 $\mu$m, respectively, the data follow two straight lines passing through the origin, as expected from the 2D theory. From their slopes, we obtain

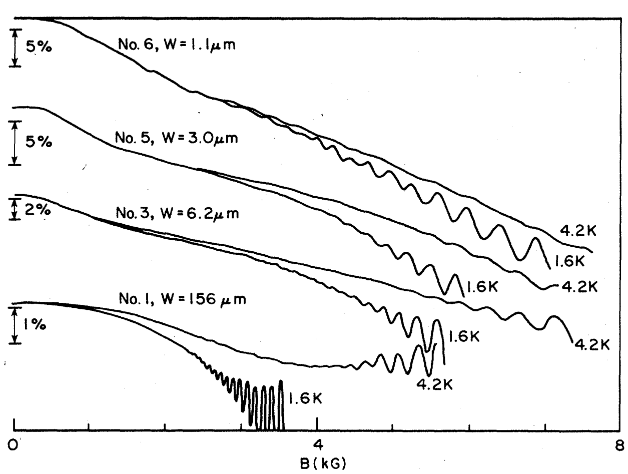


FIG. 1. The change of resistance $\Delta R/R_0$ as a function of perpendicular $B$ at 4.2 and 1.6 K. The irregular structures in the data of device No. 6 below 4 kG are reproducible; they are not Shubnikov–de Haas oscillations.

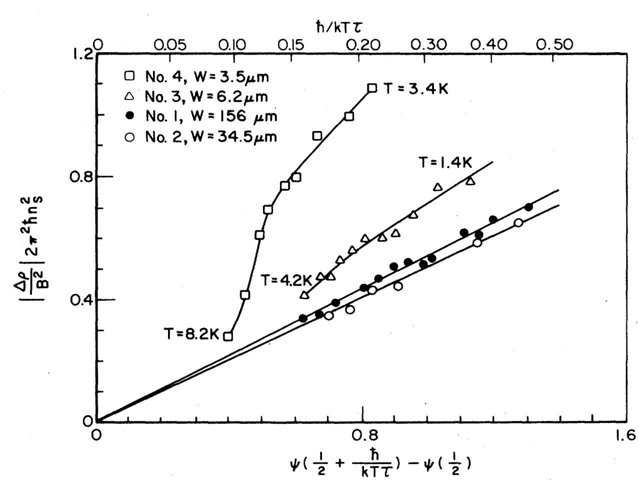


FIG. 2.

$$|\Delta\rho/B^2|2\pi^2\hbar n_s^2 \text{ vs } \left[\psi\left(\frac{1}{2}+\frac{\hbar}{kT\tau}\right)-\psi\left(\frac{1}{2}\right)\right]$$

for devices No. 1, No. 2, No. 3, and No. 4.

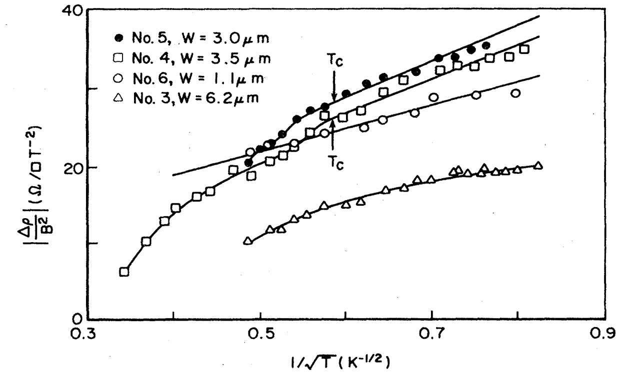


FIG. 3. $|\Delta\rho/B^2|$ vs $1/\sqrt{T}$ for devices No. 3, No. 4, No. 5, and No. 6.

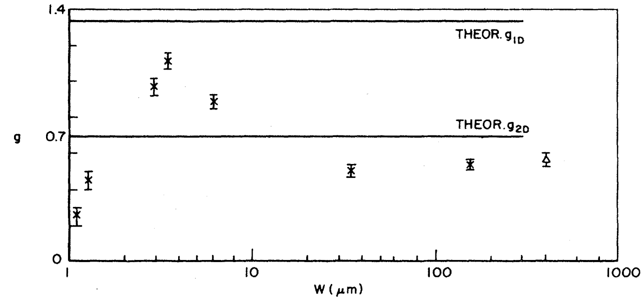


FIG. 4. Experimental values of the interaction parameter $g$ vs $w$. $g$ is extracted either from Eq. (7) or from Eq. (8) as determined by the characteristic temperature dependence of $\Delta\sigma$. Since $g$ is a weak function of $n_s$, the data show only the general trend. The data point ($\Delta$) is from Ref. 7 with $n_s = 1.17\times10^{15}$ m$^{-2}$, using Eq. (7).

$g = 0.54 \pm 0.02$ and $0.51 \pm 0.02$. These values of $g$ are close to the theoretical value of 0.7 from Eq. (2), using $F = 0.45$ for $n_s \sim 5.5\times10^{15}$ m$^2$. Within experimental uncertainties, the data show no $w$ dependence, consistent with Eq. (7). For $w = 6.2$ and 3.5 $\mu$m the data show increasing deviations from the 2D behavior as $T$ decreases. These deviations are suggestive of a transition as $T$ approaches $T_c$ (Table I), expected for the crossover from the 2D interaction effect to the 1D interaction effect.

In Fig. 3, the data from the four narrower devices are plotted as $|\Delta\rho(B)/B^2|$ vs $1/\sqrt{T}$. For $w = 1.1$ $\mu$m, the $1/\sqrt{T}$ dependence, predicted by the 1D interaction theory [Eq. (8)], is observed in the entire $T$ range. For $w = 3.0$ and 3.5 $\mu$m, this $T$ dependence is observed for $T < T_c$, whereas for $w = 6.2$ $\mu$m, the 1D characteristic is not yet fully developed, consistent with the predicted $T_c$ of 1.1 K. In addition, $|\Delta\rho|$ is observed to increase with decreasing $w$ for $w \gtrsim 3$ $\mu$m, consistent with the expected $1/w$ dependence. The $g$ values, deduced from the slopes of the curves using Eq. (8), are shown in Fig. 4, together with those of devices No. 1 and No. 2 obtained from the 2D theory using Eq. (7). The largest value of $g$, obtained from $w = 3.5$ $\mu$m, is $1.11 \pm 0.05$, close to the theoretical value $g_{1D} = 1.33$, calculated from Eq. (6) using $F = 0.45$. For $w = 6.2$ $\mu$m, $g$ deduced from the data at 1.5 K is 0.89, indicating that the device is in 2D-to-1D transisition in this temperature range. For $w < 3.5$ $\mu$m, the $g$ value begins to drop. It can be argued that when $w \lesssim l_e$ ($\sim 3$ $\mu$m), where $l_e$ is the elastic impurity scattering length, the broadening of the Landau levels due to boundary scattering becomes important. The scattering decreases the lifetime of an electron in a Landau orbit and hence decreases the orbital effect. This is reflected in a reduction of $g$ extracted from Eq. (4). Further evidence that boundary scattering is increasingly important in narrower devices is seen in Fig. 1, where the onset of the SdH oscillations occur in higher $B$ for narrower channels. In fact, it may already be significant for $w = 3.5$ $\mu$m and leads to the observed reduction of $g$ from its theoretical value.

In summary, we showed that the correction to the conductivity due to the interaction effect can be deduced from magnetoresistance measurements, regardless of the dimensionality of the electron gas. The result confirms the 1D as well as the 2D interaction theories and shows the presence of the dimensional crossover. The magnitude of the magnetoresistance first increases with decreasing $w$, indicating a 2D-to-1D transition, and then decreases for $w < 3$ $\mu$m. This decrease is believed to be the precursor of the extremely 1D case, in which electrons are localized along both the width and the depth of the channel. The *orbital* effect is expected to be absent.

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