Surface plasmon of a semiconductor superlattice

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A simple explicit dispersion relation is given for the surface plasmon of a semi-infinite layered electron gas. Two limiting cases are discussed. Effects of the optical phonons and magnetic field are discussed.

I. INTRODUCTION

Intermediate between two- and three-dimensional electron gas is layered electron gas (LEG),[1-4] which consists of a regular array of planes of two-dimensional electron gas (2D EG). The LEG model was first employed[1] to approximate graphite and intercalated transition-metal dichalcogenides, and now finds a good physical realization in semiconductor superlattices. For the discussion of intrasubband electronic collective modes of a LEG, in the simplest model one assumes that the electrons are free to move within the planes, do not tunnel to other planes, and remain in the lowest subband.

If one takes the planes of 2DEG situated at $z=ld$, $l=0, \pm 1, \ldots, \pm\infty$, then a (bulk) plasmon is obtained,[4] whose dispersion relation is conveniently expressed as (following the notation of Ref. 3)

$$b=\cos(q_z d) \ , \qquad (1)$$

with

$$b\equiv\cosh(qd)-D^0 V\sinh(qd) \ , \qquad (2)$$

where $\underset{\sim}{q}$ and q_z are the components of the plasmon wave vector parallel and perpendicular to the planes, respectively. D^0 is the polarizability of the 2DEG given by, in standard notation,

$$D^0(\underset{\sim}{q},\omega)=2\int\frac{d^2p}{(2\pi)^2}\frac{f(\underset{\sim}{p}+\underset{\sim}{q})-f(\underset{\sim}{p})}{E(\underset{\sim}{p}+\underset{\sim}{q})-E(\underset{\sim}{p})-\omega} \ ,$$

and for $qv_F \ll \omega$ (v_F is the Fermi velocity) it can be approximated by[5]

$$D^0\sim nq^2/m\omega^2 \ , \qquad (3)$$

where n is the density of the electrons per unit area and m is the effective mass. We shall assume Eq. (3) in rest of the paper. V is the $\underset{\sim}{q}$th component of the two-dimensional Fourier transform of the Coulomb interaction

$$V=2\pi e^2/\epsilon q \ , \qquad (4)$$

where ϵ is the dielectric constant of the background medium. In the above formulas, as well as in the rest of the paper, a notation is used in which often only the dependence of variables on q_z is explicitly shown and $\underset{\sim}{q}$ and ω dependence is suppressed. From the dispersion relation (1) it is clear that for fixed $\underset{\sim}{q}$ the bulk plasmon lies within the band $-1\leqslant b\leqslant 1$, which is called the bulk-plasmon band. Using Eqs. (1), (2), and (3) an explicit dispersion relation follows

$$\omega_p=\left[\frac{2\pi ne^2}{\epsilon m}q\frac{\sinh(qd)}{\cosh(qd)-\cos(q_z d)}\right]^{1/2} \ , \qquad (5)$$

which is valid for $\omega_p \gg qv_F$.

II. SURFACE PLASMON

Now let us consider a semi-infinite LEG[2,3] with planes at $z=ld$, $l=0,1,\ldots,\infty$. If the dielectric constant of the medium is ϵ for $z>0$ and ϵ_0 for $z<0$, i.e., if there is a dielectric mismatch at the surface at $z=0$, then there exists a surface plasmon whose dispersion relation has been obtained in Refs. 2 and 3 by two different methods. In both these works the dispersion relation is implicit and is solved numerically. However, by working on the formula given in Ref. 3, it is possible to write the dispersion relation for the surface plasmon as

$$b=\tfrac{1}{2}(x+x^{-1}), \quad |x|\geqslant 1 \ , \qquad (6)$$

where

$$x\equiv\frac{e^{-qd}+\alpha e^{qd}}{1+\alpha}=\cosh(qd)-\frac{\epsilon_0}{\epsilon}\sinh(qd) \ , \qquad (7)$$

$$\alpha=(\epsilon-\epsilon_0)/(\epsilon+\epsilon_0) \ , \qquad (8)$$

and b is given as before, by Eq. (2). With the help of Eq. (3) an explicit solution can be obtained,

$$\omega_{\rm sp}=\left[\frac{2\pi ne^2}{\epsilon m}q\frac{\sinh(qd)}{\cosh(qd)-\frac{1}{2}(x+x^{-1})}\right]^{1/2} . \qquad (9)$$

The properties of the surface plasmon of the semi-infinite LEG now follow immediately. (i) For $\alpha=0$ Eq. (6) becomes $b=\cosh(qd)$ which does not have any nontrivial solutions as is obvious from the definition of b. Thus no surface plasmon exists for $\epsilon=\epsilon_0$. (ii) Solutions of Eq. (6) exist only for $b\leqslant -1$ and $\geqslant 1$, i.e., the surface plasmon lies outside the bulk-plasmon band $|b|\leqslant 1$. (iii) At the boundaries of the bulk-plasmon band $b=\pm 1=x$. From the definition of x, this implies $e^{-q^*d}=\pm\alpha$. As the condition $|x|\geqslant 1$ in Eq. (6) is satisfied only for $q\geqslant q^*$, there are no surface plasmons with wave vectors less than q^*. (iv) Outside the bulk-plasmon band, which is the relevant region for surface plasmons, all α, x, and b have the same sign. Thus for positive $\alpha(\epsilon>\epsilon_0)$ the surface plasmon lies above the bulk-plasmon band, and negative $\alpha(\epsilon<\epsilon_0)$ below it. (v)

Because the surface plasmon lies outside the bulk-plasmon band, it cannot decay into bulk plasmons to the lowest approximation, and is therefore free from Landau damping. (With $\omega_{sp} > qv_F$, it cannot decay into electron-hole pairs either.)

The formal similarity between the surface and bulk-plasmon dispersion relations is striking: the former can be obtained from the latter by merely replacing $\cos(q_z d)$ by $\frac{1}{2}(x+x^{-1})$. In fact, this replacement is equivalent to substituting for q_z an imaginary quantity i/ξ for $x \geqslant 1$ and $i/\xi + \pi$ for $x \leqslant -1$, where

$$\xi = \frac{d}{\ln|x|}, \quad (|x| \geqslant 1) \quad . \tag{10}$$

The length ξ is interpreted as the penetration depth of the surface plasmon.[2]

III. ASYMPTOTICS

Now we discuss some limiting cases for the dispersion relation and the penetration depth of the surface plasmon.

(i) *Weak coupling limit.* In this limit the planes of 2DEG are so far from each other that they are only weakly coupled. This happens for $qd \to \infty$ so that $x \to \alpha e^{qd}/(1+\alpha)$ and the dispersion relation becomes

$$b = \cosh(qd) - D^0 V \sinh(qd) = \frac{1}{2}\frac{\alpha e^{qd}}{1+\alpha} \quad .$$

Using $\sinh(qd) \cong \cosh(qd) \cong \frac{1}{2}e^{qd}$ for $qd \to \infty$, we get

$$1 - D^0 V = \frac{\alpha}{1+\alpha} \quad ,$$

which yields

$$\omega_{sp}(qd \to \infty) = \left[\frac{2\pi n e^2}{\frac{1}{2}(\epsilon+\epsilon_0)m}q\right]^{1/2} \quad . \tag{11}$$

Notice that this equation is exactly the dispersion relation of the two-dimensional plasmon of the surface layer $z=0$, because on this layer the electrons interact with each other as if they were in a medium of effective dielectric constant $\frac{1}{2}(\epsilon+\epsilon_0)$. This is what one would intuitively expect because in the limit under consideration the planes are decoupled from each other.

In this limit the bulk-plasmon dispersion relation given by Eq. (5) is nothing but the dispersion relation of the two-dimensional plasmon of a 2DEG layer far from the surface

$$\omega_p = \left[\frac{2\pi n e^2}{\epsilon m}q\right]^{1/2} \quad . \tag{12}$$

Thus with Eqs. (11) and (12) it is intuitively clear in the weak coupling limit that for $\epsilon_0 > \epsilon$, the surface plasmon lies below the bulk plasmon, whereas for $\epsilon_0 < \epsilon$ it lies above. Also, for $\epsilon_0 = \epsilon$ there is no surface plasmon because the 2D plasmon of the surface layer is the same as the bulk plasmon.

The penetration depth ξ is found with the help of Eq. (10) to be q^{-1} which is much smaller than d, and therefore the surface plasmon is confined to the surface 2DEG plane only.

(ii) *Limit* $q \to q^*$. In this case it is convenient to define $Q = q - q^*$ and study the limit $Qd \to 0$. Using $\alpha = e^{-q^* d}\operatorname{sgn}\alpha$ ($\operatorname{sgn}\alpha = \alpha/|\alpha|$), and Eq. (7) we get

$$x = \frac{e^{Qd} + \alpha e^{-Qd}}{1+\alpha}\operatorname{sgn}\alpha \quad ,$$

which becomes, in the limit $Qd \to 0$,

$$x = \left[1 + \frac{\epsilon_0}{\epsilon}Qd\right]\operatorname{sgn}\alpha \quad .$$

Thus we find that right above the critical wave vector q^* the surface plasmon follows the dispersion relation

$$b = \left[1 + \frac{1}{2}\left(\frac{\epsilon_0}{\epsilon}\right)^2 (Qd)^2\right]\operatorname{sgn}\alpha \quad ,$$

or

$$\omega_{sp} = \left[\frac{2\pi n e^2}{\epsilon m} q \frac{\sinh(qd)}{\cosh(qd) + [1 + \frac{1}{2}(\epsilon_0/\epsilon)^2 (Qd)^2]\operatorname{sgn}\alpha}\right]^{1/2} , \tag{13}$$

which is coincident to first order in Qd with the bulk-plasmon boundaries, $b = 1\operatorname{sgn}\alpha$. The surface plasmon merges smoothly into the bulk-plasmon band with no discontinuity of slope.

The penetration depth is given by

$$\epsilon = \frac{d}{\ln[1 + (\epsilon_0/\epsilon)Qd]} \sim \frac{\epsilon}{\epsilon_0 Q} \quad , \tag{14}$$

and diverges linearly as $Q \to 0$ or $q \to q^*$.

IV. REMARKS

(i) It is easy to incorporate the coupling to optical phonons. One only has to replace the frequency-independent background dielectric constant ϵ by a frequency-dependent $\epsilon(\omega) = \epsilon(\infty)(\omega^2 - \omega_L^2)/(\omega^2 - \omega_T^2)$, where $\epsilon(\infty)$ is the high-frequency dielectric constant, ω_L and ω_T are the longitudinal and transverse optical-phonon frequencies. As x would now depend on frequency ω through $\epsilon(\omega)$, an explicit formula for ω_{sp} would be much more complicated.

(ii) In the presence of the magnetic field, in the long wavelength limit one has[6]

$$D^0 \sim nq^2/m(\omega^2 - \omega_c^2) \quad , \tag{15}$$

where ω_c is the cyclotron frequency. This leads to a magneto-surface plasmon whose frequency ω_{msp} is given by (in the long-wavelength limit)

$$\omega_{msp}^2 = \omega_{sp}^2 + \omega_c^2 \quad , \tag{16}$$

with ω_{sp} given in Eq. (9).

(iii) One can rewrite Eq. (9) in a form that mimics the 2D plasmon dispersion relation as follows:

$$\omega_{sp} = \left[\frac{2\pi n e^2}{\epsilon_{eff} m}q\right]^{1/2} , \tag{17}$$

where ϵ_{eff} is a wave-vector-dependent dielectric constant given by

$$\epsilon_{eff}(qd) = \frac{\epsilon^2 - \epsilon_0^2}{2[\epsilon\coth(qd) - \epsilon_0]} \quad . \tag{18}$$

For large qd, $\epsilon_{\rm eff}(\infty)=\frac{1}{2}(\epsilon+\epsilon_0)$ as pointed out earlier. On the other hand, at the critical wave vector q^*, given by $q^*d=-\ln|\alpha|$, we have

$$\epsilon_{\rm eff}(q^*d)=\epsilon_0 \ ,$$

which means that at q^*d the surface plasmon has the same energy as the two-dimensional plasmon of a 2DEG layer embedded in a dielectric ϵ_0!

(iv) All the previous formulas are correct when the surface (characterized by a dielectric mismatch) and the outermost 2DEG layer both occur at $z=0$. The theory can be generalized to the case in which the dielectric mismatch is at $z=-d'$ and 2DEG planes are at $z=0,d,2d,\ldots,$ by replacing α in Eq. (7) by $\alpha e^{-2qd'}$ and using Eq. (6) or (9) as the new surface-plasmon dispersion relation. One result that follows immediately from the new condition for critical wave vector (i.e., the wave vector below which there is no surface plasmon),

$$q^*d=-\ln|\alpha e^{-2q^*d'}| \ ,$$

is that the surface plasmon ceases to exist if $d'>\frac{1}{2}d$.

Thus in this paper a simple explicit formula [Eq. (9) or (17)] is given for the dispersion relation of the surface plasmon of a semi-infinite LEG. Modifications due to optical phonons or magnetic field are indicated. In the weak coupling limit $qd\rightarrow\infty$ the surface plasmon lends itself to an intuitive understanding: it is nothing but the two-dimensional plasmon on the surface layer. As qd decreases, more and more planes participate in the collective charge-density oscillations and the penetration depth of the surface plasmon increases until finally it diverges, i.e., the system becomes so correlated that it can no longer support any decaying surface modes but only bulk modes. This happens at $q^*d=-\ln|\alpha|$. For smaller values of qd there are no surface plasmons.

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