# Exact linear admittance of $n^+$-$n$-$n^+$ semiconductor structures

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With the self-consistent solution of the linearized Boltzmann equation in the relaxation-time approximation for a spatially inhomogeneous electron system, the admittance of $n^+$-$n$-$n^+$ semiconductor structures is studied as a function of the length $L$ of the moderately doped $n$ region. It is shown that a one-dimensional treatment of the velocity space leads to the exact, analytical solution of the problem. In addition to the conventional admittance and the geometric capacitance of the $n$ region, the equivalent circuit of the structure also includes the contact resistance and, as a new feature, the contact capacitance. For the strongly screened cases ($L \gg L_D$) the contact capacitance is approximately the permittivity $\epsilon$ of the $n$ region divided by the Debye length $L_D$ and, further, becomes exactly equal to $\epsilon L/6L_D^2$ in the weak-screening regime ($L_D \gg L$).

With the progress in the semiconductor technology the problem of electrical transport in small-scale $n^+$-$n$-$n^+$ structures has become a topic of intense research in recent years. Theoretically, if quantum effects are neglected, correct treatment of the problem evidently requires the self-consistent solution of the Boltzmann transport equation for a spatially inhomogeneous case. For dc electric potentials this solution was numerically analyzed by Baranger and Wilkins,[1] who clearly indicated the inadequacy of the conventional drift-diffusion transport calculations in submicron devices. In this article we study current transport in $n^+$-$n$-$n^+$ semiconductor structures with a variable-length $n$ region and for ac applied voltages. The distribution function of the charge carriers is taken to be dependent on the velocity component across the $n^+$-$n$-$n^+$ structure only, and the calculation is limited to linear order in the applied potentials. The one-dimensional treatment of the velocity distribution function results in an exact, analytical solution of the problem, which permits an easy interpretation of the physics involved and gives expressions for the elements of the equivalent circuit of the $n^+$-$n$-$n^+$ structure.

We start with the path-integral expression of the solution of the Boltzmann transport equation in the relaxation-time approximation[2,3]

$$f = f^{(0)} - \int_{-\infty}^{t} dt' \frac{df^{(0)}}{dt'} \exp\left[ -\int_{+t'}^{t} \frac{dt''}{\tau} \right] . \tag{1}$$

Here $\tau$ is the relaxation time and $f^{(0)}$ stands for the local equilibrium distribution.[1,4] In the linearized case and for a constant temperature and external potentials with $e^{i\omega t}$ time dependence, Eq. (1) may further be written as

$$f = f_0 - \left( \frac{\partial f_0}{\partial E} \right) (\delta E_F + e\phi) e^{i\omega t} + i\omega\tau^* \left( \frac{\partial f_0}{\partial E} \right) (\delta E_F + e\phi) e^{i\omega t} + e^{i\omega t} \int_{-\infty}^{t} dt' e^{-(t-t')/\tau^*} \left( \frac{\partial f_0}{\partial E} \right) \mathbf{v} \cdot [(1 - i\omega\tau^*) \nabla \delta E_F - i\omega e \tau^* \nabla \phi] , \tag{2}$$

where $f_0$ denotes the uniform equilibrium distribution (Fermi-Dirac or Maxwell-Boltzmann), $\phi$ the electrical potential, and $\delta E_F$ the local change of the quasi-Fermi level. Also, $1/\tau^* = 1/\tau + i\omega$. Using now the assumption that $f$ depends on the velocity component across the $n^+$-$n$-$n^+$ structure ($v_z$) only, we may directly calculate the conduction-electron current $j$ and the local change of the electron density $\delta n$ induced by the external voltage. Thus, writing

$$\Phi(z) = (1 - i\omega\tau^*) \frac{d\delta E_F}{dz} - i\omega e \tau^* \frac{d\phi}{dz} , \tag{3}$$

we obtain

$$j = \frac{e n_0 v_0}{2 k_B T} \left( \int_0^z dz' e^{-(z-z')/l} \Phi(z') + \int_z^L dz' e^{-(z'-z)/l} \Phi(z') \right) e^{i\omega t} , \tag{4}$$

and $\delta n = \delta n_0 + \delta n_1$, where

$$\delta n_0 = \frac{n_0}{k_B T} (\delta E_F + e\phi) e^{i\omega t} , \tag{5}$$

$$\delta n_1 = \frac{n_0}{k_B T} \left( -i\omega\tau^* (\delta E_F + e\phi) - \tfrac{1}{2} \int_0^z dz' e^{-(z-z')/l} \Phi(z') + \tfrac{1}{2} \int_z^L dz' e^{-(z'-z)/l} \Phi(z') \right) e^{i\omega t} . \tag{6}$$

Here the moderately doped central $n$ region extends from $z = 0$ to $z = L$. Further, for the relaxation time $\tau = l_0/|v|$, $\tau^*$'s are taken as constants under the velocity integrations, and thus $l = l_0/(1 + i\omega\tau_0)$ and $\tau^* = \tau_0/(1 + i\omega\tau_0)$, where $\tau_0 = l_0/v_0$. In addition, our basic set of equations includes the Poisson equation

$$\frac{dE}{dz} = -\frac{e}{\epsilon} (\delta n_0 + \delta n_1) , \tag{7}$$

 

and the requirement of the constancy of the total current

$$I = j + i\omega\epsilon E = \text{const} \ . \tag{8}$$

Yet, in order to satisfy Eq. (8), which is equivalent to the continuity equation, we must have $\delta n_1 = 0$, which is just the requirement of number conservation in the relaxation-time approximation.

For the solution of the equations we expand the unknown function $\Phi(z)$ in a Fourier series over the $n$-region length: $\Phi(z) = \sum \Phi(q) e^{iqz}$ $(qL = 2\pi n)$. Now it is a straightforward task to express Eqs. (4) and (7) in terms of the Fourier coefficients $\Phi(q)$, and then for $q = 0$ and $q > 0$, Eq. (8) gives

$$I = \Phi(0)\left[\frac{en_0 v_0 l}{k_B T}\left(1 - \frac{1 - e^{-L/l}}{L/l}\right) + i\omega\frac{\epsilon}{e}\right] - \frac{2en_0 v_0}{k_B T}\frac{1 - e^{-L/l}}{L}F, \quad (q = 0) \ , \tag{9a}$$

$$\left[\frac{en_0 v_0}{k_B T l} + \frac{i\omega(q^2 + 1/l^2)}{e(1 + (q/\lambda)^2 - i\omega\tau^*(q/\lambda)^2)}\right]\Phi(q) = \frac{en_0 v_0}{k_B T}\frac{1 - e^{-L/l}}{L}\left[\Phi(0) + \frac{2F}{l^2}\right], \quad (q > 0) \ , \tag{9b}$$

where $\lambda^2 = e^2 n_0/\epsilon k_B T$ and

$$F = \sum_{q(>0)} \frac{\Phi(q)}{q^2 + (1/l)^2} \ . \tag{10}$$

Eqs. (9b) and (10) are now used to eliminate $F$, and since $\Phi(0) = eV/L$, where $V$ is the total voltage across the sample, we obtain the exact expression for the linear admittance of the $n^+$-$n$-$n^+$ structures

$$Y = \frac{e^2 n_0 v_0 l}{k_B T L}\left(\frac{1 + i\omega\tau^* Q(\alpha)}{1 + 2l/L + i\omega\tau^* Q(\alpha)} + \frac{i\omega\tau^*}{\lambda^2 l^2}\right), \tag{11}$$

where $Q(\alpha) = L(\alpha \coth\alpha - 10)/2l\alpha^2$ and $\alpha = L[(\lambda l)^2 - i\omega\tau^*]^{1/2}/2l$.

Immediately it is clear that in the dc limit $(\omega \to 0)$ Eq. (11) gives the correct result for the linear conductance in the $n^+$-$n$-$n^+$ structures,[5-7] and furthermore, the exact admittance can always be presented with an equivalent circuit shown in Fig. 1, where $Y_c = e^2 n_0 v_0 l/k_B T L$, $C_g = \epsilon/L$, $G_0 = (L/l) Y_c$, and $Y_0 = i\omega\tau^* G_0 Q(\alpha)$. Evidently, $Y_c$ and $C_g$ are the conventional bulk admittance (conductance and inductance) and geometric capacitance of the central $n$ region, and $G_0$, whose value is twice the conductance given by the thermionic emission theory, can be interpreted as the contact conductance. However, in order to obtain a transparent view of the admittance $Y_0$, a more detailed study is required. It is easily seen that $\alpha$ may be written as

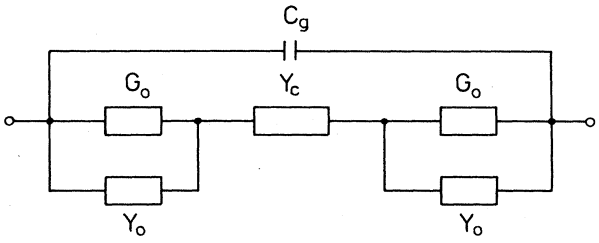


FIG. 1. The equivalent circuit of the $n^+$-$n$-$n^+$ semiconductor structure.

$\alpha = L(1 + i\omega\tau_d^*)^{1/2}/2L_D$, where $L_D = 1/\lambda$ is the Debye length and $\tau_d^*$ is a complex time constant, which is equal to the dielectric relaxation time of the $n$ region multiplied by $1 + i\omega\tau_0$. Thus, for the major part of the practical frequency spectrum the magnitude of $\alpha$ is entirely determined by the ratio $L/L_D$. Normally, $L/L_D \gg 1$, whence $\coth\alpha - 1/\alpha \approx 1$, and then $Y_0$ becomes $Y_0 = i\omega\epsilon/L_D(1 + i\omega\tau_d^*)^{1/2}$, which, at low frequencies, simply represents the contact capacitance $C_0 = \epsilon/L_D$. However, for higher $\omega$ we also get a frequency-dependent conductance in parallel with $G_0$. On the other hand, for highly resistive and extremely short central $n$ regions we may have $L/L_D \ll 1$, which implies $\coth\alpha - 1/\alpha \approx \alpha/3$ and then $Y_0 = i\omega C_0 L/6L_D$. Consequently, in this case $Y_0$ gives a small and purely capacitive contact contribution to the total admittance $Y$.

Roughly speaking, the effect of the contacts on the sample conductance is to add an extra length of one mean free path at both ends of the $n$ region, which physically reflects the fact that at least one mean free path of spatial dimension is required to establish a change in the distribution function of the charge carriers. Similarly, the charge accumulation and depletion near the $n^+$-$n$ contacts needs a distance of one Debye length at both ends of the $n$ region. This effect is accounted for by the contact capacitance. However, in the weakly screened case $(L \ll L_D)$ the carrier density is spatially only slightly modulated, which results in a small value of the contact capacitance, and, consequently, in practically constant electric field throughout the $n$ region.

The equivalent circuit obtained in this article is close to the result calculated by Lee and Shur,[5] who treated the contact effects in the spatially homogeneous Boltzmann equation as surface scattering, which results in a length-dependent, effective relaxation time. However, as a new feature, our self-consistent treatment of the inhomogeneous case brings in the contact capacitances, which must be present in the equivalent circuit. In fact, our result is completely in accord with the heuristic model of submicron semiconductor devices proposed by Barker, Ferry, and Grubin.[8]

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