Two-dimensional electron-hole fluid in a strong perpendicular magnetic field: Exciton Bose condensate or maximum density two-dimensional droplet

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The two-dimensional (2D) electron-hole fluid is studied in a strong perpendicular magnetic field. In the ideal case (simple 2D electron and hole bands), the exact ground state is a Bose condensate of noninteracting magnetic excitons. In a quantum well the asymmetry of the transverse wave functions induces an attractive interaction between excitons, and the ground state is a 2D "droplet" of maximum local density. On the other hand, virtual transitions to excited Landau levels cause a repulsive interaction, and the ground state becomes a Bose condensate of interacting excitons. This condensate is a superfluid and moves under application of an electric field in a direction perpendicular to both the electric and magnetic fields. The influence of the spin-orbit coupling between hole bands in a GaAs or Ge quantum well is examined. Both types of ground state can occur, depending on the polarization of the pumping light. The luminescence properties are also discussed, both for the case of a direct- and an indirect-gap semiconductor.

I. INTRODUCTION

The quantization of the Hall resistance is a spectacular consequence of the quasi-zero-dimensional character of the electron gas in two dimensions in a perpendicular magnetic field.[1] It is worth investigating the corresponding two-component system, the electron-hole (e-h) system in two dimensions in the presence of a perpendicular magnetic field. One can also expect for this system new physics since the kinetic energy is quenched by the magnetic field and the system's behavior is governed only by the Coulomb interaction between particles. A typical system would be created by optical pumping a gas of e-h pairs in a semiconductor quantum well. We will study the ground state of such a system, assuming the lifetime of the e-h pairs to be much longer than the relaxation time towards the equilibrium ground state.

If the following conditions are fulfilled, (i) cyclotron energies larger than the Rydberg energy, (ii) small layer thickness so that the transverse motion is quantized with confining energies larger than the cyclotron ones, then one can consider electrons and holes to be in a single Landau level and neglect the influence of the single-particle-excited states. This ideal problem has been investigated in recent years in a series of papers by Lerner and Lozovik[2] (hereafter referred to as LL) and by Bychkov and Rashba.[3] The former authors, using diagrammatic techniques, showed that the ground state of this many-body e-h system can be found exactly. It is a Bose condensate of noninteracting magnetic excitons. Our aim in this paper will be first to understand from simple arguments[4] this amazing result for the ideal system. Then we will investigate the properties of the system in more realistic situations and consider the influence of the transverse motion of the particles, of the coupling to higher Landau levels, and of the spin-orbit interaction. We will examine also the effect of an electric field and show that, under conditions such that the ground state is a Bose condensate of magnetic excitons, this fluid, although neutral, moves as a whole. This is a new aspect of the quantum Hall effect.

This paper is organized as follows. In Sec. II the ideal 2D case is investigated: the single magnetic exciton dispersion curve is derived and LL's result[2] is recovered using very simple arguments. We show in Sec. III that, if the transverse wave functions of electrons and holes are different (i.e., if the confinement along the magnetic field direction is not the same), the effective interaction between excitons is attractive. The fluid then splits into two phases: 2D "droplets" of e-h fluid with maximum density surrounded by a dilute gas of excitons. We show in Sec. IV that if one assumes the same transverse wave function for both types of particle but includes virtual transitions to excited Landau levels, the effective interaction between excitons is repulsive and the ground state is a Bose condensate of interacting magnetic excitons whose collective excitations are also computed. We investigate in Sec. V the influence of an electric field on the Bose condensate and show that the condensate acquires a superfluid velocity in a direction perpendicular to both the magnetic and electric fields. The luminescence properties of the system are studied in Sec. VI, both in the case of a direct or an indirect semiconductor. Section VII is devoted to the study of a realistic case. We show that for cubic semiconductors such as GaAs or Ge, the hole spin-orbit

coupling induces a small asymmetry between the transverse motions of electrons and holes. For high magnetic fields, the ground-state properties (Bose condensate or maximum-density 2D droplet) depend on the polarization properties of the pumping light. Finally, our results are summarized and commented on in Sec. VIII. In the Appendix the energy of the charge-density wave state is compared to that of the Bose condensate. The latter is shown to be more stable.

II. THE IDEAL SYMMETRIC MODEL

We assume here the simplest conditions: the electron and hole 2D bands split, due to the perpendicular magnetic field, into discrete Landau levels whose Zeeman splitting is big. The strong magnetic field limit means that electrons and holes are confined to the lowest Landau level, and that the typical electrostatic energy (the three-dimensional Rydberg) is much smaller than the cyclotron energies. Choosing the Landau gauge $\mathbf{A}=(0,Hx,0)$, the single-particle wave functions are

$$\psi_{k,\sigma}=L^{-1/2}\exp(iky)\phi(x+\sigma ka_0^2)\ , \tag{1}$$

where $\sigma=+1\ (-1)$ stands for electrons (holes). ϕ is the harmonic-oscillator wave function:

$$\phi(x)=(\pi^{1/2}a_0)^{1/2}\exp[-x^2/(2a_0^2)]\ .$$

The electrons and holes have a single label k which runs over $N=L^2/2\pi a_0^2$ values, where L is the simple dimension and a_0 the magnetic length; $a_0^2=\hbar c/|e|H$. Note that these wave functions depend only on a_0 and are thus independent of the three-dimensional band characteristics (effective masses, etc.). The Hamiltonian of the e-h system reduces (after dropping the gap energy) to the Coulomb interaction, electron-electron, hole-hole, and electron-hole terms

$$H=\sum_{\substack{q,k_1,k_2\\ \sigma_1,\sigma_2}} F_{\sigma_1\sigma_2}(q,k_1-k_2)c^\dagger_{\sigma_1,\sigma_1k_1-q/2}c^\dagger_{\sigma_2,\sigma_2k_2+q/2} \times c_{\sigma_2,\sigma_2k_2-q/2}c_{\sigma_1,\sigma_1k_1+q/2}\ , \tag{2}$$

where the operator $c^\dagger_{\sigma,k}$ creates a particle σ in the state $\psi_{k,\sigma}$. We will also use, when useful, the notation

$$a^\dagger_k=c^\dagger_{+1,k}\ \text{ and }\ b^\dagger_k=c^\dagger_{-1,k}\ . \tag{3}$$

The Coulomb matrix element is

$$F_{\sigma_1,\sigma_2}(K_y,k)=\frac{1}{2L^2}\sum_{K_x} v(\mathbf{K})e^{iK_xka_0^2}\sigma_1\sigma_2A_{\sigma_1}(\mathbf{K})A^*_{\sigma_2}(\mathbf{K})\ , \tag{4}$$

where $v(\mathbf{K})$ is the 2D Fourier transform of the Coulomb potential $v(\mathbf{K})=e^2/\epsilon|K|$ and

$$A_\sigma(\mathbf{K})=\int dx\,\phi^*(x-\tfrac{1}{2}\sigma a_0^2K_y)e^{-iK_xx}\phi(x+\tfrac{1}{2}\sigma a_0^2K_y)\ . \tag{5}$$

A. Single magnetic exciton

The structure of an exciton in a strong magnetic field was investigated years ago,[5] and rederived many times.[6] We present here a new derivation based on group theoretical arguments.

Let us define the magnetic translation[7] operator as usual by

$$\mathcal{T}=e^{-iy(\mathcal{T}_x/a_0^2)}e^{-[\mathcal{T}_x(\partial/\partial x)+\mathcal{T}_y(\partial/\partial y)]}$$

acting on an electron, and

$$\mathcal{T}=e^{+iy(\mathcal{T}_x/a_0^2)}e^{-[\mathcal{T}_x(\partial/\partial x)+\mathcal{T}_y(\partial/\partial y)]}\ ,$$

acting on a hole. (The phase factor comes from the translation of the Landau gauge origin.) One easily verifies that

$$\mathcal{T}c^\dagger_{\sigma,k}\mathcal{T}^\dagger=e^{-ik\mathcal{T}_y}c^\dagger_{\sigma,k-\sigma(\mathcal{T}_x/a_0^2)}\ ,$$

and consequently that the Hamiltonian commutes with any magnetic translation. Although it is well known that the magnetic translations form a ray group[7] (a group up to a phase factor), it acts on the e-h pair operators as a normal group,

$$(\mathcal{T}_1\mathcal{T}_2)(a^\dagger_{k_1}b^\dagger_{k_2})(\mathcal{T}_1\mathcal{T}_2)^\dagger=\mathcal{T}_{1+2}a^\dagger_{k_1}b^\dagger_{k_2}\mathcal{T}_{1+2}\ .$$

As a consequence, the e-h pairs can be classified according to the translation group irreducible representations

$$\mathcal{T}d^\dagger_{\mathbf{K}}\mathcal{T}^\dagger=e^{-i\mathbf{K}\cdot\mathcal{T}}d^\dagger_{\mathbf{K}}\ ,$$

with

$$d^\dagger_{\mathbf{K}}=N^{-1/2}\sum_q e^{-iK_xqa_0^2}a^\dagger_{(K_y/2)+q}b^\dagger_{(K_y/2)-q}\ . \tag{6}$$

As a further consequence, the pair states $d^\dagger_{\mathbf{K}}|\text{vac}\rangle$ are eigenstates of the Hamiltonian (which in this case is only the e-h interaction term). One gets

$$Hd^\dagger_{\mathbf{K}}|\text{vac}\rangle=E_x(\mathbf{K})d^\dagger_{\mathbf{K}}|\text{vac}\rangle\ ,$$

with

$$E_x(\mathbf{K})=\sum_{q,\sigma}F_{\sigma,-\sigma}(q,\sigma K_y)e^{-iK_x\sigma qa_0^2} =-\frac{1}{2L^2}\sum_{\substack{\mathbf{Q}\\ \sigma}} v(\mathbf{Q})A_\sigma(\mathbf{Q})A_{-\sigma}(\mathbf{Q}) \times\exp(-i\sigma a_0^2\mathbf{Q}\times\mathbf{K}\cdot\hat{\mathbf{z}})\ ,$$

which, in the ideal symmetric case, reduces to

$$E_x(\mathbf{K})=-\frac{e^2}{\epsilon a_0}\left[\frac{\pi}{2}\right]^{1/2}e^{-\frac{1}{4}K^2a_0^2}I_0(\tfrac{1}{4}K^2a_0^2)\ . \tag{7}$$

Note that the whole manifold of e-h pair states is spanned by the set of exciton wave functions. Contrary to the standard hydrogenic problem, all the e-h pairs are bound states and there are no scattering states. Note also that the exciton mass (defined as $\mathbf{K}\to 0$) depends only on the magnetic field and is independent of the original 2D band structure. The exciton wave function, written in real space, is

$$\psi_{\mathbf{K}}(\mathbf{R},\mathbf{r})=\frac{1}{Na_0(2\pi)^{1/2}}\exp(i\mathbf{K}\cdot\mathbf{R})\exp(-iXy/a_0^2)\times\exp\left[-\frac{1}{4a_0^2}[(x+a_0^2K_y)^2+(y-a_0^2K_x)^2]\right], \tag{8}$$

where the collective coordinates $\mathbf{R}$ and $\mathbf{r}$ are defined from the electron ($\mathbf{r}_e$) and hole ($\mathbf{r}_h$) ones as

$$\mathbf{R}=\frac{\mathbf{r}_e+\mathbf{r}_h}{2},\quad \mathbf{r}=\mathbf{r}_e-\mathbf{r}_h .$$

An exciton wave vector $\mathbf{K}$ thus carries an electric dipole in the direction perpendicular to $\mathbf{K}$ with a magnitude proportional to K, i.e.,

$$\langle\mathbf{r}\rangle=a_0^2\hat{\mathbf{z}}\times\mathbf{K} . \tag{9}$$

Since the exciton energy is only of electrostatic origin, one can define, in a semiclassical way, the electric field $\mathscr{E}_{\rm int}$ acting on the hole (or electron) from (9),

$$\mathscr{E}_{\rm int}=-\frac{1}{e}\frac{\partial E_x}{\partial\langle\mathbf{r}\rangle}=\frac{\mathbf{H}}{c}\times\mathbf{v}_g , \tag{10}$$

where $\mathbf{v}_g=(1/\hbar)(\partial E_x/\partial\mathbf{K})$ is the group velocity. This means that the magnetic exciton can be viewed as composed of an electron and a hole, traveling with the same velocity $\mathbf{v}_g$, each particle causing an electric field on the other one whose effect is just cancelled by the Lorentz force.

B. Many excitons

When many e-h pairs are present in the system, it is natural, as already recognized by LL,[2] to consider the system as a coherent superposition of excitons in the single exciton ground state ($\mathbf{K}=0$), i.e., as a Bose condensate. Defining the filling factor ν as the common fraction of electron or hole states occupied, we describe such a state with a BCS-type wave function

$$|\psi(\nu)\rangle=\prod_k(u+va_k^\dagger b_{-k}^\dagger)|\,\text{vac}\rangle , \tag{11}$$

with $uu^*+vv^*=1$. The number of electrons (holes) is given by

$$\left\langle\psi(\nu)\left|\sum_k a_k^\dagger a_k\right|\psi(\nu)\right\rangle=\sum_k vv^* ,$$

leading to $|v|^2=\nu$. Note that in the limit $\nu\to1$, the wave function reduces to

$$|\psi(1)\rangle=\prod_k a_k^\dagger b_{-k}^\dagger|\,\text{vac}\rangle ,$$

i.e., to the Slater determinant describing the full electron and hole Landau levels. Note further that, since there is no kinetic energy, u and v are independent of k. The expectation value of the energy is obtained straightforwardly as

$$\langle\psi(\nu)|H|\psi(\nu)\rangle=v^2\sum_{q,k}[F_{+-}(q,0)+F_{-+}(q,0)]+v^4\sum_{\substack{q,k\\ \sigma_1,\sigma_2}}[F_{\sigma_1,\sigma_2}(0,k)-F_{\sigma_1,\sigma_2}(q,0)] .$$

Owing to the definition of the Coulomb matrix element $F_{\sigma_1,\sigma_2}(k,q)$, Eq. (4), one gets

$$\sum_{k,\sigma_1,\sigma_2}F_{\sigma_1,\sigma_2}(0,k)=\frac{1}{2L^2}\sum_{\substack{K_x,k\\ \sigma_1,\sigma_2}}v(K_x,0)\sigma_1\sigma_2e^{iK_xka_0^2}\times A_{\sigma_1}(K_x,0)A_{\sigma_2}^*(K_x,0)$$

$$=\frac{1}{\pi a_0^2}v(0)\sum_\sigma\sigma_1\sigma_2A_{\sigma_1}(0)A_{\sigma_2}(0) .$$

But, from the definition of $A_\sigma(\mathbf{K})$, Eq. (5), we get $A_\sigma(0)=1$; thus,

$$\sum_{k,\sigma_1,\sigma_2}F_{\sigma_1,\sigma_2}(0,k)=0 ,$$

which is equivalent to the statement of charge neutrality. The other v^4 term can be written as

$$\sum_{q,\sigma_1,\sigma_2}F_{\sigma_1,\sigma_2}(q,0)=\frac{1}{2L^2}\sum_{\substack{\mathbf{Q}\\ \sigma_1,\sigma_2}}v(\mathbf{Q})\sigma_1\sigma_2A_{\sigma_1}(\mathbf{Q})A_{\sigma_2}^*(\mathbf{Q})$$

$$=\frac{1}{2L^2}\sum_{\mathbf{Q}}v(\mathbf{Q})[A_+(\mathbf{Q})-A_-(\mathbf{Q})]\times[A_+^*(\mathbf{Q})-A_-^*(\mathbf{Q})]$$

and is positive, unless $A_+(\mathbf{Q})=A_-(\mathbf{Q})$, as occurs in our ideal e-h symmetric case. In this case, the energy per pair

$$\mathscr{E}(\nu)=\frac{\langle\psi(\nu)|H|\psi(\nu)\rangle}{\nu N}$$

becomes

$$\mathscr{E}(\nu)=\sum_q F_{+-}(q,0)+F_{-+}(q,0)=E_x(\mathbf{K}=0) .$$

It does not depend on the filling factor and is just the single exciton energy. This amazing result has been shown by LL to be an exact result, using diagrammatic arguments. It can be, however, understood from very simple arguments. One method is to show that all matrix elements of H connecting the variational ground state with excited states in the lower Landau level vanish in the ideal case.[4] Another approach will be given here.

It can be easily checked that the following commutation relation holds:

$$[H,d_0^\dagger]=E_x(0)d_0^\dagger ,$$

which implies

$$[H,(d_0^\dagger)^n]=nE_x(0)(d_0^\dagger)^n$$

or

$$H(d_0^\dagger)^n|\,\text{vac}\rangle=nE_x(0)(d_0^\dagger)^n|\,\text{vac}\rangle .$$

This means that excitons in the $\mathbf{K}=0$ state do not interact and, thus, that the ground state of n e-h pairs is formed by n noninteracting excitons in the $\mathbf{K}=0$ state. Note, however, that $d_0^\dagger$ and d_0 do not obey simple Bose commutation relations in the presence of other e-h pairs:

$$[d_0,d_0^\dagger]=1-\frac{1}{N}\sum_k (a_k^\dagger a_k+b_k^\dagger b_k) .$$

Now the BCS-type variational ground-state, Eq. (11), can be written

$$\begin{aligned}|\psi(v)\rangle &= \prod_k (u+va_k^\dagger b_{-k}^\dagger)\,|\,\text{vac}\rangle \\ &= u^N \exp\left[\frac{v}{u}\sum_k a_k^\dagger b_{-k}^\dagger\right]|\,\text{vac}\rangle \\ &= u^N \exp\left[\frac{v}{u}N^{1/2}d_0^\dagger\right]|\,\text{vac}\rangle \\ &= u^N \sum_{n=0}^{N}\left[\frac{v}{u}\right]^n \frac{N^{n/2}}{n!}(d_0^\dagger)^n\,|\,\text{vac}\rangle .\end{aligned}$$

Thus, as is well known, the BCS-type wave function describes a coherent mixture of states with a different number of excitons in the $\mathbf{K}=0$ state. (This number is conjugate to the phase of the complex number v.) Applying the total Hamiltonian, we get

$$\begin{aligned}H\,|\,\psi(v)\rangle &= u^N\frac{v}{u}N^{1/2} \\ &\quad\times E_x(0)d_0^\dagger\left[\sum_{p=0}^{N}\left[\frac{v}{u}\right]^p\frac{N^{p/2}}{p!}(d_0^\dagger)^p\right]|\,\text{vac}\rangle ,\end{aligned}$$

i.e.,

$$H\,|\,\psi(v)\rangle=\frac{v}{u}N^{1/2}E_x(0)d_0^\dagger\,|\,\psi(v)\rangle .$$

As a consequence, the Hamiltonian does not create from the variational ground state any excited exciton pairs, as it would do in a non-ideal case. It only changes the phase coherence factors between states of different exciton numbers. The expectation value of the energy becomes

$$\begin{aligned}\langle\psi(v)\,|\,H\,|\,\psi(v)\rangle &= \frac{v}{u}E_x(0)N^{1/2}\langle\psi(v)\,|\,d_0^\dagger\,|\,\psi(v)\rangle \\ &= \frac{v}{u}E_x(0)uv^*N=N\nu E_x(0) ,\end{aligned}$$

as before.

The physical origin of the lack of interaction between $\mathbf{K}=0$ excitons stems from the fact that, in the ideal case, the electron and hole wave functions are identical. A $\mathbf{K}=0$ exciton is just made up of an electron and a hole on top of one another; it is a perfectly neutral object. Its charge density vanishes at any point. As a consequence, two of these objects do not interact and the ground state of the system is a condensate of noninteracting excitons. Note, however, that excited excitons carry an electric dipole and thus interact.

This ideal Bose character can be recovered while studying the collective excitations of the condensate described by the BCS-type wave function. Let us define $\mathbf{K}$-dependent density operators: electron density,

$$\rho_e(\mathbf{K})=\frac{1}{N^{1/2}}\sum_q e^{-iK_x qa_0^2}a_{q+K_y/2}^\dagger a_{q-K_y/2} , \tag{12a}$$

hole density,

$$\rho_h(\mathbf{K})=\frac{1}{N^{1/2}}\sum_q e^{iK_x qa_0^2}b_{q+K_y/2}^\dagger b_{q-K_y/2} , \tag{12b}$$

and condensate density,

$$d^\dagger(\mathbf{K})=\frac{1}{N^{1/2}}\sum_q e^{-iK_x qa_0^2}a_{K_y/2+q}^\dagger b_{K_y/2-q}^\dagger . \tag{12c}$$

Following Anderson's treatment of collective excitation in superconductivity,[8] we derived the random-phase approximation (RPA) equations by linearization of the equations of motion and obtained, in the symmetric case,

$$i\hbar\frac{\partial}{\partial t}\langle\rho_e(\mathbf{K})\rangle=i\hbar\frac{\partial}{\partial t}\langle\rho_h(\mathbf{K})\rangle=uv^*[E_x(\mathbf{K})-E_x(0)][\langle d(-\mathbf{K})\rangle-\langle d^\dagger(\mathbf{K})\rangle] ,$$

$$i\hbar\frac{\partial}{\partial t}\langle d^\dagger(\mathbf{K})\rangle=(2\nu-1)[E_x(\mathbf{K})-E_x(0)]\langle d^\dagger(\mathbf{K})\rangle+uv^*[E_x(0)-E_x(\mathbf{K})][\langle\rho_e(\mathbf{K})\rangle+\langle\rho_h(\mathbf{K})\rangle] .$$

The first equation shows that $\langle\rho_e(\mathbf{K})\rangle-\langle\rho_h(\mathbf{K})\rangle$ is a constant of motion, i.e., that there is no plasma oscillation in the system—an unusual fact which is easily understood when one realizes that electrons and holes cannot have any kinetic energy when confined to a single Landau level. The dispersion relation of the collective excitations is given by $\omega=E_x(\mathbf{K})-E_x(0)$, i.e., simply the single exciton dispersion relation—a result underlying again the ideal Bose condensate character of the ground state. The remarkable fact that the exact ground state of the ideal symmetric model is known allows us to describe below the physics of more complicated situations as perturbations of the ideal symmetric model.

III. THE NONSYMMETRIC CASE

In a more realistic model, one must include the deviations from exact 2D character. A 2D e-h system could be realized by optical pumping a semiconductor quantum well whose thickness, however, is always finite. Due to the different offsets of the valence and conduction bands and the different masses, the wave function of electrons and holes will spread differently within the barriers. Assuming confinement and cyclotron energies bigger than the Rydberg, the wave functions for the lowest Landau levels take the form

$$\psi_{k,+}(\mathbf{r})=L^{-1/2}e^{iky}\Theta_+(z)\phi(x+ka_0^2)$$

for electrons and

$$\psi_{k,-}(\mathbf{r})=L^{-1/2}e^{iky}\Theta_-(z)\phi(x-ka_0^2)$$

for holes, and the Hamiltonian is identical to (2) with different matrix elements:

$$F_{\sigma_1,\sigma_2}(K_y,k)=\frac{1}{2L^2}\sum_{K_x,K_z} v(\mathbf{K})e^{iK_xka_0^2} \times\sigma_1\sigma_2 A_{\sigma_1}(\mathbf{K})A^*_{\sigma_2}(\mathbf{K}) , \tag{13}$$

where $v(\mathbf{K})$ is now the three-dimensional Fourier transform of the Coulomb potential $v(\mathbf{K})=e^2/\epsilon K^2$ and

$$A_\sigma(\mathbf{K})=\int dz\,\Theta^*_\sigma(z)e^{-iK_zz}\Theta_\sigma(z) \times\int dx\,\phi^*(x-\tfrac{1}{2}a_0^2\sigma K_y) \times e^{-iK_xx}\phi(x+\tfrac{1}{2}a_0^2\sigma K_y) . \tag{14}$$

This means that here the electron-electron, hole-hole, and electron-hole matrix elements are no longer equal.

The same symmetry arguments as before determine the single-exciton wave function as

$$|\psi_{\mathbf{K}}\rangle=N^{-1/2}\sum_q e^{-iK_xqa_0^2}a^\dagger_{(K_y/2)+q}b^\dagger_{(K_y/2)-q}|\text{vac}\rangle ,$$

where the $\mathbf{K}$ vector remains 2D and parallel to the quantum-well plane. The energy is obtained in the same way:

$$E_x(\mathbf{K})=\sum_{q,\sigma}F_{\sigma,-\sigma}(q,\sigma K_y)e^{-iK_x\sigma qa_0^2} .$$

In this case, however, it cannot be written in a closed form. In this nonsymmetric model, the electron and hole wave functions are different, so the $\mathbf{K}=0$ exciton is no longer perfectly neutral and these excitons will now interact with each other.

Still assuming a variational ground state described by the BCS-type wave function

$$|\psi(\nu)\rangle=\prod_k(u+va^\dagger_kb^\dagger_{-k})|\text{vac}\rangle ,$$

we get the expectation value of the energy as

$$\langle\psi(\nu)|H|\psi(\nu)\rangle=v^2\sum_{q,k,\sigma}F_{\sigma,-\sigma}(q,0) +v^4\sum_{\substack{q,k\\ \sigma_1,\sigma_2}}[F_{\sigma_1,\sigma_2}(0,k)-F_{\sigma_1,\sigma_2}(q,0)] .$$

The term $\sum_{k,\sigma_1,\sigma_2}F_{\sigma_1,\sigma_2}(0,k)$ again vanishes, due to global charge neutrality, and the other v^4 term can be rewritten as a positive-definite quantity

$$\sum_{q,\sigma_1,\sigma_2}F_{\sigma_1,\sigma_2}(q,0)=\frac{1}{2L^2}\sum_{\mathbf{Q}}v(\mathbf{Q})|A_+(\mathbf{Q})-A_-(\mathbf{Q})|^2=B .$$

As a final result, the energy per pair takes the form

$$\mathscr{E}(\nu)=E_x(0)-B\nu , \tag{15}$$

which shows that the interaction between excitons is attractive. This conclusion agrees with that of Bychkov and Rashba,[3] although in (15) the origin of the attraction is in the exchange rather than in the electrostatic energies. These authors, however, argued that in this nonsymmetric case, the ground state should be a charge-density wave. We checked the energy per e-h pair of such a state for the symmetric ideal model and it is substantially larger than the Bose condensate ground-state one. A brief account is given in the Appendix. This shows that, at least for a not too asymmetric case, the Bose condensate is favored.

Actually, as $\mathscr{E}(\nu)$ is a decreasing function of ν, whatever the filling factor, the excitonic system will split into two phases: an e-h maximum-density 2D droplet with local filling factor $\nu=1$ (and thus with a vanishing pairing order parameter) surrounded by a dilute gas of excitons at a small but finite temperature. At the 2D droplet's boundary, the filling factor changes in a small distance from one to zero. In this region, the pairing order parameter

$$\frac{1}{N^{1/2}}\langle d_0^\dagger\rangle=uv^*=[\nu(1-\nu)]^{1/2}$$

reaches a maximum for the local filling factor value $\nu=\frac{1}{2}$.

IV. THE INFLUENCE OF HIGHER LANDAU LEVELS IN THE SYMMETRIC CASE

Let us go back to the symmetric system and study, using second-order perturbation theory, the influence of higher Landau energy levels (both electron and holes levels) on the nature of the ground state. In simple systems without spin-orbit coupling, transitions are allowed only between Landau levels with the same spin. We assume also, for simplicity, that the electron and hole masses are equal, i.e., that both cyclotron frequencies take the same value Ω_c. The wave functions corresponding to (1) are

$$\psi_{k,\sigma,n}(\mathbf{r})=L^{-1/2}e^{iky}\phi_n(x+\sigma ka_0^2) ,$$

where ϕ_n is the nth harmonic-oscillator wave function. The Hamiltonian of the system becomes rather complicated, as it involves a kinetic-energy part

$$T=\sum_{n,\sigma,k}n\hbar\Omega_cc^\dagger_{\sigma,n,k}c_{\sigma,n,k}$$

and Coulomb interactions between all the particles in different levels. The only term of interest at low temperature ($K_BT \ll \hbar\Omega_c$) is the interaction coupling particles in the ground Landau levels ($n=0$, $\sigma=\mp 1$) with excited states ($n\neq 0$). A careful but tedious analysis of the matrix elements shows[2,6] that the nonvanishing terms only promote two particles (which can be of different species) to Landau levels bearing the same index n. This takes the form

$$V_{n0}=\sum_{\substack{q,k_1,k_2\\ \sigma_1\sigma_2}} F_{\substack{n,0\\ \sigma_1,\sigma_2}}(q,k_1-k_2)c^{\dagger}_{\sigma_1,n,\sigma_1k_1-q/2}c^{\dagger}_{\sigma_2,n,\sigma_2k_2+q/2} \times c_{\sigma_2,\sigma_2k_2-q/2}c_{\sigma_1,\sigma_1k_1+q/2}$$

and

$$V_{0n}=V^{\dagger}_{n0} = \sum_{\substack{Q,k_3,k_4\\ \sigma_3,\sigma_4}} F_{\substack{0,n\\ \sigma_3,\sigma_4}}(Q,k_3-k_4)c^{\dagger}_{\sigma_3,\sigma_3k_3-Q/2}c^{\dagger}_{\sigma_4,\sigma_4k_4+Q/2} \times c_{n,\sigma_4,\sigma_4k_4-Q/2}c_{n,\sigma_3,\sigma_3k_3+Q/2} ,$$

where the Coulomb matrix elements are

$$F_{\substack{p,q\\ \sigma_1,\sigma_2}}(K_y,k)=\frac{\sigma_1\sigma_2}{2L^2}\sum_{K_x} v(\mathbf{K})e^{iK_xka_0^2} \times A_{\sigma_1,p,q}(\mathbf{K})A^{*}_{\sigma_2,p,q}(\mathbf{K}) , \quad (16)$$

with

$$A_{\sigma,p,q}(\mathbf{K})=\int dx\, \phi^{*}_p(x-\tfrac{1}{2}\sigma a_0^2K_y)e^{-iK_xx} \times\phi_q(x+\tfrac{1}{2}\sigma a_0^2K_y) . \quad (17)$$

We then obtain, from second-order perturbation theory, an effective Hamiltonian describing the interactions of particles in the lowest Landau levels through virtual transitions to excited states as

$$\tilde{H}=V_{00}-\sum_{n\neq 0} V_{0n}\frac{1}{2n\hbar\Omega_c}V_{n0} .$$

The factor $2n\hbar\Omega_c$ comes from the fact that there are always two particles in the intermediate state, each having the bare kinetic energy $n\hbar\Omega_c$. These two particles, created by the interaction V_{n0}, can recombine to their original state through V_{0n} or, if they belong to the same species, can exchange before recombining. This gives two terms. The overall result for the effective Hamiltonian takes the form

$$\tilde{H}=\sum_{\substack{q,k_1,k_2\\ \sigma_1,\sigma_2}} \tilde{F}_{\sigma_1,\sigma_2}(q,k_1-k_2)c^{\dagger}_{\sigma_1,\sigma_1k_1-q/2}c^{\dagger}_{\sigma_2,\sigma_2k_2+q/2} \times c_{\sigma_2,\sigma_2k_2-q/2}c_{\sigma_1,\sigma_1k_1+q/2} ,$$

with

$$\tilde{F}_{\sigma_1,\sigma_2}(q,k)=F_{\substack{00\\ \sigma_1,\sigma_2}}(q,k)-\sum_{K,n\neq 0}\left[F_{\substack{0n\\ \sigma_1,\sigma_2}}\left[q-K,k-(\sigma_1+\sigma_2)\frac{q+K}{2}\right]+F_{\substack{0,n\\ \sigma_2,\sigma_1}}\left[q-K,k-(\sigma_1+\sigma_2)\frac{q+K}{2}\right]\right] \times\frac{1}{2n\hbar\Omega_c}F_{\substack{n,0\\ \sigma_2,\sigma_1}}\left[q+K,k+(\sigma_1+\sigma_2)\frac{q-K}{2}\right] .$$

The effective Hamiltonian has the same general form as in the asymmetric case and we get formally similar results, as exemplified in Secs. IV A, IV B, and IV C.

A. Single-exciton energy

We have

$$\tilde{E}_x(\mathbf{K})=\sum_{q,\sigma}\tilde{F}_{\sigma,-\sigma}(q,\sigma K_y)e^{-i\sigma K_xqA_0^2} .$$

In particular, the $\mathbf{K}=0$ exciton energy takes the form

$$\tilde{E}_x(0)=E_x(0)+\Delta E_x(0) ,$$

with

$$\Delta E_x(0)=-\sum_{n\neq 0}\frac{\left|\sum_{Q,\sigma} F_{\substack{0,n\\ \sigma,-\sigma}}(Q,0)\right|^2}{2n\hbar\Omega_c} ,$$

i.e., the binding energy of the exciton increases.

B. Energy per e-h pair

We have

$$\mathscr{E}(\nu)=\tilde{E}_x(0)-B\nu ,$$

with

$$B=\sum_{q,\sigma_1,\sigma_2}\tilde{F}_{\sigma_1,\sigma_2}(q,0) .$$

Taking account of the symmetries of the matrix elements $F_{\substack{p,q\\ \sigma_1,\sigma_2}}(K,Q)$ which can be inferred from (16) and (17), B can be rewritten as

$$B=-\frac{1}{2}\sum_{\substack{n\neq 0\\ \sigma_1,\sigma_2\\ Q_1,Q_2}}\frac{\left[F_{\substack{n,0\\ \sigma_2,\sigma_1}}\left[Q_1,\frac{\sigma_1+\sigma_2}{2}Q_2\right]+F_{\substack{n,0\\ \sigma_1,\sigma_2}}\left[Q_1,\frac{\sigma_1+\sigma_2}{2}Q_2\right]\right]\left[F_{\substack{n,0\\ \sigma_1,\sigma_2}}\left[Q_2,\frac{\sigma_1+\sigma_2}{2}Q_1\right]+F_{\substack{n,0\\ \sigma_2,\sigma_1}}\left[Q_2,\frac{\sigma_1+\sigma_2}{2}Q_1\right]\right]^*}{2n\hbar\Omega_c}$$

$$=-\frac{1}{16L^4\hbar\Omega_c}\sum_{\substack{n\neq 0\\ \sigma_1,\sigma_2\\ \mathbf{Q}_1,\mathbf{Q}_2}}\frac{v(\mathbf{Q}_1)v(Q_2)\exp\left[i\frac{\sigma_1+\sigma_2}{2}\mathbf{Q}_1\times\mathbf{Q}_2\cdot\hat{\mathbf{z}}\right]}{n}[A_{\sigma_1,n,0}(\mathbf{Q}_1)A^*_{\sigma_2,n,0}(\mathbf{Q}_1)+A_{\sigma_2,n,0}(\mathbf{Q}_1)A^*_{\sigma_1,n,0}(\mathbf{Q}_1)]$$

$$\times[A_{\sigma_1,n,0}(\mathbf{Q}_2)A^*_{\sigma_2,n,0}(\mathbf{Q}_2)+A_{\sigma_2,n,0}(\mathbf{Q}_2)A^*_{\sigma_1,n,0}(\mathbf{Q}_2)]\ ,$$

a quantity which can be shown, after careful inspection, to be negative. As a consequence, the virtual transitions induce a repulsive interaction among excitons, and the energy per pair $\mathscr{E}(\nu)$ is a linearly increasing function of the filling factor. The ground state is thus a Bose condensate of weakly repulsive excitons. A similar conclusion was reached by LL.[2]

C. Collective modes

To examine the analogy with the standard Bose fluid, we calculate the dispersion relations of the collective modes. We will use the same techniques as for the ideal case and linearize the operator equation of motion $i\hbar(\partial O/\partial E)=[O,H-\mu N]$, where $\mu=(d/d\nu)[\nu\mathscr{E}(\nu)]$ is the chemical potential for e-h pairs and $N=\sum_{\sigma,k}c^\dagger_{k\sigma}c_{k\sigma}$, the particle number operator. We get

$$\omega\langle\rho_e(\mathbf{K})\rangle=\omega\langle\rho_h(\mathbf{K})\rangle=[\nu(1-\nu)]^{1/2}[\tilde{E}_x(\mathbf{K})-\tilde{E}_x(0)][\langle d(-\mathbf{K})\rangle-\langle d^\dagger(\mathbf{K})\rangle]\ ,$$

$$\omega\langle d^\dagger(\mathbf{K})\rangle=(2\nu-1)[\tilde{E}_x(\mathbf{K})-\tilde{E}_x(0)]\langle d^\dagger(\mathbf{K})\rangle$$
$$+[\nu(1-\nu)]^{1/2}[\tilde{E}_x(0)-\tilde{E}_x(\mathbf{K})+B(\mathbf{K})-\mathscr{V}(\mathbf{K})][\langle\rho_e(\mathbf{K})\rangle+\rho_h(\mathbf{K})\rangle]\ ,$$

where the two potential energies are

$$B(\mathbf{K})=\sum_{q,\sigma_1,\sigma_2}\tilde{F}_{\sigma_1,\sigma_2}(q,K_y)e^{-iK_xqa_0^2}\ ,$$

$$\mathscr{V}(\mathbf{K})=\sum_{q,\sigma_1,\sigma_2}\tilde{F}_{\sigma_1,\sigma_2}(K_y,q)e^{-iK_xqa_0^2}\ .$$

We again find no plasma oscillations. The dispersion relations are easily derived and take the form

$$\omega=\{[\tilde{E}_x(\mathbf{K})-\tilde{E}_x(0)]^2+4\nu(1-\nu)[\tilde{E}_x(\mathbf{K})-\tilde{E}_x(0)]\times[\mathscr{V}(\mathbf{K})-B(\mathbf{K})]\}^{1/2}\ . \quad (18)$$

Noting that $\mathscr{V}(0)=0$, because of charge neutrality, and that $B(0)=B$ is negative as shown previously, the dispersion relation reduces for small $\mathbf{K}$ to

$$\omega=2[\nu(1-\nu)]^{1/2}(-B)^{1/2}[\tilde{E}_x(\mathbf{K})-\tilde{E}_x(0)]^{1/2}$$
$$=2[\nu(1-\nu)]^{1/2}(-B)^{1/2}(2M)^{1/2}\hbar K+\cdots\ ,$$

where M is the single-exciton mass. The result (18) should be compared with the dispersion relation of a standard Bose gas with repulsive interaction:[9]

$$\omega=\left[\left[\frac{\hbar^2K^2}{2m}\right]^2+\frac{\hbar^2K^2}{m}n_0\bar{V}(\mathbf{K})\right]^{1/2}\ ,$$

where m, n_0, and $\bar{V}(\mathbf{K})$ are, respectively, the single boson mass, the condensate density, and the Fourier transform of the repulsive interaction. We get a complete analogy by identifying $\tilde{E}_x(\mathbf{K})-\tilde{E}_x(0)$ with the single boson kinetic energy,

$$\nu(1-\nu)=|\langle d_0^\dagger\rangle|^2/N$$

with the condensate density and $2[\mathscr{V}(\mathbf{K})-B(\mathbf{K})]$ with the Fourier transform of the effective boson-boson interaction. Thus, for small $\mathbf{K}$, the collective excitations are simply exciton density fluctuations, with sound velocity

$$v_s=[\nu(1-\nu)]^{1/2}\left[\frac{-2B}{M}\right]^{1/2}\ .$$

For high values of $\mathbf{K}$ they reduce to single-exciton excitations.

The existence of a finite sound velocity raises the possibility of a finite critical velocity for superflow of the exciton condensate. Note that a simple application of the Landau criterion[10] is not possible in this case since the asymptotic limit of $\omega(K)$ as $K\rightarrow\infty$ is a constant. We would, however, point out that at a small superflow velocity the dissipative processes implied by the violation of the Landau criterion involve the creation of a widely

separated e-h pair (i.e., an exciton with a large K value), and the matrix element for any localized perturbation creating such an excitation will be vanishingly small [$\sim\exp(-\frac{1}{2}K^2a_0^2)$]. Therefore, we believe that the critical superflow velocity is finite. The next question is how to give a velocity to the exciton condensate. This, we will show, can be achieved by an electric field.

V. INFLUENCE OF AN ELECTRIC FIELD PERPENDICULAR TO THE MAGNETIC FIELD

A. A single magnetic exciton in an applied electric field

An electric field $\mathscr{E}$ parallel to $\hat{\mathbf{x}}$ creates two extra potentials:

$$T_e = e\int \psi_e^\dagger(\mathbf{r})\mathscr{E}_x x\psi_e(\mathbf{r})d^3r \ ,$$

acting on the electrons, and

$$T_h = -e\int \psi_h^\dagger(\mathbf{r})\mathscr{E}_x x\psi_h(\mathbf{r})d^3r \ ,$$

acting on the holes. Therefore, the total Hamiltonian is no longer translationally invariant. However, the extra potential can be written as

$$T = -\mathscr{E}_x e a_0^2 \sum_{K_y} K_y \left[\sum_q a^\dagger_{(K_y/2)+q}a_{(K_y/2)+q} + b^\dagger_{(K_y/2)-q}b_{(K_y/2)-q}\right] ,$$

i.e., the matrix elements depend only on the e-h separation in the x direction within a pair, which in turn is proportional to the K_y component of the pair wave vector. In other words, the exciton creation operators $d^\dagger_{\mathbf{K}}$ still commute with the Hamiltonian, and thus still generate the pair eigenstates of the system. The energy of the exciton is now

$$E_x(\mathbf{K},\mathscr{E}_x) = E_x(\mathbf{K},0) - \mathscr{E}_x e a_0^2 K_y \ .$$

The extra energy which is proportional to the electric field is easily identified with the energy of the electric dipole carried by the exciton. The group velocity becomes

$$\mathbf{v}_g(\mathscr{E}_x) = \mathbf{v}_g(0) - \mathscr{E}_x a_0^2 \frac{e}{\hbar}\hat{\mathbf{y}}$$

and, taking account of the internal electric field $\mathscr{E}_{\rm int}$ [Eq. (10)] acting on the hole from the electron, one gets

$$\mathscr{E}_{\rm int} + \mathscr{E} = -\mathbf{v}_g \times \frac{\mathbf{H}}{c} \ .$$

This means that the magnetic exciton can again be understood as being composed of an electron and a hole travelling with the same velocity $\mathbf{v}_g$, such that the electric and magnetic forces cancel (Hall effect). As a consequence, applying an electric field on the $\mathbf{K}=0$ exciton produces a velocity $c(\mathscr{E}/H)$ perpendicular to both electric and magnetic fields. Although the exciton is a neutral composite particle, it reacts to the application of $\mathscr{E}$ and H as a single charge or a rigid classical electric dipole.

B. The exciton condensate in an electric field

Let us compute the energy of a Bose condensate of excitons, all of them having the same wave vector $\mathbf{K}$, in the presence of the electric field $(\mathscr{E}_x,0)$. Such a condensate is described by the BCS-type wave function,

$$|\psi(\nu,\mathbf{K})\rangle = \prod_q (u + \nu e^{-iK_x q a_0^2} \times a^\dagger_{(K_y/2)+q} b^\dagger_{(K_y/2)-q})\,|\,\text{vac}\rangle \ ,$$

where $u^2+\nu^2=1$, and u and ν are determined from

$$\langle a^\dagger_{k_1}a_{k_2}\rangle = \langle b^\dagger_{k_1}b_{k_2}\rangle = \nu^2\delta_{k_1,k_2} \ ,$$

$$\langle a^\dagger_{k_1}b^\dagger_{k_2}\rangle = \delta_{k_1+k_2,K_y} u\nu e^{iK_x(k_1-k_2)a_0^2/2} \ .$$

The expectation value of the energy is straightforwardly evaluated as

$$\begin{aligned}\langle\psi(\nu,\mathbf{K})\,|\,\tilde{H}+T\,|\,\psi(\nu,\mathbf{K})\rangle &= \sum_{\sigma_1,\sigma_2,k}\tilde{F}_{\sigma_1,\sigma_2}(0,k)\nu^4 - \sum_{\sigma,q}\tilde{F}_{\sigma,\sigma}(q,0)\nu^4 \\ &\quad + \sum_{\sigma,q}\tilde{F}_{\sigma,-\sigma}(q,\sigma K_y)u^2\nu^2 e^{-iK_x\sigma q a_0^2} \\ &\quad - e a_0^2 \mathscr{E}_x K_y \nu^2 N \ .\end{aligned}$$

The first term is the Hartree term, which vanishes due to charge neutrality. The other terms are, respectively, the exchange, the pairing, and the electric-field-induced terms. Remembering that

$$B = \sum_{q,\sigma,\sigma_2}\tilde{F}_{\sigma_1,\sigma_2}(q,0) \ ,$$

the energy per e-h pair takes the form

$$\mathscr{E}(\nu,\mathbf{K}) = \tilde{E}_x(\mathbf{K}) - \nu[B + \tilde{E}_x(\mathbf{K}) - E_x(0)] - \mathscr{E}_x e a_0^2 K_y \ ,$$

and the condensate group velocity $\mathscr{V}_c$ is

$$\mathscr{V}_c = \mathbf{v}_g(1-\nu) - \mathscr{E}_x \frac{e a_0^2}{\hbar}\hat{\mathbf{y}} \ ,$$

where $\mathbf{v}_g$ is the group velocity of the single exciton with the same wave vector as the condensate.

Hence, the $\mathbf{K}=0$ condensate, which was the ground state of the system in absence of the electric field, acquires the Hall velocity $-\mathscr{E}_x e a_0^2/\hbar\hat{\mathbf{y}}$ when the electric field is applied. As long as this Hall velocity is smaller than the superfluid critical velocity discussed above, the exciton condensate cannot dissipate its kinetic energy and behaves like a superfluid.

VI. LUMINESCENCE PROPERTIES

In discussing the luminescence we must distinguish between direct- and indirect-gap semiconductors. In the former case (e.g., GaAs), there can be a direct decay of the $\mathbf{K}=0$ exciton with the emission of a photon. In the latter case (e.g., Ge or Si), luminescence decay is possible only

with the emission of a phonon as well, to conserve the wave vector. In this case there are no restrictions on the momentum of the decaying exciton. We will consider first the latter case.

A. Indirect-gap semiconductor

We consider first the decay of a single exciton with 2D wave vector $\mathbf{K}$ (measured from the wave vector connecting the two band extrema). The luminescence process can be calculated by the golden rule with an intensity

$$I_{\rm ex}(\omega)=\frac{2\pi}{\hbar}\,|\langle \text{vac}|\mathscr{L}d_{\mathbf{K}}^{\dagger}|\text{vac}\rangle|^2\delta(\hbar\omega-E_x(\mathbf{K})+\hbar\omega_{\rm ph})\ ,$$

where the photon energy $\hbar\omega$ is measured from the energy difference between the electron and hole Landau levels and $\omega_{\rm ph}$ is the phonon energy which we take as a constant independent of $\mathbf{K}$. The luminescence operator $\mathscr{L}$ is represented conveniently in terms of the annihilation operators $\tilde{a}_{\mathbf{k}}$ ($\tilde{b}_{\mathbf{k}}$) of electrons (holes) in 2D Bloch states in the absence of the magnetic field,

$$\mathscr{L}=g\sum_{\mathbf{k},\mathbf{K}}\tilde{a}_{\mathbf{k}+\frac{1}{2}\mathbf{K}}\tilde{b}_{-\mathbf{k}+\frac{1}{2}\mathbf{K}}\ .$$

Here, g is a coupling constant. The Landau-level operators a_k (b_k) introduced in (3) are from (1) related by

$$a_k=\sum_{k_x}\mathscr{A}(k_x,k)\tilde{a}_{\mathbf{k}},\quad b_{-k}=\sum_{k_x}\mathscr{A}^*(k_x,k)\tilde{b}_{-\mathbf{k}}\ ,$$

with $\mathbf{k}=(k_x,k)$ and

$$\mathscr{A}(k_x,k)=L^{-1/2}\exp(ik_xka_0^2-k_x^2a_0^2/2)\ .$$

The exciton creation operator $d_{\mathbf{K}}^{\dagger}$ can be written from (6) as

$$d_{\mathbf{K}}^{\dagger}=N^{-1/2}\sum_q e^{-iK_xqa_0^2}\sum_{k_x,k_x'}\mathscr{A}\left[k_x,\frac{K_y}{2}+q\right]\times\mathscr{A}\left[-k_x',-\frac{K_y}{2}+q\right]\times\tilde{a}^{\dagger}_{k_x,K_y/2+q}\tilde{b}^{\dagger}_{-k_x',K_y/2-q}\ .$$

It is straightforward to evaluate the luminescence intensity and one finds

$$I_{\rm ex}(\omega)=\frac{2\pi}{\hbar}g^2e^{-\mathbf{K}^2a_0^2/2}\delta(\hbar\omega-E_x(\mathbf{K})+\hbar\omega_{\rm ph})\ ,$$

i.e., there is a Gaussian weighting factor in the luminescence intensity. The origin of this factor is clear. The luminescence intensity in an indirect-gap semiconductor is proportional to the probability of finding the electron and hole at the same point,[11] but from (8) the dipole of the exciton leads to

$$|\psi_{\mathbf{K}}(\mathbf{R},\mathbf{r}=0)|^2\propto\exp\left[-\frac{K^2a_0^2}{2}\right],$$

i.e., to a Gaussian decay of the e-h overlap and thus of the luminescence efficiency.

Turning now to the many-exciton case we wish to examine the decay process starting from the wave function (11). There are two types of decay processes possible. One is a direct decay of a $\mathbf{K}=0$ exciton which will change the number of $\mathbf{K}=0$ excitons from $N\nu$ to $N\nu-1$:

$$I_{\rm coh}=\frac{2\pi}{\hbar}\left|\left\langle\psi\left[\nu-\frac{1}{N}\right]\middle|\mathscr{L}\middle|\psi(\nu)\right\rangle\right|^2\delta(\hbar\omega-E_x(0)+\hbar\omega_{\rm ph})\ .$$

It is simple to evaluate the matrix element, using the form of the wave function, as $\propto uv$, leading, for this coherent decay process, to

$$I_{\rm coh}(\omega)=\frac{2\pi}{\hbar}g^2u^2v^2N\delta(\hbar\omega-E_x(0)+\hbar\omega_{\rm ph})=\frac{2\pi}{\hbar}g^2\nu(1-\nu)N\delta(\hbar\omega-E_x(0)+\hbar\omega_{\rm ph})\ .$$

This process reaches its maximum efficiency at an exactly half-filled Landau level and disappears both as $\nu\to0$ and $\nu\to1$.

The second (incoherent) process arises when the decay process involves an electron and a hole from different terms in the product wave function. The final state is now an excited state with $N\nu-2$ excitons in the $\mathbf{K}=0$ state and one exciton is a $\mathbf{K}\neq0$ state. After some algebra, we find the intensity

$$I_{\rm inc}(\omega)=\frac{2\pi N}{\hbar}g^2\sum_{\mathbf{K}}e^{-K^2a_0^2/2}4v^2\times\delta(\hbar\omega-2E_x(0)+E_x(\mathbf{K})+\hbar\omega_{\rm ph})\ .$$

The incoherent term grows in efficiency as ν^2, until as $\nu\to1$, all processes are incoherent. The total luminescence intensity is the sum of the two terms, with the coherent process more important as $\nu\to0$ and the incoherent one dominating as $\nu\to1$. The total intensity is shown schematically in Fig. 1 for the ideal symmetric model. In the case of repulsive interactions between the excitons, the collective mode or excitation spectrum has a linear dispersion as $\mathbf{K}\to0$ (see Sec. IV) and thus will reduce the strength of the incoherent process as $\mathbf{K}\to0$ or $(\hbar\omega+\hbar\omega_{\rm ph})\to E_x(0)$.

B. Direct-gap semiconductor

In this case, the $\mathbf{K}=0$ excitons are coupled directly to the photon and the matrix element in general is larger, as no phonon is involved. Since the many-exciton condensate wave function is essentially a superposition of $\mathbf{K}=0$ noninteracting excitons, the coherent process is an example of an ideal two-level system with a nonzero matrix element of the electric dipole between these two levels. This will cause a phase locking between the exciton condensate and the photon field. In the absence of any other decay channel, the system will oscillate in a time determined by the strength of the electric dipole matrix element[12] and the average number of excitons will be an oscillatory function of the time. Thus, we can expect a very strong lasing action in this case. Note that the coherent process in the indirect case will not lase so easily because of the necessity to build up and maintain a coherent population of pho-

nons as well. Note also that, in the direct case, there is no analog of the incoherent process since the total wave vector is required to be conserved. As a result, there is only the coherent process, and since this process vanishes as $\nu \rightarrow 1$, the maximum-density e-h 2D droplet will have no bulk luminescence decay process in the direct-band-gap case.

VII. A MORE REALISTIC MODEL: MAGNETIC EXCITONS IN QUASI-TWO-DIMENSIONAL GALLIUM ARSENIDE

In reality, a 2D semiconductor is made by an epitaxial slice of a small-gap semiconductor (e.g., GaAs) sandwiched between two barriers of higher-gap semiconductors (e.g., AlAs) and both the electrons and holes are confined in quantum wells. In the strong magnetic field limit, the wave functions of electrons are

$$|m,n,k,\tau\rangle = \Theta_m(z) e^{ik_y} \phi_n(x + ka_0^2) \otimes \tau \ ,$$

where the quantum numbers are m for the transverse confining level, n for the Landau level within a given m level, k for the wave vector along y which labels the x coordinate of the center of a cyclotron orbit, and τ for the spin projection along the z axis (parallel to the magnetic field and to the growth direction). The corresponding energies are

$$E_{m,n,\sigma} = E_m + (n + \tfrac{1}{2})\hbar\Omega_e + \tau g \mu_B H \ ,$$

where E_m, Ω_e, and g are, respectively, the confining energy, the electron cyclotron frequency, and the conduction-band effective Landé factor. This simple labeling of the states comes from the s character of the electron states around the Brillouin-zone center.

The situation is far more complicated for the hole states, due to the spin-orbit coupling. Following Luttinger,[13] we write the effective Hamiltonian for the lowest hole states using the effective mass approximation as

$$H_{\text{hole}} = \frac{-1}{m_0} \left[(\gamma_1 + \tfrac{5}{2}\overline{\gamma}) \frac{\mathbf{P}^2}{2} - \overline{\gamma} (\mathbf{P}\cdot\mathbf{J})^2 + \kappa \frac{e}{c} \mathbf{J}\cdot\mathbf{H} \right] .$$

Here, $\mathbf{P}$ is the operator $[(\hbar/i)\nabla - (e/c)\mathbf{A}]$, m_0 is the free-electron mass, and $\mathbf{J}$ is the angular momentum vector operator acting within the $j = \frac{3}{2}$ hole manifold. γ_1, $\overline{\gamma}$, and κ are Luttinger parameters. We assumed for simplicity a spherical symmetry for the bulk semiconductor, neglecting the cubic anisotropy. Actually the effective parameter $\overline{\gamma}$ (Ref. 14) amounts to $\overline{\gamma} = (3\gamma_3 + 2\gamma_2)/5$, where γ_2 and γ_3 are the cubic Luttinger coefficients. A better approximation would be to use the so-called axial model,[15] but this would not change qualitatively our results. Scaling the energies by the free-electron cyclotron energy $\hbar\omega_0 = \hbar eH/m_0 c$ and lengths by the magnetic length $a_0 = (\hbar c/eH)^{1/2}$, and assuming the wave functions of the form

$$\psi(x,y,z) = e^{ik_y} \sum_{m=-3/2}^{3/2} f_m(z) \phi_m(x - ka_0^2) \left| \tfrac{3}{2}, m \right\rangle ,$$

and using new scaled coordinates defined as $X = (1/a_0)(x - ka_0^2)$, $Y = y/a_0$, $Z = z/a_0$, then the Luttinger Hamiltonian can be written as shown at right:

$$H_{\text{well}} = \hbar\omega_0 \begin{array}{|cccc|} \hline (\gamma_1 - 2\overline{\gamma})\dfrac{P_z^2}{2} + (N + \frac{1}{2})(\gamma_1 + \overline{\gamma}) + \frac{3}{2}\kappa & -i\overline{\gamma}\sqrt{6}P_Z a^\dagger & \overline{\gamma}\sqrt{3}a^{\dagger 2} & 0 \\ i\overline{\gamma}\sqrt{6}P_Z a & P_Z^2(\gamma_1 + 2\overline{\gamma}) + (N + \frac{1}{2})(\gamma_1 - \overline{\gamma}) + \frac{1}{2}\kappa & 0 & \overline{\gamma}\sqrt{3}a^{\dagger 2} \\ \overline{\gamma}\sqrt{3}a^2 & 0 & P_Z^2(\gamma_1 + 2\overline{\gamma}) + (N + \frac{1}{2})(\gamma_1 - \overline{\gamma}) - \frac{1}{2}\kappa & -i\overline{\gamma}\sqrt{6}P_Z a^\dagger \\ 0 & \overline{\gamma}\sqrt{3}a^2 & i\overline{\gamma}\sqrt{6}P_Z a & \dfrac{P_Z^2}{2}(\gamma_1 - 2\overline{\gamma}) + (N + \frac{1}{2})(\gamma_1 + \overline{\gamma}) - \frac{3}{2}\kappa \\ \hline \end{array} \tag{19}$$

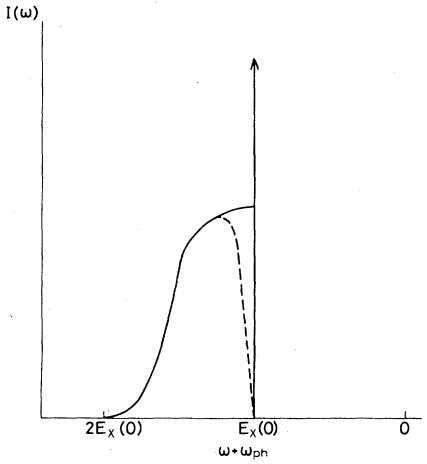

FIG. 1. Schematic form of the luminescence intensity for an indirect-gap semiconductor. The coherent process leads to the δ function at $E_x(0)$, while the incoherent process has a final-state energy between $E_x(0)$ and $2E_x(0)$. The relative weight of the processes is $\nu/(1-\nu)$. Dashed line: form of the incoherent luminescence for the interacting case.

Here,

$$P_Z=\frac{1}{i}\frac{\partial}{\partial Z},\quad a=\frac{1}{\sqrt{2}}\left[X+\frac{\partial}{\partial X}\right],$$

$$a^\dagger=\frac{1}{\sqrt{2}}\left[X-\frac{\partial}{\partial X}\right],\quad N=a^\dagger a$$

and the matrix is written in the basis $|\frac{3}{2},\frac{3}{2}\rangle$, $|\frac{3}{2},\frac{1}{2}\rangle$, $|\frac{3}{2},-\frac{1}{2}\rangle$, $|\frac{3}{2},-\frac{3}{2}\rangle$. The eigenfunctions clearly take the form

$$\psi(X,Y,Z)=e^{ika_0Y}\sum_{l=0}^{3}\Theta^{(n)}_{3/2-l}(Z)\phi_{n-l}(X)\,|\,\tfrac{3}{2},\tfrac{3}{2}-l\rangle\ , \quad (20)$$

where $\phi_n(X)$ is the nth harmonic-oscillator wave function. The eigenfunctions must also be eigenstates of the barrier effective Hamiltonian, which takes the same form as $H_{\rm well}$ (with its own Luttinger parameters), except for an additive diagonal term Δ which is the offset energy between the valence bands of the two bulk materials. One has to solve this set of coupled differential equations subject to the conditions of continuity of both wave functions and probability current densities at the two interfaces. Although this problem has been solved numerically for superlattices,[16] this is a tremendous task and we will make approximations to proceed further.

Let us assume that, (i) the offset parameter Δ is large, which implies that the wave functions do not spread within the barrier, and (ii) the thickness d of the well is rather small. In this situation of the confinement energy is rather large and the dominant terms in $H_{\rm well}$ are those proportional to P_Z^2. The confinement will be stronger for heavy holes than for light ones and, within this approximation, the low-lying states are

$$\psi_{+,k_y}=L^{-1/2}e^{ika_0Y}\left[\frac{2}{d}\right]^{1/2}\cos\left[\frac{\pi Z}{d}\right]\Phi_0(X)\,|\,\tfrac{3}{2},\tfrac{3}{2}\rangle,$$

with energy

$$E_+=\hbar\omega_0\frac{\gamma_1-2\bar\gamma}{2}\left[\frac{\pi}{d}\right]^2+\gamma_1+\bar\gamma+\tfrac{3}{2}\kappa\ ,$$

and

$$\psi_{-,k}=L^{-1/2}e^{ika_0Y}\left[\frac{2}{d}\right]^{1/2}\cos\left[\frac{\pi Z}{d}\right]\phi_0(X)\,|\,\tfrac{3}{2},-\tfrac{3}{2}\rangle\ ,$$

with energy

$$E_-=\hbar\omega\frac{\gamma_1-2\bar\gamma}{2}\left[\frac{\pi}{d}\right]^2+\gamma_1+\bar\gamma-\tfrac{3}{2}\kappa\ ,$$

the thickness d of the well being measured in units of a_0. Note that $\psi_{+,k}$ is an exact eigenstate of $H_{\rm well}$.

There are, however, relevant corrections to ψ_- which can be obtained by considering P_Z terms as a perturbation. (This corresponds to expanding in powers of d.) The P_Z term couples the $\phi_0(X)\,|\,-\frac{3}{2}\rangle$ component to a $\phi_1(X)\,|\,\frac{1}{2}\rangle$ component from a constant term in $H_{\rm well}$ which we will drop. Within the $\phi_1(X)\,|\,-\frac{1}{2}\rangle,\phi_0(X)\,|\,-\frac{3}{2}\rangle$ manifold, the effective Hamiltonian which determines the transverse wave functions $\Theta^{(n)}_m(Z)$ [see Eq. (20)] is

$$H_{\rm eff}=\begin{vmatrix} -\left[\frac{\gamma_1+2\bar\gamma}{2}\right]\frac{\partial^2}{\partial Z^2}+\frac{3}{2}(\gamma_1-\bar\gamma)-\frac{1}{2}\kappa & -\bar\gamma\sqrt{6}\frac{\partial}{\partial Z} \\ \bar\gamma\sqrt{6}\frac{\partial}{\partial Z} & -\left[\frac{\gamma_1-2\bar\gamma}{2}\right]\frac{\partial^2}{\partial Z^2}+(\gamma_1+\bar\gamma)-\frac{3}{2}\kappa \end{vmatrix}.$$

The ground state can be easily found, assuming that the wave functions vanish at the well edges $Z=\pm(d/2)$. It is described by a wave function of the form

$$\psi_-\sim[\cos(q_1Z)+a\cos(q_2Z)]\phi_0(X)e^{ikYa_0}\,|\,-\tfrac{3}{2}\rangle$$
$$+[b\sin(q_1Z)+c\sin(q_2Z)]\phi_1(X)e^{ikYa_0}\,|\,-\tfrac{1}{2}\rangle\ ,$$

where q_1 and q_2 both differ from $q=\pi/d$.

To be specific, let us list some data[17] for GaAs, assuming an applied magnetic field $H=10$ T: magnetic length 80 Å, electron cyclotron energy 17 meV, heavy-hole cyclotron energy $\hbar\omega_0(\gamma_1+\bar\gamma)=12$ meV, hole Zeeman splitting $3K\hbar\omega_0=4$ meV, magnetic exciton energy $(e^2/\epsilon a_0)(\pi/2)^{1/2}\approx 9$ meV, electron Zeeman splitting

negligible (<1 meV), and three-dimensional Rydberg energy 4 meV. With these data, the energy splitting between ψ_+ and ψ_- appears to be independent (to 1% accuracy) of the well thickness d ($d<1.35$). To estimate the change of the wave function due to the spin-orbit perturbation, we computed the projection c of ψ_- on the unperturbed transverse wave function $(2/d)^{1/2}\cos(\pi Z/d)$. c evidently decreases with increasing d, and amounts to 0.992 for $d=1.35$, which means that the perturbation of the wave function is extremely small. When comparing numerically the $|-\frac{3}{2}\rangle$ component of ψ_- with the unperturbed wave function, one sees that the coupling with the $|-\frac{1}{2}\rangle$ state produces a slight concentration of the wave function towards the center of the layer: the hole described by ψ_- is slightly more 2D than the one described by ψ_+. We got similar results for Ge.

It is clear from GaAs data that the ground state of a many e-h pair system must now be computed within the manifold of the Zeeman split lowest Landau levels of electrons and holes. The up and down spin electrons, as well as the up (i.e., $+\frac{3}{2}$) hole have the same transverse wave function. Let us define the creation operators $c^\dagger_{\sigma,s,k}$ which create a particle σ in the state

$$L^{-1/2}e^{iky}f_{\sigma,s}(Z)\phi_0(x+\sigma ka_0^2)\,|\,\sigma,s\rangle\ ,$$

where s labels the spin part of the wave function, i.e., $|11\rangle$ and $|1,-1\rangle$ are the electronic states $|s\uparrow\rangle$ and $|s\downarrow\rangle$, $|-11\rangle$ and $|-1,-1\rangle$ are the hole states $|\frac{3}{2}\rangle=|-i(x+iy)/\sqrt{2}\uparrow\rangle$ and $|-\frac{3}{2}\rangle=|i(x-iy)/\sqrt{2}\downarrow\rangle$. As the Coulomb interaction does not change the spin part of the wave functions, the Hamiltonian takes the form

$$H=\sum_k E_Z c^\dagger_{-1,1,k}c_{-1,1,k}+\sum_{\sigma_1,\sigma_2,s_1,s_2,q,k_1,k_2}\ \sum_{s_1,s_2}F_{\sigma_1,\sigma_2}(q,k_1-k_2)c^\dagger_{\sigma_1,s_1,\sigma_1k_1-q/2}c^\dagger_{\sigma_2,s_2,\sigma_2k_2+q/2}c_{\sigma_2,s_2,\sigma_2k_2-q/2}c_{\sigma_1,s_1,\sigma_1k_1+q/2}\ ,$$

where E_z is the hole-splitting energy $E_z\sim 3\kappa\hbar\omega_0$, where the electron Zeeman splitting has been neglected. The set of Coulomb matrix elements $\{F\}$ is defined by generalizing Eqs. (13) and (14). The same calculation as in Sec. IV leads to the following set of four excitons, with wave functions

$$\psi_{x,s_1,s_2}=N^{-1/2}\sum_q e^{-iK_xqa_0^2}a^\dagger_{s_1,K_y/2+q}b^\dagger_{s_2,K_y/2-q}\,|\,\mathrm{vac}\rangle$$

and energies

$$E_{x,s_1,s_2}=\delta_{s_{2,1}}E_z+\sum_{q,\sigma}\ \sum_{s_1,s_2}F_{\sigma,-\sigma}(q,\sigma K_y)e^{-iK_x\sigma qa_0^2}\ .$$

Knowing that the most general BCS wave function involves a single pairing type,[18] we choose a variational ground-state wave function of the form

$$|\,\psi(\nu)\rangle=\prod_k\{u+v[\cos(\alpha_k)e^{i\gamma_k}a^\dagger_{k\uparrow}+\sin(\alpha_k)e^{-i\gamma_k}a^\dagger_{k_\downarrow}][\cos(\beta_k)e^{i\delta_k}b^\dagger_{-k\uparrow}+\sin(\beta_k)e^{-i\delta_k}b^\dagger_{-k\downarrow}]\}\,|\,\mathrm{vac}\rangle\ ,$$

where the angles α_k, β_k, γ_k, and δ_k specify the spin direction of the electron and the hole involved in a given pair. Obviously, for a filling factor ν smaller than one, if no external constraint is imposed, the ground state corresponds to $\beta_k=\pi/2$, i.e., the pairs involve only $|-\frac{3}{2}\rangle$ holes. It is easily found that the energy per e-h pair does not depend on the electron-spin direction and is

$$\mathscr{E}(\nu)=\sum_{k,\sigma}F^{(-3/2)}_{\sigma,-\sigma}(q,0)-\nu\sum_{q,\sigma_1,\sigma_2}F^{(-3/2)}_{\sigma_1,\sigma_2}(q,0)\ .$$

Here the Coulomb matrix elements $F^{(-3/2)}_{\sigma_1,\sigma_2}(q,0)$ do not depend on the spin direction of the electrons, because both spin electrons have the same orbital wave function. Thus, we can apply the result of the simple asymmetric case studied in Sec. III, i.e., there is an attractive interaction between the excitons. The repulsive part of the interaction, due to virtual transitions to excited Landau levels, scales roughly as $1/a_0^2\Omega_c$, i.e., is independent of the magnetic field. On the other hand, the attractive part of the interaction is an increasing function of the asymmetry between electron and hole transverse wave functions, i.e., an increasing function of the magnetic field for a given well thickness. As a consequence, one can expect a crossover between the two types of ground states. A Bose condensate of excitons will occur for relatively small fields (but still big enough so that the cyclotron energies remain bigger than the 3D Rydberg energy). At higher magnetic fields, a maximum-density 2D droplet of e-h pairs will occur when the attractive interaction between excitons due to the asymmetry overcomes the repulsive interaction due to virtual transitions.

In practice, the e-h pairs will be produced by optical pumping and this has important consequences for the e-h states. If the incident light propagates parallel to the magnetic field with circular polarization and with a frequency small enough to prevent creation of light holes, two different cases arise.[19]

(i) Right circular polarization (σ_-): the excited electrons are up spin and the holes are $|-\frac{3}{2}\rangle$. We have just discussed this case above.

(ii) Left circular polarization (σ_+): the excited electrons are down spin and the holes are $|+\frac{3}{2}\rangle$. The trans-

verse wave functions of the two types of particle are the same and the system is exactly symmetric. Thus, the effective interaction between excitons is repulsive whatever the ratio of the layer thickness to the magnetic length, and the ground state is an exciton Bose condensate.

These arguments presuppose that the spin lifetimes of both types of particle are rather long. It is well known that, in three dimensions and without any applied magnetic field, the spin lifetime of holes in GaAs, etc., is extremely small compared with the recombination time. This is due to the fact that the hole wave functions are not pure spin at finite **K** vector.[20] As a consequence, interactions with phonons can change the hole spin projection, leading to a very short hole spin lifetime. This argument is no longer valid in our case, because of the quasi-zero-dimensional energy spectrum which imposes severe restrictions on possible phonon transitions. Thus, the assumption of a long spin lifetime should be reasonable in our case.

VIII. CONCLUSIONS

In this paper we have examined the two-component system of electrons and holes confined to move in 2D in a strong perpendicular magnetic field. The quantum Hall effect and particularly the fractional quantum Hall effect are exceptional properties of the one-component system. The properties of the two-component system are also exceptional but in a different way. The ground state of the ideal two-component system with symmetry between electrons and holes is exactly soluble as ideal 2D Bose gas. This result was obtained some years ago by LL.[2] In this paper a simpler, and hopefully more transparent, derivation is given of this remarkable result. In more realistic models, there are corrections from any asymmetry between electrons and holes and from virtual transitions to higher Landau levels. The first type of correction favors lower densities and therefore a uniform density exciton Bose condensate, while the latter favors the maximum possible density and therefore a 2D droplet with the Landau levels locally filled in the interior. As we discussed in Sec. V, superflow of the condensate is possible in an electric field. This could in principle be observed by an experiment in which the condensate was created by optical pumping in one part of the sample and observed by luminescence in a different part. The spectral form of the luminescence from the exciton Bose condensate and from the maximum e-h density pancake was examined in Sec. VI. The essential conclusions are that a sharp condensate emission line should occur for both direct- and indirect-gap semiconductors with maximum intensity at a half-filled Landau level. The maximum e-h density 2D droplet cannot decay directly in a direct-band-gap semiconductor but only via phonon-assisted decay processes in both direct- and indirect-band-gap cases.

Lastly, in Sec. VII we examine in some detail the forms of the hole wave functions in GaAs and consequently the form of the e-h system under different pumping conditions. It appears that it should be possible to obtain examples of both symmetric and asymmetric systems. Experiments to test these theoretical results would be most welcome. On the theoretical side, the treatment of the temperature dependence is an interesting, largely open question with the possibility of a Kosterlitz-Thouless transition as suggested by LL.[2]

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APPENDIX: THE CHARGE-DENSITY WAVE STATE

It has been argued[3] that, for the asymmetric model (see Sec. III), the ground state of the e-h system is a charge-density wave (CDW). We will compute here the energy per e-h pair $E_{\rm CDW}(\nu)$ for such a state, for the ideal symmetric model, and show that it is larger than the energy $E_x(0)$ of the Bose condensate state. This implies that, at least for small asymmetry, the Bose condensate state remains the ground state of the e-h system.

Following Yoshioka and Lee,[21] we will assume a triangular CDW lattice with primitive translations

$$\tau_1=\frac{2\pi}{Q_0}(1,-1/\sqrt{3}),\quad \tau_2=\frac{2\pi}{Q_0}\left[0,\frac{2}{\sqrt{3}}\right],$$

whose Bravais lattice is generated by the wave vectors

$$\mathbf{Q}_{n,m}=Q_0(n+m/2,m\sqrt{3}/2),$$

where m and n are integers.

The commensurability relation, which forces a large gap at the Fermi level,[22] is

$$\nu|\tau_1\times\tau_2|=2\pi a_0^2,\quad \text{i.e., } Q_0^2a_0^2=\frac{4\pi\nu}{\sqrt{3}}, \tag{A1}$$

which fixes the lattice period.

Let us define $\rho_\sigma(\mathbf{Q})$ ($\sigma=\pm1$) as charge-density operators for both particles [see Eq. (12)], and assume the following:

(i) The same density pattern for electrons and holes, which seems natural owing to their mutual attraction.

(ii) Order parameters $\langle\rho_\sigma(\mathbf{Q}_{nm})\rangle$ depending only on the modulus of the wave vector $\mathbf{Q}$: $\langle\rho_\sigma(\mathbf{Q})\rangle=\Delta(Q)$. Then the Hartree-Fock Hamiltonian for the CDW state takes the form

$$H_{\rm CDW}=\sum_{\mathbf{Q},\sigma}E_x(Q)\Delta(Q)\rho_\sigma(-\mathbf{Q})-\sum_{\mathbf{Q}}E_x(Q)|\Delta(Q)|^2,$$

where the direct Coulomb part vanishes due to charge neutrality.

Let us specialize to a single harmonic for the CDW.[22] One then gets

$$H_{\mathrm{CDW}}=E_x(0)\nu\sum_{k,\sigma}c^{\dagger}_{\sigma,k}c_{\sigma,k}+E_x(Q_0)\Delta(Q_0)\sum_{\sigma}\sum_{|\mathbf{Q}|=Q_0}\rho_\sigma(-\mathbf{Q})$$

$$-NE_x(0)\nu^2-6E_x(Q_0)|\Delta(Q_0)|^2 ,$$

because $\Delta(0)=\nu N^{1/2}$ and there are six different wave vectors fulfilling $|\mathbf{Q}|=Q_0$. It appears that the Q_0-dependent part of H_{CDW},

$$H_{\mathrm{CDW}}(Q_0)=E_x(Q_0)\Delta(Q_0)\sum_{\sigma}\sum_{|\mathbf{Q}|=Q_0}\rho_\sigma(-\mathbf{Q}) ,$$

scales on $E_x(Q_0)\Delta(Q_0)$. This implies that the spectrum of H_{CDW} for a given type of particle is the same (up to a scaling constant) as in the one-component system.[22] Furthermore, the solution of the self-consistency relation

$$\Delta(Q_0)=\langle\rho_\sigma(\mathbf{Q})\rangle_{|\mathbf{Q}|=Q_0}$$

is also the same, which means that the one-component density pattern does not depend on the effective interaction $E_x(\mathbf{Q})$.

The energy per e-h pair is

$$E_{\mathrm{CDW}}(\nu)=E_x(0)\nu+\frac{6}{\nu}E_x(Q_0)\Delta^2(Q_0) . \tag{A2}$$

Knowing that for $\nu<\frac{1}{2}$ the charge densities are the same (within a few percent) as in the magnetic Wigner crystal[22]

$$\Delta_w(Q)=N^{1/2}\nu\exp(-\tfrac{1}{4}a_0^2Q^2) ,$$

taking account of (7), (A1), and (A2), one gets

$$\frac{E_{\mathrm{CDW}}(\nu)}{E_x(0)}=\nu+6\nu e^{-\pi\nu\sqrt{3}}I_0\left[\frac{\pi\nu}{\sqrt{3}}\right] \tag{A3}$$

for $0\le\nu\le0.5$. For $\nu>0.5$, the electron charge-density pattern corresponds to a hole Wigner crystal.[22] From the e-h symmetry, one can show that the energy per e-h pair fulfills the relation[4]

$$\nu E(\nu)=(2\nu-1)E(\nu=1)+(1-\nu)E(1-\nu) ,$$

which knowing that $E(\nu=1)=E_x(0)$ defines, from (A3), $E_{\mathrm{CDW}}(\nu)$ for $\nu>0.5$.

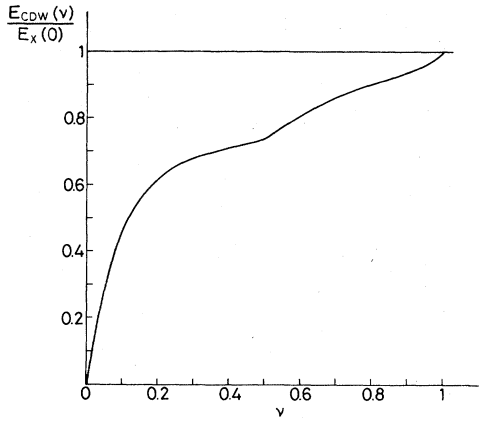

FIG. 2. Scaled energy per e-h pair $E_{\mathrm{CDW}}(\nu)/E_x(0)$ as a function of the filling factor ν for the ideal symmetric model. This function is always <1, demonstrating that the exciton Bose condensate state is the stable one.

The function $E_{\mathrm{CDW}}(\nu)/E_x(0)$ is drawn in Fig. 2 for $0\le\nu\le1$. This quantity remains smaller than one. This implies that, whatever the filling factor, the CDW energy remains higher than the Bose condensate energy. A better estimation would include higher harmonics. For the single-component problem, Yoshioka and Lee[21] showed that the higher harmonics lower the CDW cohesive energy only by a few percent. It is thus extremely unlikely that these terms could stabilize the CDW state in the ideal e-h problem.

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