Reduced quantum fluctuations in the Josephson junction

K. Wódkiewicz
Department of Physics and Astronomy, University of Rochester, Rochester, New York, 14627
*and Institute of Theoretical Physics, Warsaw University, Warsaw 00-681, Poland**
(Received 18 January 1985)

We predict that a Josephson junction can exhibit, under certain conditions, squeezed or reduced quantum fluctuations of the tunneling current. Using the formal analogy of the Josephson junction to a system of two-level atoms, we show that these reduced fluctuations are closely related to the Bloch-state description of the Bardeen-Cooper-Schrieffer wave function and to the angular-momentum algebra of the proper dynamical variables describing the tunneling process.

One of the main properties of the quantum fluctuations of two dynamical variables $\hat{A}$ and $\hat{B}$ is that they satisfy the uncertainty principle:

$$\Delta A \Delta B \geqslant \tfrac{1}{2}|[\hat{A},\hat{B}]| \ . \tag{1}$$

From this fundamental principle it follows that there is no basic restriction on the magnitude of the individual fluctuations ΔA and ΔB as long as the inequality (1) is satisifed.

This fact has led to the investigation of the so-called squeezed or reduced quantum fluctuations where we can have

$$\Delta A^2 < \tfrac{1}{2}|[\hat{A},\hat{B}]| \text{ or } \Delta B^2 < \tfrac{1}{2}|[\hat{A},\hat{B}]| \ ; \tag{2}$$

i.e., one of the individual fluctuations of $\hat{A}$ or $\hat{B}$ is smaller than their joint uncertainty measured by $\frac{1}{2}|[\hat{A},\hat{B}]|$.

The theoretical importance of such squeezed fluctuations has been quite widely recognized in quantum optics,[1] optical communications,[2] and gravitational-wave detector schemes.[3] In optics $\hat{A}$ can be identified as the amplitude of one of the quadrature phases of the electromagnetic field or atomic dipole operators. A number of possible schemes for the generation of such squeezed states have been proposed, utilizing a wide variety of nonlinear effects in quantum optics including, for example, parametric amplifiers, two-photon transitions, three- or four-wave mixing, or resonance fluorescence.[1] So far there has been no experimental manifestation of squeezed states of light in any of the above mentioned systems.

It is the purpose of this Brief Report to show that squeezed fluctuations also occur in the Josephson junction and that the quantity that can exhibit reduced quantum fluctuations is the Josephson current. These squeezed fluctuations of the Josephson current are mathematically closely related to reduced quantum fluctuations of the radiating dipoles in the strong-field resonance fluorescence of a two-level atom.[4] These formal similarities between the Josephson junction and the two-level dynamics have their source in the *two-level* tunneling model due to Feynman, Leighton, and Sands.[5] Since then the two-level analogy of the Josephson junction has been extensively used in the literature.[6–8] One of the main results of this research has been the fact that pseudospin or the angular-momentum algebra describes remarkably well the tunneling of electron pairs between the superconductors and the dynamics of systems of two-level atoms.

In the angular-momentum notation the tunneling of the Cooper pairs from the left to the right superconductors is described by the operator[9]

$$\hat{S} = \frac{1}{\sqrt{2m}} \sum_{k,q} \hat{\sigma}_k^{\dagger} \hat{\sigma}_q \ , \tag{3}$$

where $\hat{\sigma}_k^{\dagger}(\hat{\sigma}_q)$ are the Anderson pseudospin operators[10] for the creation and annihilation of a pair on the left (right) electrode. In this definition m is the number of pair states between the Fermi energy and the Debye energy. Similarly, we can describe the inverse process by $\hat{S}^{\dagger} = (\hat{S})^{\dagger}$. These two operators, combined with the following operator,

$$\hat{S}_3 = \tfrac{1}{2}\left(\sum_k \hat{\sigma}_{k3} - \sum_q \hat{\sigma}_{q3}\right) \ , \tag{4}$$

which counts the number of excess pairs in the left superconductor ($\hat{\sigma}_{3k}$ and $\hat{\sigma}_{3q}$ are the Anderson-pair population operators[10]), satisfy the following commutation relations:

$$[\hat{S}_3,\hat{S}^{\dagger}] = \hat{S}^{\dagger}, \quad [\hat{S}_3,\hat{S}] = -\hat{S} \ . \tag{5}$$

The tunneling Hamiltonian describing the transfer of the Cooper pairs may be written as follows:[11]

$$\hat{H}_T = -\frac{\hbar I_0}{4em}(\hat{S}^{\dagger} + \hat{S}) = -\frac{\hbar I_0}{2em}\hat{S}_1 \ , \tag{6}$$

where I_0 is the experimentally measured maximal Josephson tunneling current and $\hat{S}_1 = \frac{1}{2}(\hat{S}^{\dagger} + \hat{S})$. The tunneling current is given by the following expression:

$$\hat{I} = -2e\frac{d\hat{S}_3}{dt} = -\frac{2e}{i\hbar}[\hat{S}_3,\hat{H}_T] = \frac{I_0}{m}\hat{S}_2 \ , \tag{7}$$

where $\hat{S}_2 = (1/2i)(\hat{S}^{\dagger} - \hat{S})$.

From the general relation (1), the two Hermitian operators $\hat{S}_1$ and $\hat{S}_2$ associated with the tunneling energy and current, respectively, satisfy the uncertainty principle in the following form:

$$\Delta S_1 \Delta S_2 \geqslant \tfrac{1}{2}|[\hat{S}_1,\hat{S}_2]| \ . \tag{8}$$

From Eq. (2) we see that reduced fluctuations of the $\hat{S}_2$ operator associated with the Josephson current are possible if

$$\Delta S_2^2 < \tfrac{1}{2}|[\hat{S}_1,\hat{S}_2]| \ . \tag{9}$$

From the definitions of the pair transition operators we calculate that

$$[\hat{S}_1,\hat{S}_2]=\frac{1}{2i}[\hat{S},\hat{S}^\dagger]$$
$$=\frac{i}{2m^2}\left(\sum_{q'qk}\hat{\sigma}^\dagger_{q'}\hat{\sigma}_q\hat{\sigma}_{k3}-\sum_{k'kq}\hat{\sigma}^\dagger_{k'}\hat{\sigma}_k\hat{\sigma}_{q3}\right) . \quad (10)$$

For a Josephson tunnel junction, the wave function $|\psi\rangle$ that represents the two pieces of superconductor, both in their ground states, can be written as

$$|\psi\rangle=|\psi_{\rm BCS};L\rangle\otimes|\psi_{\rm BCS};R\rangle , \quad (11)$$

where at zero temperature the Bardeen-Cooper-Schrieffer (BCS) wave function of the left electrode is given by[12]

$$|\psi_{\rm BCS};L\rangle=\prod_k(u_k+v_k\hat{\sigma}_k)|0\rangle , \quad (12)$$

where $v_k(u_k)$ is the probability amplitude that the pair state is occupied (unoccupied). A similar expression holds for the right BCS wave function $|\psi_{\rm BCS};R\rangle$.

It is well known[13,14] that the junction wave function (11) is analogous to the Bloch state $|\theta,\phi\rangle$, i.e., the coherent state of the angular-momentum algebra[15] with ϕ being the phase difference between the two sides of the junction and the angle θ describing the phase correlation between the two superconductors. The many-body wave function (11) corresponds to a Bloch state with the total angular momentum J equal to m.[16]

For such a state we have the following expectation value of the $\hat{S}_2$ operator:

$$\langle\hat{S}_2\rangle=m\sin\theta\sin\phi ; \quad (13)$$

accordingly we obtain the famous Josephson relation:[17]

$$\langle I\rangle=I_J\sin\phi , \quad (14)$$

where $I_J=I_0\sin\theta$.

If the correlation between the left- and right-hand electrodes is sufficiently weak, the right-hand side of the commutator (10) is simply $\hat{S}_3$ and the Josephson dynamics is fully described by angular-momentum operators.[7,9,16,18,19] In this case the condition for reduced quantum fluctuations of $\hat{S}_2$ [see Eq. (9)] has the form $\Delta S_2^2<\frac{1}{2}|\langle\hat{S}_3\rangle|$. For the BCS states of the superconductors we have

$$\Delta S_2^2=\frac{J}{2}(1-\sin^2\theta\sin^2\phi)$$

and

$$|\langle\hat{S}_3\rangle|=\frac{J}{2}|\cos\theta| ,$$

and reduced or squeezed fluctuations of the Josephson current are possible if[20]

$$1-\sin^2\theta\sin^2\phi<|\cos\theta| . \quad (15)$$

This inequality with the proper interpretation of the Bloch angles θ and ϕ is the main clue for the investigation of squeezed fluctuations in a Josephson junction. Note that the condition (15) is independent of the number of Cooper pairs m involved in the problem.

In the steady state the Josephson junction has fixed values of θ, m, and $\dot{\phi}=2e/\hbar V_0$ (where V_0 is the electrostatic voltage).[21] A Josephson junction biased at zero voltage for which $\theta=\pi/2$ gives the greatest correlation between the two superconductors. It is clear from the inequality (15) that in this case no reduced fluctuations of the current can be observed. Now squeezed fluctuations can occur also for the other case of completely phase uncorrelated superconductors with $\theta=0$ or π. The only region where a reduction of fluctuations can occur corresponds to situations where there is a large deviation of the Bloch angle θ from $\pi/2$. Junctions composed of ultrathin superconducting films or junctions consisting of two different superconductors with different transition temperatures can lead, under certain conditions, to states in which θ deviates significantly from $\pi/2$.[16]

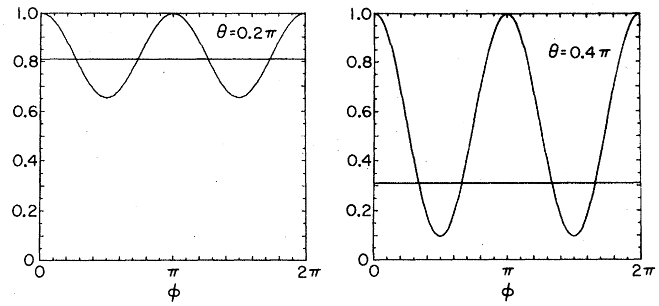

FIG. 1. Plots of $1-\sin^2\theta\sin^2\phi$ (the dispersion ΔS_2^2 scaled in units of $J/2$) as a function of ϕ for two different values of the Bloch angle: $\theta=0.4\pi$ and $\theta=0.2\pi$. The straight line corresponds to $|\cos\theta|$.

Figure 1 shows two examples of the steady-state fluctuations of S_2 as a function of the Josephson angle ϕ for two different values of the Bloch angle: $\theta=0.4\pi$ and $\theta=0.2\pi$. The straight line corresponds to $|\cos\theta|$. Regions of reduced fluctuations [see the inequality (15)] occur clearly for certain values of ϕ. Note that for a given configuration of the junction (with $\theta\neq\pi/2$) reduced quantum fluctuations occur always for the minimal values of the Josephson current fluctuations at $\phi=\pi/2$, i.e., for $\langle\hat{I}\rangle=I_J$.

Much larger deviations of the Bloch angle θ from the value $\pi/2$ can be observed in transient situations in which a Josephson junction is irradiated by nearly resonant electromagnetic radiation.[14,16,22] But for the purpose of this Brief Report we limit our discussion to steady-state situations only. Possible measurements of the Josephson current fluctuation (at zero temperature) for different junction configurations (different θ) as a function of a dc voltage offer several experimental advantages compared with the similar attempts in optics. The difficult task of obtaining a well-controlled phase ϕ in optics is relatively simpler for the Josephson junction, where for the dc or the ac voltage there is an available method for an accurate measurement of ϕ (e.g., the Shapiro steps[23]). For these reasons one can hope that a measurement of the current fluctuations in a Josephson junction at zero temperature can offer first evidence of reduced quantum fluctuations associated with the uncertainty relation (8).

The author would like to thank Professor J. H. Eberly for his hospitality at the University of Rochester where this work was performed and to Dr. J. Cresser for helpful comments about the final form of the manuscript. This research was partially supported by the U.S. Office of Naval Research.

*Permanent address.

[1]See, for example, the review article by D. F. Walls, Nature **306**, 141 (1983).
[2]H. P. Yuen and J. H. Shapiro, IEEE Trans. Inf. Theory **IT-24**, 657 (1978); **IT-26**, 78 (1980).
[3]C. M. Caves, Phys. Rev. D **23**, 1693 (1981).
[4]D. F. Walls and P. Zoller, Phys. Rev. Lett. **47**, 709 (1981).
[5]R. P. Feynman, R. B. Leighton, and M. Sands, *The Feynman Lectures on Physics* (Addison-Wesley, Reading, 1965), Vol. III, Chap. 21.
[6]D. Rogovin, Ann. Phys. (N.Y.) **90**, 18 (1975).
[7]See the review article by D. Rogovin and M. Scully, Phys. Rep. **25C**, 177 (1976).
[8]A. Barone and G. Paterno, *Physics and Applications of the Josephson Effect* (Wiley, New York, 1982).
[9]P. A. Lee and M. O. Scully, Phys. Rev. B **3**, 769 (1971).
[10]P. W. Anderson, Phys. Rev. **112**, 1900 (1958).
[11]M. Cohen, L. Falicov, and J. Phillips, Phys. Rev. Lett. **8**, 316 (1962).
[12]J. Bardeen, L. Cooper, and J. Schrieffer, Phys. Rev. **108**, 1175 (1957).
[13]D. Rogovin, Phys. Rev. B **12**, 130 (1975).
[14]A. DiRienzo, D. Rogovin, and M. O. Scully, in *Coherence and Quantum Optics IV*, edited by L. Mandel and E. Wolf (Plenum, New York, 1978), p. 15.
[15]F. T. Arecchi, E. Courtens, R. Gilmore, and H. Thomas, Phys. Rev. A **6**, 2211 (1972).
[16]D. Rogovin, M. O. Scully, and A. DiRienzo, Phys. Rev. B **18**, 3231 (1978).
[17]B. D. Josephson, Phys. Lett. **1**, 251 (1962).
[18]L. A. Lugiato and M. Milani, Nuovo Cimento **55B**, 417 (1980).
[19]R. A. Ferrell, Phys. Rev. B **25**, 496 (1982).
[20]A similar condition holds for the reduced fluctuations of the dipole-moment operator of two-level atoms coupled to a strong pulsed-laser interaction; see K. Wódkiewicz, Opt. Commun. **51**, 198 (1984); K. Wódkiewicz and J. H. Eberly, J. Opt. Soc. Am. **B2**, 458 (1985).
[21]We omit here the very small pulling of the electrostatic frequency. See, for details, P. A. Lee and M. O. Scully, Phys. Rev. Lett. **22**, 23 (1969).
[22]R. Bonifacio, D. Rogovin, and M. O. Scully, Opt. Commun. **21**, 293 (1977).
[23]S. Shapiro, Phys. Rev. Lett. **11**, 80 (1963).