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Pinning of incommensurate spin-density waves by impurities

P. F. Tua and J. Ruvalds
Physics Department, University of Virginia, Charlottesville, Virginia 22901
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The pinning of incommensurate spin-density waves (SDW's) by both nonmagnetic and magnetic impurities is investigated in quasi-one-dimensional systems. Nonmagnetic impurities pin the SDW weakly by inducing a distortion of the total electron density near the impurity sites. Magnetic impurities have a stronger effect. Because of the local magnetic field produced by the SDW, the energy density has a periodicity equal to half the SDW wavelength. The dependence of the energy density on an external magnetic field is analyzed and predictions are made on the possibility of observing narrow-band noise in "sliding" SDW's.

I. INTRODUCTION

In recent years, there has been a considerable interest in inorganic chain compounds which exhibit a phase transition associated with the formation of an incommensurate charge-density wave (CDW).[1] Their unusual transport properties are due to the collective response of electrons condensed in the CDW state. In one dimension, the electron density for a given spin direction γ is

$$\rho_\gamma^{\mathrm{CDW}}(x)=\tfrac{1}{2}\rho_0+\rho_1\cos(2k_F x-\phi)\ , \tag{1}$$

where ρ_0 is the total (linear) electron density, $2\rho_1$ is the CDW amplitude, k_F is the Fermi wave number in the direction of the chain, and ϕ determines the position of the CDW.

The first evidence for collective CDW dynamics was the observation[2] of nonlinear dc conductivity in $NbSe_3$ at very low applied dc electric fields E. Non-Ohmic conduction was later shown to occur only after a well-defined threshold electric field E_T is exceeded.[3] For low applied dc fields the CDW is assumed to be pinned by impurities, and only when the electric field exceeds the pinning force does the CDW become mobile and contribute to the conductivity. A particularly striking feature in the nonlinear regime is the presence of coherent current oscillations[3] (or narrow-band noise) whose fundamental frequency is proportional to the current carried by the condensate.[4]

The "sliding" motion of the CDW was originally proposed by Fröhlich,[5] and subsequently analyzed by Lee *et al.*,[6] who showed that the translational electron-phonon collective mode, or phason mode, is responsible for the CDW motion. Despite intense theoretical and experimental work, the microscopic process of CDW depinning and conduction is still unclear. Bardeen[7] has suggested that CDW motion arises from Zener tunneling of the CDW condensate through an impurity pinning gap. Grüner *et al.*[8] have proposed a simple explanation in which the CDW is considered as a rigid object moving in a washboard impurity potential. Other classical theories[9–12] take into account internal degrees of freedom of the CDW. Bardeen's theory gives the best overall fit to the current-voltage characteristics. It must also be mentioned that the narrow-band noise can be generated by a Josephson-type mechanism,[13] although a quite different approach attributes the origin of the narrow-band noise to the contact regions.[14,15]

Lee *et al.*[6] have speculated that a similar sliding motion was expected for incommensurate spin-density wave (SDW) systems, even if the formation of a SDW condensate arises out of direct electron-electron interaction. The existence of SDW's was first proposed by Overhauser[16] who showed that, in the Hartree-Fock approximation, the paramagnetic state of an electron gas is unstable with respect to the formation of a static SDW below a critical temperature T_c. In one dimension, the electron density for a given spin direction γ is

$$\rho_\gamma^{\mathrm{SDW}}(x)=\tfrac{1}{2}\rho_0+\rho_1\cos\left[2k_F x+\gamma\frac{\pi}{2}-\phi\right]\ , \tag{2}$$

where $\gamma=1$ and -1 for spin-up and spin-down bands, respectively. In the incommensurate case, the SDW condensate is not supported by the underlying lattice and has uniform density. Chromium[17–22] has provided much of the early interest as an example of antiferromagnetism associated with static SDW's in three dimensions.

Recently, Takada[23] has shown that in one dimension the inclusion of the translational spin-wave mode leads to a collective conductivity analogous to the CDW case, but without electron-mass renormalization. Experimental studies[24] of the frequency dependence of the conductivity of tetramethyl tetraselenafulvalene-hexafluorophosphate [$(TMTSF)_2PF_6$] in the microwave region have provided some evidence for a collective response of SDW's in a linear chain compound. Nevertheless, no appreciable threshold electric field for the onset of nonlinear dc conductivity was found and, in the nonlinear regime, there was no evidence of the narrow-band noise typical of incommensurate CDW's. These results have been explained by assuming that nonmagnetic impurities couple weakly to SDW's.[23] In the approximation up to the first order in the electron-impurity interaction, there is no pinning because the total electron density is constant in the SDW case. Nevertheless, the interaction between impurities and conduction electrons might generate a small distortion of the charge density near the impurity sites in the form of Friedel oscillations which, in turn, could produce a very weak pinning.[25,26] In the present paper, we show that the inclusion of second-order terms gives rise to a weakly pinned SDW. These terms represent the Josephson-type mechanism originally proposed by Barnes and Zawadowski[13] for CDW's. They showed that the CDW condensate can be thought of as the superposition of two macroscopic quantum states. Each state is composed of electron-hole pairs with, respectively, total momentum $\pm 2k_F$ in the chain direction, where k_F is the Fermi momentum. Two subsequent impurity scatterings with large momentum transfer can scatter coherently an electron-hole pair with momentum $2k_F$ into an electron-hole pair with momentum $-2k_F$, and hence the two macroscopic quantum states are weakly coupled in analogy with the Josephson effect. In the present paper, we consider only the case $\rho_1 \ll \rho_0$ and hence we neglect the effects due to the second-order CDW harmonic that appears when $\rho_1 \lesssim \rho_0$ even in the absence of impurities. These effects were considered by Lee and Rice[25] to explain the weak pinning of the SDW in Cr by Ta impurities.[27]

A different situation arises when magnetic impurities are included.[23] Even in the approximation of undistorted SDW oscillations, magnetic impurities can pin the SDW condensate. In the present paper we show that this effect occurs because the orientation of the impurity spins is affected by the local magnetic field generated by the SDW. As a consequence, the energy density is periodic in space with periodicity equal to half the SDW wavelength. Nevertheless, when a strong magnetic field is applied so that all the impurity spins are aligned in the same direction, the energy density regains the same periodicity of the SDW. Furthermore, we consider the distortion produced by magnetic impurities on the total electron density, and we compare the results with the CDW case.[28]

We safely assume that the magnitude of the SDW order parameter is independent of the magnetic field. This assumption is corroborated by the fact that the transition temperature in chromium is not dependent on the applied magnetic field.[21] It must also be noted that in $NbSe_3$ the transition temperature is not affected by magnetic fields up to 206 kG,[29] even if the threshold electric field is strongly suppressed at temperatures below 20 K.[30] To explain the anomalous resistance in $NbSe_3$ at very low temperatures, Coleman[29] has proposed the coexistence of SDW and CDW states.

We develop the formalism for SDW's in Sec. II, we discuss the case of nonmagnetic and magnetic impurities in Secs. III and IV, respectively, and we summarize our results in Sec. V.

II. MODEL HAMILTONIAN

The SDW condensate is assumed to consist of a single domain analogous to the domains proposed by Fukuyama and Lee[31] for CDW's. Neglecting the electron-phonon interaction, we consider the following one-dimensional Hamiltonian for the incommensurate case:

$$H_0 = \sum_\gamma \int dx\, \psi_\gamma^\dagger(x) \left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} \right] \psi_\gamma(x) + \frac{g}{2} \sum_\gamma \int dx\, \psi_\gamma^\dagger(x) \psi_{-\gamma}^\dagger(x) \psi_{-\gamma}(x) \psi_\gamma(x) \,, \tag{3}$$

where $\psi_\gamma^\dagger(x)$ and $\psi_\gamma(x)$ are field operators with spin index $\gamma = \pm 1$. The Coulomb interaction has been replaced by a repulsive contact potential ($g > 0$) between electrons with opposite spin and on opposite sides of the Fermi surface. The field operator $\psi_\gamma(x)$ can be split into "right" (R) and "left" (L) parts as

$$\psi_\gamma(x) = \psi_{R,\gamma}(x) + \psi_{L,\gamma}(x) \,, \tag{4a}$$

$$\psi_{\alpha,\gamma}(x) = l^{-1/2} \sum_{\alpha k\,(>0)} e^{ikx} a_{k,\gamma} \quad (\alpha = R, L) \,, \tag{4b}$$

where α as a multiplicative factor means $+$ or $-$ for $\alpha = R$ or L, respectively, l is the (linear) size of the domain, and $a_{k,\gamma}$ ($a_{k,\gamma}^\dagger$) is the destruction (creation) operator of an electron in the state (k,γ). The relevant Green's functions are defined by the relation

$$G_\gamma^{(\alpha,\beta)}(x_1, x_2, t_1 - t_2) = -i \langle T \psi_{\alpha,\gamma}(x_1, t_1) \psi_{\beta,\gamma}^\dagger(x_2, t_2) \rangle \,, \tag{5}$$

where the angular brackets denote averaging over the Gibbs distribution with Hamiltonian H_0. The mean-field approximation of H_0 for a SDW instability is

$$H_{\mathrm{SDW}} = \sum_\gamma \int dx\, \psi_\gamma^\dagger(x) \left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} \right] \psi_\gamma(x) - \sum_\gamma \left[i\gamma\Delta \int dx\, e^{i2k_F x} \psi_{R,\gamma}^\dagger(x) \psi_{L,\gamma}(x) + \mathrm{H.c.} \right] + 2|\Delta|^2 l/g \,, \tag{6}$$

where the complex order parameter Δ is given by the gap equation

$$\Delta = -g\gamma G_\gamma^{(R,L)}(0, 0, t = 0^-) \,. \tag{7}$$

At zero temperature, one obtains $|\Delta(T=0)| \simeq 2\xi e^{-1/\lambda}$ with $\lambda = g/2\pi v_F$, where ξ is an appropriate energy cutoff, λ is the dimensionless coupling constant, and v_F is the Fermi velocity. It must be noted that in $(TMTSF)_2PF_6$ the critical temperature[24] is about 11 K, which is considerably smaller than the critical temperatures[1] for CDW instability in linear chain compounds.

The energy spectrum of H_{SDW} is independent of the phase of Δ which, on the other hand, determines the position of the SDW. Setting $\Delta = |\Delta| e^{-i\phi}$, the electron density for a given spin orientation γ is

$$\rho_\gamma^{(0)}(x) = \tfrac{1}{2}\rho_0 + \rho_1 \cos\left[2k_F x + \gamma\frac{\pi}{2} - \phi \right] , \tag{8}$$

where $\rho_1 = 2|\Delta|/g$. Alternatively, one can assume Δ real and positive in Eqs. (6) and (7) and then make the following gauge transformation:

$$\psi_{\alpha,\gamma}(x) \rightarrow e^{i\phi_\alpha}\psi_{\alpha,\gamma}(x) , \tag{9}$$

where the phases are different for R and L. The electron density in the form of Eq. (8) is recovered by setting $\phi = \phi_L - \phi_R$. There is no such gauge invariance associated with a commensurate SDW.

With the help of Eq. (9), the Green's functions of Eq. (5) can be rewritten in the form

$$G_\gamma^{(\alpha,\beta)}(x_1, x_2, t_1 - t_2) = e^{i(\phi_\alpha - \phi_\beta)} e^{ik_F(\alpha x_1 - \beta x_2)} e^{i\gamma(\alpha-\beta)\pi/4} \overline{G}^{(\alpha,\beta)}(x_1 - x_2, t_1 - t_2) , \tag{10}$$

where the $\overline{G}$'s are spin and gauge independent.

III. NONMAGNETIC IMPURITIES

The interaction between conduction electrons and nonmagnetic impurities is given by the Hamiltonian

$$H_{\rm NM} = V \sum_{j,\gamma} \psi_\gamma^\dagger(R_j)\psi_\gamma(R_j) , \tag{11}$$

where V is the strength of the interaction, R_j is the position of the jth impurity, and the impurity distribution is random. Only the backscattering part of $H_{\rm NM}$ is kept, i.e.,

$$H'_{\rm NM} = V \sum_{j,\gamma} [\psi_{R,\gamma}^\dagger(R_j)\psi_{L,\gamma}(R_j) + \text{H.c.}] , \tag{12}$$

because the diagonal part only yields a damping term, which is not essential to the present analysis.

The first-order term of the Gibbs average of the interaction energy vanishes because the SDW is undistorted at this order of approximation. Assuming the case of time-independent phases and keeping only anomalous pairings, the second order term is

$$\begin{aligned} U_{\rm UM}^{(2)} &= \frac{1}{i\hbar} \int_{-\infty}^{t} dt' \langle [H'_{\rm NM}(t), H'_{\rm NM}(t')] \rangle \\ &= -\frac{8V^2}{\hbar} \sum_{j,l} \cos[2\phi - 2k_F(R_j + R_l)] \int_{-\infty}^{0} dt' \, \text{Im}[\overline{G}^{(R,L)}(R_l - R_j, t')\overline{G}^{(R,L)}(R_j - R_l, -t')] . \end{aligned} \tag{13}$$

This expression contains single-impurity ($j = l$) as well as pair contributions.

The single-impurity ($j = l$) term implies evaluation only of the function $\overline{G}^{(R,L)}(x = 0, t)$, which is easily calculated by analogy with the expression for the Josephson current.[32] The summation over the impurity distribution gives

$$\sum_j e^{-i4k_F R_j} = \overline{C}_2 e^{-2i\phi_2} \quad (\overline{C}_2 \text{ real}) , \tag{14}$$

where, for a random distribution, $\overline{C}_2$ is proportional to $N_{\rm imp}^{1/2}$, $N_{\rm imp}$ being the total number of nonmagnetic impurities in the SDW domain. Thus, the single-impurity term has the simple form

$$U_{{\rm UM},S}^{(2)} = -\overline{U}_{{\rm NM},S}^{(2)} \cos[2(\phi - \phi_0)] , \tag{15a}$$

where

$$\overline{U}_{{\rm NM},S}^{(2)} = \frac{V^2 \overline{C}_2 |\Delta|}{2v_F^2 \hbar^2} \tanh\left[\frac{|\Delta|}{2k_B T}\right] , \tag{15b}$$

and T is the temperature.

The contribution from impurity pairs may be comparable to the single-impurity term, if the (amplitude) coherence length is larger than the average distance between impurities. Nevertheless, the pair term is still proportional to $N_{\rm imp}^{1/2}$.

Equation (15) is similar to the corresponding expression for CDW's,[13] because the distortion produced by nonmagnetic impurities on the condensate is qualitatively the same in both cases. Therefore, one would expect the existence of a threshold field and of a narrow-band noise which have not been observed yet. A possible explanation is that the SDW oscillations are generally much smaller than the CDW oscillations, and, hence, the coupling turns out to be very weak.

If we consider the special case of a single impurity at the origin, the dependence of the interaction energy on the SDW phase ϕ is a consequence of the change in the electron density at the origin which is

$$\delta\rho_\gamma(x = 0, \phi)/\rho_1 = -\frac{Vg}{8v_F^2\hbar^2} \tanh\left[\frac{|\Delta|}{2k_B T}\right] \cos(2\phi) . \tag{16}$$

The change in the electron density at the origin is the same for both spins, and this feature is expected in all higher-odd orders in the impurity potential V. Assuming $v_F \approx 10^7$ cm sec^{-1}, $g \approx 3 \times 10^{-8}$ eV cm, and $V \approx 0.1g$, it follows that (i) at $T \approx 0$, $\max|\delta\rho_\gamma| \approx 0.26\rho_1$, which is consistent with our perturbation approach, and (ii) at

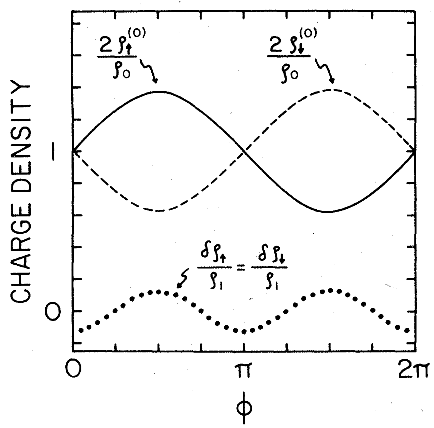

FIG. 1. Change $\delta\rho_\gamma$ in the electron density at the impurity site as a function of the SDW phase ϕ ($V>0$). The amplitude ρ_γ of the undistorted SDW at the impurity site is also shown for the sake of comparison.

$T \lesssim T_c$, $\max|\delta\rho_\gamma| \ll \rho_1$, and this effect may be too small to be detected experimentally. Figure 1 shows the change in electron density at the impurity site as a function of the phase ϕ.

Equation (15) predicts that, when a dc electric field is applied, the fundamental frequency ν of the coherent current oscillations depends on the average current $I_{\rm SDW}$ carried by the SDW condensate through the relation

$$I_{\rm SDW}/\nu=\rho_0\lambda \ , \tag{17}$$

where $\lambda=\pi/2k_F$. This is in contrast with the CDW case where the fundamental frequency is one-half the above value because of the contribution from the undistorted component of the CDW condensate.

We notice that the change in electron density near the impurity sites modifies the effects of the electron-electron interaction. This variation leads to a spatially dependent order parameter $\Delta(x)$. A self-consistent calculation of $\Delta(x)$ can be performed analytically only at zero temperature, as in the CDW case,[26] and it does not modify Eq. (15) because it involves only higher-order terms.

Teisseron *et al.*[27] have shown that, in the SDW phase of Cr doped with Ta, to one part in 10^8 the spin density at individual Ta sites is essentially random and does not take a unique value. This result indicates a weak coupling so that the individual impurities do not minimize the local SDW-impurity interaction energy as in the strong pinning, but rather the SDW phase only pins to large-scale fluctuations in the impurity density. Unfortunately, the experiment of Teisseron *et al.*[27] cannot detect small distortions of the spin density at the impurity sites if such distortions depend on the SDW phase, and hence it cannot confirm the present theory.

IV. MAGNETIC IMPURITIES

In this section, we consider the interaction of a SDW with magnetic impurities in the presence of an external and constant magnetic field H. If the magnetic field H is small (compared with the SDW energy gap), its only effect on the SDW condensate is an energy shift between the spin-up and spin-down electron bands. Consequently, the electron Green's functions defined in Eq. (5) have now the form

$$G_\gamma^{(\alpha,\beta)}(x_1,x_2,t_1-t_2)=e^{i(\phi_\alpha-\phi_\beta)}e^{ik_F(\alpha x_1-\beta x_2)}e^{i\gamma(\alpha-\beta)\pi/4}e^{-i(\gamma/2)h_e(t_1-t_2)}\bar{G}^{(\alpha,\beta)}(x_1-x_2,t_1-t_2) \ , \tag{18}$$

where $h_e=g_e\mu_B H$, g_e is an effective electron g factor, μ_B is the Bohr magneton, and H is in the z direction.

Quite generally, the interaction between conduction electrons and magnetic impurities is represented by the Hamiltonian

$$H_{\rm M}=-J\sum_j[\psi_\gamma^\dagger(R_j)\sigma_{\gamma\delta}\psi_\delta(R_j)]\cdot\mathbf{S}_j \ , \tag{19}$$

where J is the strength of the interaction, and $\mathbf{S}_j$ is the spin operator of the jth impurity. For the sake of brevity, the impurity spins are assumed to be $\frac{1}{2}$. As in the preceding section, only the backscattering part of $H_{\rm M}$ is kept, i.e.,

$$H'_{\rm M}=-J\sum_j[\psi_{\gamma,R}^\dagger(R_j)\sigma_{\gamma\delta}\psi_{\delta,L}(R_j)+\text{H.c.}]\cdot\mathbf{S}_j \ . \tag{20}$$

The magnetic field H interacts with the impurity spins through the Hamiltonian

$$H'_{\rm imp}=\sum_j g_i\mu_B H S_j^z \ , \tag{21}$$

where g_i is an effective impurity g factor. In calculating the thermal average over the impurity spins, the external magnetic field as well as the local magnetic field generated by the SDW must be considered. Assuming an undistorted SDW, the thermal average over the impurity spins is determined by the Hamiltonian

$$H_{\rm imp}=\sum_j h_j S_j^z \ , \tag{22a}$$

where

$$h_j=h_E+h_{J,j} \ , \tag{22b}$$

$$h_E=g_i\mu_B H \ , \tag{22c}$$

and

$$h_{J,j}=-(4J|\Delta|/g)\sin(\phi-2k_F R_j) \ . \tag{22d}$$

For an undistorted SDW, the thermal average of the interaction energy is

$$U_{\mathrm{M}}^{(1)}=-(4J\,|\Delta|/g)\sum_j \sin(\phi-2k_FR_j)\langle S_j^z\rangle$$
$$=(2J\,|\Delta|/g)\sum_j \sin(\phi-2k_FR_j)\tanh\left[\left(g_i\mu_BH-\frac{4J\,|\Delta|}{g}\sin(\phi-2k_FR_j\right)/2k_BT\right]. \tag{23}$$

The summation over the impurity distribution gives

$$\sum_j e^{-in2k_FR_j}=\bar{C}_n e^{-in\phi_n} \quad (n\geq 1 \text{ and } \bar{C}_n \text{ real})\ , \tag{24}$$

where, for a random distribution, the $\bar{C}_n$'s are proportional to $(N'_{\mathrm{imp}})^{1/2}$, N'_{imp} being the total number of magnetic impurities in the SDW domain. We notice that, in general, $\phi_n\neq\phi_m$ for $n\neq m$. Thus, Eq. (23) can be rewritten in the general form

$$U_{\mathrm{M}}^{(1)}(\phi,T,H)=U_{\mathrm{M},0}^{(1)}(T,H)+\sum_{n\,(\geq 1)} D_n(T,H)\sin[n(\phi-\phi'_n)]\ , \tag{25}$$

where $U_{\mathrm{M},0}^{(1)}\sim N'_{\mathrm{imp}}$ and $D_n\sim\bar{C}_n\sim(N'_{\mathrm{imp}})^{1/2}$. In the following, the constant term $U_{\mathrm{M},0}^{(1)}$ will be neglected because it does not contribute to the SDW dynamics.

If there is no external magnetic field, the odd harmonics of Eq. (25) vanish. Therefore, the interaction energy has a periodicity equal to half the SDW wavelength, and the fundamental frequency ν of the narrow-band noise is again given by Eq. (17) with $\lambda=\pi/2k_F$, as in the case of nonmagnetic impurities.

In the limits $T\simeq 0$ and $T\simeq T_c$, where T_c is the transition temperature, the interaction energy has the simple forms

$$U_{\mathrm{M}}^{(1)}(\phi,\,T\sim 0,\,H=0)=\frac{8J\,|\Delta|}{g\pi}\sum_{n=1}^{\infty}\frac{\bar{C}_{2n}}{4n^2-1}\times\cos[2n(\phi-\phi_{2n})] \tag{26}$$

and

$$U_{\mathrm{M}}^{(1)}(\phi,\,T\lesssim T_c,\,H=0)=\left[\frac{2J\,|\Delta|}{g}\right]^2\frac{\bar{C}_2}{2k_BT_c}\times\cos[2(\phi-\phi_2)]\ . \tag{27}$$

These two limiting cases show that, as the temperature is decreased, the contribution of the higher harmonics of the pinning potential increases. It follows that in the narrow-band noise the ratio of the amplitudes between the higher harmonics and the fundamental one increases as the temperature is lowered.

When the external magnetic field is turned on, the term $n=1$ in Eq. (25) gives a nonzero contribution. Therefore, the interaction energy has the same periodicity of the SDW, and the fundamental frequency ν of the narrow-band noise is now given by Eq. (17) with $\lambda=\pi/k_F$. For relatively small values of the magnetic field, the amplitude of the first harmonic of the narrow-band noise should be smaller than the amplitude of the second harmonic. In the opposite limit of a strong magnetic field, i.e., $H\gg 4J\,|\Delta|\,/g_i\mu_Bg$, $U_{\mathrm{M}}^{(1)}$ can be written as

$$U_{\mathrm{M}}^{(1)}(\phi,T,H)=(2J\,|\Delta|\,\bar{C}_2/g)\times\tanh\left[\frac{g_i\mu_BH}{2k_BT}\right]\sin(\phi-\phi_1)\ . \tag{28}$$

Unfortunately, a meaningful prediction about the relative amplitudes of the narrow-band noise in a strong magnetic field cannot be made because, as we shall see later, the distortion of the electron density at the impurity sites might give a contribution comparable to that of Eq. (28). We simply notice that, at $T\lesssim T_c$, $|U_{\mathrm{M}}^{(1)}|\propto|\Delta|$ in a strong magnetic field, while $|U_{\mathrm{M}}^{(1)}|\propto|\Delta|^2$ in absence of an external magnetic field.

Let us now consider the first correction to the interaction energy due to the distortion of the SDW around the impurities. This correction corresponds to the Josephson-type mechanism proposed by Barnes and Zawadowski[13] for CDW's. Following the same approximations of the preceding section, we obtain

$$U_{\mathrm{M}}^{(2)}=\frac{4J^2}{\hbar}\sum_{j,l}\sum_{\gamma,\delta}\cos[2\phi-2k_F(R_j+R_l)-(\gamma+\delta)\pi/2]\int_{-\infty}^{0}dt\,\mathrm{Im}\{\bar{G}^{(L,R)}(R_l-R_j,\,t)\bar{G}^{(L,R)}(R_j-R_l,\,-t)\times\langle[\sigma_{\gamma\delta}\cdot\mathbf{S}_j(0)][\sigma_{\delta\gamma}\cdot\mathbf{S}_l(t)]\rangle\exp[-(i/2)h_e(\gamma-\delta)t]\}\ . \tag{29}$$

This expression contains single-impurity ($j=l$) as well as pair contributions. Since the impurity concentration is generally low in the cases of experimental importance, we neglect correlations between the spins of two different impurities.

Following the same method of the preceding section, Eq. (29) becomes

$$U^{(2)}_{\mathrm{M},S} = -4J^2 \sum_j \cos(2\phi - 4k_F R_j) \int_{-\infty}^{+\infty} d\omega' d\omega'' n_F(\omega') n_F(\omega'') B(\omega') B(\omega'') \times \left[-\frac{1}{2}\frac{P}{\omega'+\omega''} + P_{\uparrow,j}\frac{P}{\omega'+\omega''+(h_j-h_e)} + P_{\uparrow,j}\frac{P}{\omega'+\omega''-(h_j-h_e)} \right], \tag{30a}$$

where

$$B(\omega) = \frac{1}{2\pi\hbar v_F}\frac{|\Delta|}{(\omega^2 - |\Delta|^2)^{1/2}} \mathrm{sgn}(\omega)\Theta(|\omega| - |\Delta|), \tag{30b}$$

$$P_{\uparrow,j} = y_j/(y_j + 1/y_j) = 1 - P_{\uparrow,j}, \tag{30c}$$

$$y_j = \exp(-h_j/2k_B T), \tag{30d}$$

and $n_F(\omega)$ is the Fermi factor.

If there is no external magnetic field, and if $J \ll g$, Eq. (26) can be approximated up to J^2 by the following expression:

$$U^{(2)}_{\mathrm{M},S}(\phi, T, H=0) = \frac{J^2|\Delta|\bar{C}_2}{8\hbar^2 v_F^2}\tanh\left[\frac{|\Delta|}{2k_B T}\right] \times \cos[2(\phi - \phi_2)] + O(J^3). \tag{31}$$

Assuming $v_F \approx 10^7$ cm sec^{-1}, $g \approx 3\times 10^{-8}$ eV cm, and $J \approx 0.1g$, $U^{(1)}_{\mathrm{M}}$ as given in Eqs. (26) and (27) is comparable in magnitude to $U^{(2)}_{\mathrm{M},S}$. The large uncertainty of the parameters makes it unclear which term dominates, but the general features discussed above for the case of no magnetic field are still valid.

In the limit of a strong magnetic field, i.e., $H \gg 4J|\Delta|/g_i\mu_B g$, $U^{(2)}_{\mathrm{M},S}$ strongly depends on the difference $g_i - g_e$ between the impurity and electron g factors. Only in the special case $g_i = g_e$ is $U^{(2)}_{\mathrm{M},S}$ independent of the magnetic field, with the same form given in Eq. (31). This result is analogous to the corresponding expression for the CDW case.[28] At zero temperature, $U^{(2)}_{\mathrm{M},S}$ can be expanded in powers of $\mu_B H/|\Delta|$ to obtain

$$U^{(2)}_{\mathrm{M},S}(\phi, T=0, H) = \frac{J^2|\Delta|\bar{C}_2}{8\hbar^2 v_F^2}\cos[2(\phi-\phi_2)] \times \left[1 - \eta\frac{(g_i - g_e)\mu_B|H|}{2|\Delta|} + \frac{1}{2}\left[\frac{(g_i-g_e)\mu_B H}{2|\Delta|}\right]^2 + \cdots \right], \tag{32}$$

where η is a constant of the order of unity. The magnetic field enhances $U^{(2)}_{\mathrm{M},S}$ if $g_e > g_i$, but depresses it if $g_i > g_e$. This behavior can be shown to occur also at finite temperatures. Because of the large uncertainty of the parameters, we cannot safely determine the ratio between the fundamental and the second harmonic in the narrow-band noise when a strong magnetic field is present.

To the best of our knowledge, the analysis[33,34] of the effects of Fe impurities in Cr represents the only experimental study of magnetic impurities embedded in a SDW matrix. Fe in Cr has a localized moment of about $2\mu_B$ above the critical temperature,[33,35] but the moment seems to be reduced slightly below it.[33] Susceptibility measurements[33] suggest that the localized moments are coupled very weakly with the SDW of Cr and the corresponding interaction energy E_i is of the order of 1 K. Choosing $J \approx 0.1g$ and $|\Delta| \approx 10^2$ K in Eq. (26), we obtain $E_i \gtrsim 10$ K at low temperatures, i.e., a value at least one order of magnitude larger. We cannot explain this discrepancy, but we simply point out that the SDW state in Cr has a three-dimensional character, and the conduction electrons not in the condensate might play an important role.

V. CONCLUSIONS

We have shown that magnetic as well as nonmagnetic impurities can pin a SDW. Nonmagnetic pinning is produced only by distortions of the electron density near the impurity sites and has a periodicy equal to half the wavelength of the SDW. In general, nonmagnetic pinning is small in comparison with the CDW case. In the approximation of unperturbed SDW oscillations, magnetic impurities can pin the SDW condensate because the orientation of the impurity spins is affected by the local magnetic field generated by the SDW. Consequently, the pinning energy is periodic in space with periodicity equal to half the wavelength of the SDW, as in the case with nonmagnetic impurities. When a sufficiently large magnetic field is applied, the pinning energy has the same periodicity of the SDW, and, hence, the fundamental frequency of the narrow-band noise is one-half the value expected in the zero-magnetic-field limit. The threshold electric field must also exhibit a dependence on the magnetic field. At present, there are no experimental data regarding magnetic impurity pinning of SDW's and new experiments are needed to test the present theory. Finally, we must emphasize that, in the perturbation series, terms of higher order in the exchange coupling may be relevant and further study is in progress.

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