

# Spontaneous emission by two atoms with different resonance frequencies near a metal surface

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The interaction between electromagnetic radiation and two two-level atoms with different resonance frequencies near a perfectly conducting metal surface is considered. The atom-atom and atom-surface separations are assumed to be smaller than the corresponding mean resonance wavelength. A quantum-mechanical version of the image method is adopted to study the spontaneous emission by such an atomic pair. Within the framework of this approach, each individual atom and its corresponding image are kinematically correlated, while dynamically they are in effect independent. The total radiation rate of the atomic system is calculated as a function of time for various values of frequency difference. Explicit results are given for several different initial states of the atomic system. Some of them exhibit superradiance, and some initially act as photon-trapping states and eventually are able to undergo radiative decay. Oscillations as a manifestation of beating appear in the time evolution of the radiation rate in all cases of the various initial states, when the frequency difference becomes larger than a critical value given as twice the mean halfwidth of the atomic resonance lines.

## I. INTRODUCTION

Over a decade ago, with the technical advances in fatty-acid monolayer assembly,<sup>1-3</sup> a series of measurements was successfully made on the fluorescent lifetime of oriented dye molecules held at well-controlled distances from a metal mirror.<sup>4-6</sup> It was found that for large distances from the metal surface the fluorescent lifetime oscillates as a function of distance, while for small distances the lifetime approaches zero monotonically.<sup>7</sup> A theoretical basis for these experimental observations was provided by Kuhn,<sup>7,8</sup> who utilized Sommerfeld's classical electromagnetic treatment<sup>9</sup> for radio waves propagating along the earth's surface. Within the framework of this theory, the emitting molecule acts as an oscillating dipole near a partially absorbing and partially reflecting surface. The interference between the reflected wave and the initial wave gives rise to the observed oscillations in the lifetime as a function of distance. A number of researchers<sup>10-13</sup> have further developed the calculations along the same line of arguments for more detailed energy transfers between the molecule and various surfaces, obtaining good agreement with experimental results.<sup>4-7,14,15</sup>

On the other hand, Morawitz<sup>16</sup> adopted a quantum-mechanical viewpoint to investigate the emission by a two-level atom at a distance comparable to the radiation wavelength from a metal surface. Assuming the metal to be a perfect conductor, i.e., with an infinite conductivity, he replaced the metal mirror by an image behind the mirror, at a distance equal to that between the source atom and the mirror. A symmetric or antisymmetric linear combination of the atomic excited and ground levels can then be used to represent the initial state of the emitting atom in front of the mirror, corresponding to, respectively, the perpendicular (in phase) or parallel (out of phase) dipole transition to the mirror plane. The associated physical interpretation is simply that the emitted photon

from the atom carries no information as to which process has occurred, be it by either direct emission or by reflection from the surface, so that it can be considered to be emitted from a fictitious image atom. The same results were obtained as those from the classical approach.<sup>16</sup>

This quantum-mechanical treatment was later extended by Milonni and Knight<sup>17</sup> in the consideration of spontaneous emission of an atom between two infinite plane mirrors parallel to each other. Previous theoretical approaches<sup>18,19</sup> for this two-mirror problem usually involve an expansion of the electromagnetic field in appropriate mode functions satisfying the boundary conditions imposed by the mirrors. Again, both approaches lead to the identical results, lending support to the quantum-mechanical treatment for the radiative emission of atoms near a metal surface. As a matter of fact, it has motivated the application of the more sophisticated quantum-electrodynamic theory to such a problem.<sup>20,21</sup>

In this paper, we shall consider the spontaneous emission from a system of two atoms with different resonance frequencies in the presence of a metal surface. It is important to note that the spontaneous emission of radiation has interesting properties related to the source atom's environment. Perhaps the most famous example of such an environmental influence is Dicke's superradiance,<sup>22</sup> which has stimulated a large number of investigations, both theoretically and experimentally, on cooperative radiation for atomic-gas<sup>23-29</sup> and condensed-matter<sup>30,31</sup> systems. One of our aims here is to study a somewhat related effect of interest, namely, the influence of images on the radiation of a system of atoms, which may be in a stage of cooperative radiation, when they are in the neighborhood of a metal surface. For this purpose, the quantum-mechanical approach is indispensable. Another important feature underlying the present problem is that the resonance frequencies of the atoms are different. As shown in the case of free space,<sup>32</sup> namely, in the absence of the met-

al, the emission rate of the radiation from two atoms with different resonance frequencies has distinctively different patterns, depending on the ratio of frequency difference to the single-particle decay rate. The situation is expected to become more complicated when there is a metal surface nearby. We shall apply the image method<sup>16,17</sup> to this problem for which the atom and the corresponding image are treated on the same footing. In effect, the atoms and images can be treated as if they were dynamically independent of each other, as far as the interaction with the radiation is concerned. The spontaneous emission rate for the system can thus be calculated by standard methods.<sup>24</sup> When the frequency difference is set equal to zero, the case for two identical atoms on a metal surface ensues naturally.

This paper is organized as follows. In the next section, we introduce the atomic and radiation-field operators. This is followed by the construction of atom-image correlated states and the effective Hamiltonian for the system, where physical arguments are provided. The equations of motion are given in Sec. III. Here a conservation law is established which is useful for expressing the total emission rate solely in terms of the atomic operators, and subsequently a hierarchy of coupled equations are solved with the help of appropriate approximations. Finally, we are able to relate the emission rate to all expectation values of initial states. For Sec. IV, a variety of possible initial states which can be prepared experimentally are considered, and the corresponding emission rates are computed with respect to time and frequency differences, accompanied by a discussion of the results.

## II. THEORY

### A. Two-level atoms and the quantized electromagnetic field

We shall restrict ourselves to the case where each atom near a metal surface has just two energy levels. We let  $|-\mu\rangle$  denote the ground level for the atom labeled by the index  $\mu$  and  $|+\mu\rangle$  the excited level, where normalization and orthogonality require that  $\langle +\mu | +\mu \rangle = \langle -\mu | -\mu \rangle = 1$  and  $\langle +\mu | -\mu \rangle = 0$ . We designate the frequency corresponding to the energy difference between the two levels as  $\omega_\mu$ , where  $\omega_1 \neq \omega_2$ . Although we shall concentrate on the situation in which the distance between the atoms is much smaller than their radiation wavelengths, we shall assume that the wave functions of the two atoms do not overlap, i.e.,  $\langle \pm\mu | \pm\nu \rangle = 0$  for  $\mu \neq \nu$ , so that the atoms are distinguishable. The transitions between the ground and excited levels can be facilitated by introducing excitation (raising) and deexcitation (lowering) operators  $c_\mu^\dagger$  and  $c_\mu$ , respectively, with the properties<sup>33</sup> that  $c_\mu^\dagger |-\mu\rangle = |+\mu\rangle$ ,  $c_\mu^\dagger |+\mu\rangle = 0$ ,  $c_\mu |-\mu\rangle = 0$ , and  $c_\mu |+\mu\rangle = |-\mu\rangle$ . As a result, it can be readily recognized that  $c_\mu$  and  $c_\mu^\dagger$  obey the anticommutation relation, i.e.,

$$\{c_\mu, c_\mu^\dagger\} = 1. \quad (2.1)$$

It is also required by the nonoverlapping property of the

two atoms that operators representing different atoms commute, i.e.,

$$\begin{aligned} [c_\mu^\dagger, c_\nu^\dagger] &= [c_\mu, c_\nu] = 0, \\ [c_\mu, c_\nu^\dagger] &= 0 \text{ for } \mu \neq \nu. \end{aligned} \quad (2.2)$$

Thus the atoms, according to the above descriptions, can be modeled as dipoles. Since our main concern here is the total radiation rate of the system, we shall neglect the dipole-dipole interaction between the atoms, which is expected to only shift the emitted radiation frequencies.<sup>34</sup> Therefore, the atoms are interacting with each other via the common radiation fields.

Each mode of the radiation field can be treated as a quantum oscillator with frequency  $\Omega_\alpha$ . We further distinguish the  $n_\alpha$  different quantum oscillators with the same frequency  $\Omega_\alpha$  by a second index  $\beta$ . For example, in the case of a plane wave,  $\alpha$  may refer to the frequency (or  $|\mathbf{k}|$ , where  $\mathbf{k}$  is the wave vector), while  $\beta$  refers to direction of  $\mathbf{k}$  and the polarization. Transitions between occupation number states  $|n_{\alpha\beta}\rangle$  are described by annihilation and creation operators  $a_{\alpha\beta}$  and  $a_{\alpha\beta}^\dagger$  with the well-known properties<sup>35</sup>

$$a_{\alpha\beta} |n_{\alpha\beta}\rangle = (n_{\alpha\beta})^{1/2} |n_{\alpha\beta}-1\rangle$$

and

$$a_{\alpha\beta}^\dagger |n_{\alpha\beta}\rangle = (n_{\alpha\beta}+1)^{1/2} |n_{\alpha\beta}+1\rangle.$$

These operators obey the commutation relations

$$[a_{\alpha\beta}, a_{\alpha'\beta'}^\dagger] = \delta_{\alpha\alpha'} \delta_{\beta\beta'}, \quad (2.3)$$

and the occupation number states  $|n_{\alpha\beta}\rangle$  form the orthonormal set, namely,  $\langle n_{\alpha\beta} | n_{\alpha'\beta'} \rangle = \delta_{\alpha\alpha'} \delta_{\beta\beta'}$ . All atomic operators  $c_\mu$  and  $c_\mu^\dagger$  commute with all operators ( $a_{\alpha\beta}$  and  $a_{\alpha\beta}^\dagger$ ) for the quantized electromagnetic oscillator.

### B. Atom-image correlation in the presence of a metal surface

In the present work, we shall assume the metal to have a perfect conductivity. Therefore, the role played by the metal surface with respect to radiation is nothing more than a reflecting mirror. The radiation emitted from an atom close to the surface can reach another atom by either direct transmission or through reflection from the surface, as though it were emitted by the image of the first atom (see Fig. 1). There is therefore a possibility of interference between the radiation fields from two processes, as in the case of a Lloyd's mirror.<sup>36</sup> Since these fields carry no information about which process has occurred, it leads one to consider the atom and the corresponding image on the same footing.<sup>16,17</sup> Namely, we may describe the emission by a single atom in the vicinity of a metal surface as a two-atom problem with complete uncertainty as to which atom is excited. We can thus write the initial (excited) state incorporating such ambiguity for atom  $\mu$  as

$$|e_\mu\rangle_\pm = \frac{1}{\sqrt{2}} (|+A_\mu\rangle | -I_\mu\rangle \pm | -A_\mu\rangle | +I_\mu\rangle), \quad (2.4)$$

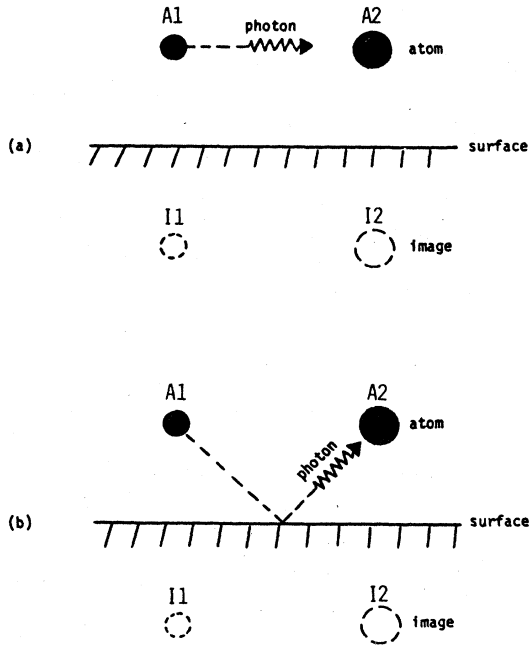


FIG. 1. Two two-level atoms with different resonance frequencies  $\omega_1$  and  $\omega_2$  located near a perfectly conducting metal surface. Both electronic transition dipoles are assumed to be perpendicular to the surface plane. (a) Direct photon transmission; (b) photon transmission via reflection from the surface.

where the indices  $A$  and  $I$  refer to the atom and image, respectively, while the meanings of  $|+\rangle$ ,  $|-\rangle$  are the same as those given in Sec. II A. State  $|e_\mu\rangle_+$  is used if the atomic transition dipole is perpendicular to the plane of the metal surface, because the dipoles of the atom and its image have the same phase. On the other hand, state  $|e_\mu\rangle_-$  is used when the transition dipole is parallel to the plane of the metal surface where dipole has a phase opposite to that of its image. The physical picture embedded in Eq. (2.4) represents a quantum-mechanical version of the classical description of an oscillating dipole and its image. The ground state is given by

$$|g_\mu\rangle = |-_{A\mu}\rangle |-_{I\mu}\rangle. \quad (2.5)$$

### C. Hamiltonian and emission rate

From the preceding discussion, we see that the effect of the metal surface can be cast in terms of the correlation between the atoms and their corresponding images. The latter can be treated effectively as independent but identical to their respective source atoms, in the sense that the source atom and its image are *kinematically correlated* but in effect *dynamically independent*. Thus, one can immediately write down an effective Hamiltonian for the system of two atoms with different frequencies  $\omega_1$  and  $\omega_2$  near a metal surface as

$$H_{\text{eff}} = \hbar \sum_{\mu=1}^2 \omega_\mu (c_{A\mu}^\dagger c_{A\mu} + c_{I\mu}^\dagger c_{I\mu}) + \hbar \sum_{\alpha} \Omega_{\alpha} \sum_{\beta=1}^{n_{\alpha}} (a_{\alpha\beta}^\dagger a_{\alpha\beta} + \frac{1}{2}) - \hbar g \sum_{\mu} \sum_{\alpha} \sum_{\beta} [a_{\alpha\beta}^\dagger (c_{A\mu} + c_{I\mu}) + (c_{A\mu}^\dagger + c_{I\mu}^\dagger) a_{\alpha\beta}], \quad (2.6)$$

where  $g$  is the radiation-atom coupling constant and  $c_{A\mu}^\dagger$  ( $c_{A\mu}$ ) is the excitation (deexcitation) operator for the  $\mu$ th atom and  $c_{I\mu}^\dagger$  ( $c_{I\mu}$ ) for the corresponding image, which obey the operator algebras given in Sec. II A. In writing Eq. (2.6) we have assumed that the atom-atom and atom-surface separations are both smaller than any of the radiation wavelengths, so that there is no phase difference at different positions of atoms and images. Equation (2.6), in fact, describes effectively the same physical properties as those of four atoms, two of them with resonance frequency  $\omega_1$  and the other two with a different resonance frequency  $\omega_2$ , except that two with the same frequency are correlated to each other rather than completely free atoms.

The quantity of central interest is the total emission rate of the system,  $\Gamma(t)$ , which is given by

$$\Gamma(t) = \frac{d}{dt} \sum_{\alpha} \sum_{\beta} \langle a_{\alpha\beta}^\dagger(t) a_{\alpha\beta}(t) \rangle, \quad (2.7)$$

where  $a_{\alpha\beta}^\dagger$  and  $a_{\alpha\beta}$  are expressed in the Heisenberg picture. (From hereon, for simplicity of presentation, the time dependence of the operators will not be displayed unless necessary.) The advantage of working in the Heisenberg picture is that the expectation value in Eq. (2.7) is taken on the initial state of the whole system,  $|\Phi(0)\rangle$ , which consists of the atomic and radiation-field states. Since we are only interested in the spontaneous emission, it is understood that the radiation-field part of the state involved here is the vacuum state. Thus, we only have to pay attention to the various *atomic* initial states of the system.

## III. CALCULATIONS

### A. Equations of motion

Using the effective Hamiltonian of Eq. (2.6) and Heisenberg's equation of motion, one can obtain the following equations of motion:

$$\dot{a}_{\alpha\beta} + i\Omega_{\alpha} a_{\alpha\beta} = ig \sum_{\mu} (c_{A\mu} + c_{I\mu}), \quad (3.1)$$

$$\dot{c}_{A\mu} + i\omega_{\mu} c_{A\mu} = ig [c_{A\mu}, c_{A\mu}^\dagger] \sum_{\alpha} \sum_{\beta} a_{\alpha\beta}, \quad (3.2)$$

$$\dot{c}_{I\mu} + i\omega_{\mu} c_{I\mu} = ig [c_{I\mu}, c_{I\mu}^\dagger] \sum_{\alpha} \sum_{\beta} a_{\alpha\beta}, \quad (3.3)$$

where the dot over an operator signifies the time derivative. In order to simplify the calculation, we begin by eliminating the rapidly oscillating part in the operator by choosing a frequency  $\omega_0$  between  $\omega_1$  and  $\omega_2$  and defining the operators  $\mathcal{A}$  and  $\mathcal{C}$  through the relations

$$\sum_{\beta=1}^{n_{\alpha}} a_{\alpha\beta}(t) = \mathcal{A}_{\alpha}(t) e^{-i\omega_0 t}, \quad (3.4)$$

$$c_{A\mu}(t) = \mathcal{C}_{A\mu}(t) e^{-i\omega_0 t}, \quad (3.5)$$

$$c_{I\mu}(t) = \mathcal{C}_{I\mu}(t) e^{-i\omega_0 t}. \quad (3.6)$$

These new operators are clearly less oscillatory in time. To write Eqs. (3.1)–(3.3) in terms of these new operators, we sum (3.1) over  $\beta$ ,

$$\sum_{\beta} \dot{a}_{\alpha\beta} + i\Omega_{\alpha} \sum_{\alpha} a_{\alpha\beta} = ign_{\alpha} \sum_{\mu} (c_{A\mu} + c_{I\mu}), \quad (3.7)$$

and then transform it by using Eqs. (3.4)–(3.6) into

$$\dot{\mathcal{A}}_{\alpha} + i(\Omega_{\alpha} - \omega_0) \mathcal{A}_{\alpha} = ign_{\alpha} \sum_{\mu} (\mathcal{C}_{A\mu} + \mathcal{C}_{I\mu}), \quad (3.8)$$

where we have used the fact that there are  $n_{\alpha}$  modes of radiation oscillators having the same frequency  $\Omega_{\alpha}$ . Similarly, we have, from Eq. (3.2),

$$\dot{\mathcal{C}}_{A\mu} + i(\omega_{\mu} - \omega_0) \mathcal{C}_{A\mu} = ig[\mathcal{C}_{A\mu}, \mathcal{C}_{A\mu}^{\dagger}] \sum_{\alpha} \mathcal{A}_{\alpha}, \quad (3.9)$$

and, from Eq. (3.3),

$$\dot{\mathcal{C}}_{I\mu} + i(\omega_{\mu} - \omega_0) \mathcal{C}_{I\mu} = ig[\mathcal{C}_{I\mu}, \mathcal{C}_{I\mu}^{\dagger}] \sum_{\alpha} \mathcal{A}_{\alpha}. \quad (3.10)$$

Equation (3.8) can be integrated to give

$$\begin{aligned} \mathcal{A}_{\alpha} &= e^{-i(\Omega_{\alpha} - \omega_0)t} \mathcal{A}_{\alpha}(0) \\ &+ ign_{\alpha} \sum_{\mu} \int_0^t dt' e^{i(\Omega_{\alpha} - \omega_0)(t' - t)} (\mathcal{C}_{A\mu} + \mathcal{C}_{I\mu}). \end{aligned} \quad (3.11)$$

Summing over all  $\alpha$ , we obtain

$$\sum_{\alpha} \mathcal{A}_{\alpha} = \sum_{\alpha} e^{-i(\Omega_{\alpha} - \omega_0)t} \mathcal{A}_{\alpha}(0) + igF(t), \quad (3.12)$$

where

$$F(t) = \sum_{\mu} \int_0^t dt' (\mathcal{C}_{A\mu} + \mathcal{C}_{I\mu}) \int d\Omega_{\alpha} \rho(\Omega_{\alpha}) e^{i(\Omega_{\alpha} - \omega_0)(t' - t)}, \quad (3.13)$$

and  $\rho(\Omega_{\alpha})$  is the density of radiation oscillators per unit frequency. Following the arguments of Dillard and Robl<sup>24</sup> that the interaction is confined to some frequency range  $\omega_0 - \delta\omega \leq \Omega_{\alpha} \leq \omega_0 + \delta\omega$  with  $\delta\omega \ll \omega_0$ , the double integrals in Eq. (3.13) can be performed to yield

$$F(t) = \pi\rho(\omega_0) \sum_{\mu} (\mathcal{C}_{A\mu} + \mathcal{C}_{I\mu}). \quad (3.14)$$

Consequently, Eq. (3.12) becomes

$$\sum_{\alpha} \mathcal{A}_{\alpha} = \sum_{\alpha} e^{-i(\Omega_{\alpha} - \omega_0)t} \mathcal{A}_{\alpha}(0) + i\pi g\rho(\omega_0) \sum_{\mu} (c_{A\mu} + c_{I\mu}), \quad (3.15)$$

or, with Eqs. (3.4)–(3.6),

$$\sum_{\alpha} \sum_{\beta} a_{\alpha\beta} = \sum_{\alpha} e^{-i\Omega_{\alpha} t} \sum_{\beta} a_{\alpha\beta}(0) + i\pi g\rho(\omega_0) \sum_{\mu} (c_{A\mu} + c_{I\mu}). \quad (3.16)$$

At this stage we are ready to obtain the total emission rate by determining a means of finding the expectation value of  $a_{\alpha\beta}^{\dagger} a_{\alpha\beta}$  [see the right-hand side of Eq. (2.7)]. We

multiply Eq. (3.1) on the left by  $a_{\alpha\beta}^{\dagger}$  and the Hermitian adjoint of Eq. (3.1) on the right by  $a_{\alpha\beta}$ , and the two results are summed to give

$$\begin{aligned} \frac{d}{dt} \sum_{\alpha} \sum_{\beta} a_{\alpha\beta}^{\dagger} a_{\alpha\beta} &= ig \sum_{\alpha} \sum_{\beta} \sum_{\mu} [a_{\alpha\beta}^{\dagger} (c_{A\mu} + c_{I\mu}) \\ &- (c_{A\mu}^{\dagger} + c_{I\mu}^{\dagger}) a_{\alpha\beta}]. \end{aligned} \quad (3.17)$$

Substituting Eq. (3.16) and its Hermitian adjoint into Eq. (3.17) and taking the expectation value on the initial state, we obtain

$$\begin{aligned} \frac{d}{dt} \sum_{\alpha} \sum_{\beta} \langle a_{\alpha\beta}^{\dagger} a_{\alpha\beta} \rangle &= 2\pi g^2 \rho(\omega_0) \\ &\times \sum_{\mu} \sum_{\nu} \langle (c_{A\mu}^{\dagger} + c_{I\mu}^{\dagger})(c_{A\nu} + c_{I\nu}) \rangle \\ &= \gamma \sum_{\mu} \sum_{\nu} \langle (c_{A\mu}^{\dagger} + c_{I\mu}^{\dagger})(c_{A\nu} + c_{I\nu}) \rangle, \end{aligned} \quad (3.18)$$

where  $\gamma = 2\pi g^2 \rho(\omega_0)$  is the transition rate for spontaneous emission by an isolated excited atom.<sup>24,37</sup> Equation (3.18) can be further written as

$$\begin{aligned} \frac{d}{dt} \sum_{\alpha} \sum_{\beta} \langle a_{\alpha\beta}^{\dagger} a_{\alpha\beta} \rangle &= \gamma \sum_{\mu} \langle c_{A\mu}^{\dagger} c_{A\mu} + c_{I\mu}^{\dagger} c_{I\mu} \rangle \\ &+ \gamma \sum_{\mu \neq \nu} \langle (c_{A\mu}^{\dagger} + c_{I\mu}^{\dagger})(c_{A\nu} + c_{I\nu}) \rangle \\ &+ \gamma (c_{A1}^{\dagger} c_{I1} + c_{A2}^{\dagger} c_{I2} + c_{I1}^{\dagger} c_{A1} + c_{I2}^{\dagger} c_{A2}), \end{aligned} \quad (3.19)$$

whose solution then yields the total emission rate.

### B. Conservation law

We digress for a moment here to derive a conservation law, which will prove to be helpful solving the overall problem. From Eq. (3.2) and its adjoint, one can write

$$\begin{aligned} \frac{d}{dt} \sum_{\mu} c_{A\mu}^{\dagger} c_{A\mu} &= ig \sum_{\mu} c_{A\mu}^{\dagger} [c_{A\mu}, c_{A\mu}^{\dagger}] \sum_{\alpha} \sum_{\beta} a_{\alpha\beta} \\ &- ig \sum_{\alpha} \sum_{\beta} a_{\alpha\beta}^{\dagger} \sum_{\mu} [c_{A\mu}, c_{A\mu}^{\dagger}] c_{A\mu}. \end{aligned} \quad (3.20)$$

From Sec. II A, we know that  $c_{A\mu}^{\dagger} [c_{A\mu}, c_{A\mu}^{\dagger}] = c_{A\mu}^{\dagger}$  and  $[c_{A\mu}, c_{A\mu}^{\dagger}] c_{A\mu} = c_{A\mu}$ . As a consequence, Eq. (3.20) becomes

$$\frac{d}{dt} \sum_{\mu} c_{A\mu}^{\dagger} c_{A\mu} = ig \sum_{\mu} c_{A\mu}^{\dagger} \sum_{\alpha} \sum_{\beta} a_{\alpha\beta} - ig \sum_{\alpha} \sum_{\beta} a_{\alpha\beta}^{\dagger} \sum_{\mu} c_{A\mu}. \quad (3.21)$$

Similarly, from Eq. (3.3) and its adjoint, we obtain

$$\frac{d}{dt} \sum_{\mu} c_{I\mu}^{\dagger} c_{I\mu} = ig \sum_{\mu} c_{I\mu}^{\dagger} \sum_{\alpha} \sum_{\beta} a_{\alpha\beta} - ig \sum_{\alpha} \sum_{\beta} a_{\alpha\beta}^{\dagger} \sum_{\mu} c_{I\mu}. \quad (3.22)$$

The addition of Eqs. (3.21) and (3.22) gives

$$\begin{aligned} \frac{d}{dt} \sum_{\mu} (c_{A\mu}^{\dagger} c_{A\mu} + c_{I\mu}^{\dagger} c_{I\mu}) = ig \sum_{\alpha} \sum_{\beta} \sum_{\mu} (c_{A\mu}^{\dagger} + c_{I\mu}^{\dagger}) a_{\alpha\beta} \\ - ig \sum_{\alpha} \sum_{\beta} \sum_{\mu} a_{\alpha\beta}^{\dagger} (c_{A\mu} + c_{I\mu}). \end{aligned} \quad (3.23)$$

In comparison with Eq. (3.17), we find the following conservation law:

$$\frac{d}{dt} (N_{\text{ph}} + N_A + N_I) = 0. \quad (3.24)$$

$N_{\text{ph}}$  represents the total number of photons, defined as

$$N_{\text{ph}} = \sum_{\alpha} \sum_{\beta} a_{\alpha\beta}^{\dagger} a_{\alpha\beta}, \quad (3.25)$$

$N_A$  represents the total number of excited atoms, defined as

$$N_A = \sum_{\mu} c_{A\mu}^{\dagger} c_{A\mu}, \quad (3.26)$$

and  $N_I$  represents the total number of excited images, defined as

$$N_I = \sum_{\mu} c_{I\mu}^{\dagger} c_{I\mu}. \quad (3.27)$$

Equation (3.24) states that the total number of the photons and the excited atoms and images is constant. Note that it is an operator equation and therefore holds for expectation values on any states. In other words,  $N_{\text{ph}} + N_A + N_I$  is a constant of motion.

### C. Solutions

Having set up the conservation law, we immediately recognize that, from Eqs. (3.24)–(3.27),

$$\frac{d}{dt} \sum_{\alpha} \sum_{\beta} a_{\alpha\beta}^{\dagger} a_{\alpha\beta} = - \frac{d}{dt} \sum_{\mu} (c_{A\mu}^{\dagger} c_{A\mu} + c_{I\mu}^{\dagger} c_{I\mu}). \quad (3.28)$$

Thus, with Eq. (2.7), we have the total radiation rate given by

$$\Gamma(t) = - \frac{d}{dt} \sum_{\mu} \langle c_{A\mu}^{\dagger} c_{A\mu} + c_{I\mu}^{\dagger} c_{I\mu} \rangle. \quad (3.29)$$

Furthermore, by combining Eqs. (3.19) and (3.28) we have an equation involving only the atomic operators,

$$\begin{aligned} \frac{d}{dt} \sum_{\mu} \langle c_{A\mu}^{\dagger} c_{A\mu} + c_{I\mu}^{\dagger} c_{I\mu} \rangle + \gamma \sum_{\mu} \langle c_{A\mu}^{\dagger} c_{A\mu} + c_{I\mu}^{\dagger} c_{I\mu} \rangle \\ = -\gamma \sum_{\mu \neq \nu} \langle (c_{A\mu}^{\dagger} + c_{I\mu}^{\dagger})(c_{A\nu} + c_{I\nu}) \rangle \\ - \gamma \langle c_{A1}^{\dagger} c_{I1} + c_{A2}^{\dagger} c_{I2} + c_{I1}^{\dagger} c_{A1} + c_{I2}^{\dagger} c_{A2} \rangle. \end{aligned} \quad (3.30)$$

At this point, it should be emphasized that the expectation value of off-diagonal terms on the right-hand side of the above equation does not necessarily vanish. This is the place where the coherence comes into play. It is in the form of a dipole-dipole interaction between atoms and images, as induced by the common radiation with which the atoms interact in phase, resulting from the fact that the atom-atom and atom-surface separations are smaller than the mean resonance wavelength. Were we to include the direct dipole-dipole interaction, we would have an additional term of the same form but with a direct dipole-dipole coupling coefficient other than  $\gamma$  on the right-hand side of Eq. (3.30). However, one should note that the expectation value of the direct dipole-dipole interaction vanishes, since it only has nonvanishing off-diagonal matrix elements due to the selection rule of electric dipole transitions. The direct dipole-dipole interaction is therefore expected to affect the frequency shift only.<sup>34</sup>

To solve Eq. (3.30), we define  $M$ ,  $P$ , and  $Q$ :

$$M = \sum_{\mu} (c_{A\mu}^{\dagger} c_{A\mu} + c_{I\mu}^{\dagger} c_{I\mu}), \quad (3.31)$$

$$P = c_{A1}^{\dagger} c_{A2} + c_{A1}^{\dagger} c_{I2} + c_{I1}^{\dagger} c_{A2} + c_{I1}^{\dagger} c_{I2}, \quad (3.32)$$

and

$$Q = c_{A1}^{\dagger} c_{I1} + c_{A2}^{\dagger} c_{I2} + c_{I1}^{\dagger} c_{A1} + c_{I2}^{\dagger} c_{A2}. \quad (3.33)$$

Thus, Eq. (3.30) becomes

$$\dot{m} + \gamma m = -\gamma(p + p^* + q), \quad (3.34)$$

where the lower-case letters represent the expectation values of the corresponding operators of capital letters defined in Eqs. (3.31)–(3.33), i.e.,  $p = \langle P \rangle$ , etc.

In order to obtain  $p$  and  $q$ , we consider the equations of motion satisfied by  $P$  and  $Q$ ,

$$\begin{aligned} \dot{P} + (2\gamma + i\Delta)P = -\gamma[M + Q - c_{A1}^{\dagger} c_{A1} (c_{A2}^{\dagger} + c_{I1}^{\dagger} + c_{I2}^{\dagger})(c_{A2} + c_{I2}) - c_{I1}^{\dagger} c_{I1} (c_{A1}^{\dagger} + c_{A2}^{\dagger} + c_{I2}^{\dagger})(c_{A2} + c_{I2}) \\ - (c_{A1}^{\dagger} + c_{I1}^{\dagger})(c_{A1} + c_{A2} + c_{I1}) c_{I2}^{\dagger} c_{I2} - (c_{A1}^{\dagger} + c_{I1}^{\dagger})(c_{A1} + c_{I1} + c_{I2}) c_{A2}^{\dagger} c_{A2}] \end{aligned} \quad (3.35)$$

and

$$\begin{aligned} \dot{Q} + Q = -\gamma[M + P + P^{\dagger} - c_{A1}^{\dagger} c_{A1} (c_{A2}^{\dagger} + c_{I1}^{\dagger} + c_{I2}^{\dagger}) c_{I1} - c_{A2}^{\dagger} c_{A2} (c_{A1}^{\dagger} + c_{I2}^{\dagger} + c_{I1}^{\dagger}) c_{A2} - c_{I1}^{\dagger} c_{I1} (c_{A1}^{\dagger} + c_{A2}^{\dagger} + c_{I2}^{\dagger}) c_{A1} \\ - c_{I2}^{\dagger} c_{I2} (c_{A1}^{\dagger} + c_{A2}^{\dagger} + c_{I1}^{\dagger}) c_{I2} - c_{A1}^{\dagger} (c_{A1} + c_{A2} + c_{I2}) c_{I1}^{\dagger} c_{I1} - c_{A2}^{\dagger} (c_{A1} + c_{A2} + c_{I1}) c_{I2}^{\dagger} c_{I2} \\ - c_{I1}^{\dagger} (c_{A2} + c_{I1} + c_{I2}) c_{A1}^{\dagger} c_{A1} - c_{I1}^{\dagger} (c_{A1} + c_{I1} + c_{I2}) c_{A2}^{\dagger} c_{A2}], \end{aligned} \quad (3.36)$$

where  $\Delta = \omega_2 - \omega_1$ .

The off-diagonal terms of four multiplicative atomic operators in the last two equations, such as  $c_{A1}^\dagger c_{A1} c_{A2}^\dagger c_{I2}$ , etc., are nonvanishing provided that the two atoms exchange a photon with a third one in its excited level. If we neglect such terms, it is equivalent to saying that we are neglecting small terms on the order of  $\gamma/\omega_0 < 1$ . Under this approximation we have

$$\dot{P} + (2\gamma + i\Delta)P = -\gamma(M + Q - 2R_1) \quad (3.37)$$

and

$$\dot{Q} + \gamma Q = -\gamma(M + P + P^\dagger - 2R_2), \quad (3.38)$$

where

$$R_1 = (c_{A1}^\dagger c_{A1} + c_{I1}^\dagger c_{I1})(c_{A2}^\dagger c_{A2} + c_{I2}^\dagger c_{I2}) \quad (3.39)$$

and

$$R_2 = 2(c_{A1}^\dagger c_{A1} c_{I1}^\dagger c_{I1} + c_{A2}^\dagger c_{A2} c_{I2}^\dagger c_{I2}). \quad (3.40)$$

Here we have made use of the fact that  $[c_{A\alpha} c_{A\alpha}, c_{I\alpha} c_{I\alpha}] = 0$ , etc., to write  $R_1$  and  $R_2$ , respectively, in the compact form in Eqs. (3.39) and (3.40). The corresponding equations for the expectation values of Eqs. (3.37) and (3.38) are

$$\dot{p} + (2\gamma + i\Delta)p = -\gamma(m + q - 2r_1) \quad (3.41)$$

and

$$\dot{q} + \gamma q = -\gamma(m + p + p^* - 2r_2). \quad (3.42)$$

From Eq. (3.41), we further obtain

$$\dot{s} + 2\gamma s + i\Delta d = -\gamma(2m + q + q^* - 2r_1 - 2r_1^*) \quad (3.43)$$

and

$$\dot{d} + 2\gamma d + i\Delta s = -2\gamma(r_1^* - r_1), \quad (3.44)$$

where  $s = p + p^*$  and  $d = p - p^*$ . Taking the time derivative of Eq. (3.43), we get

$$\ddot{s} + 2\gamma \dot{s} + i\Delta \dot{d} = -\gamma(2\dot{m} + \dot{q} + \dot{q}^* - 2\dot{r}_1 - 2\dot{r}_1^*). \quad (3.45)$$

We note here that  $q = q^*$ , since  $q$  and  $q^*$  satisfy the same equation (3.42), due to the fact that  $R_2$ , as defined in Eq. (3.40), is a Hermitian operator and therefore  $r_2 = r_2^*$ . Thus we can rewrite Eq. (3.45) as

$$\ddot{s} + 2\gamma \dot{s} + i\Delta \dot{d} = -2\gamma(\dot{m} + \dot{q} - \dot{r}_1 - \dot{r}_1^*). \quad (3.46)$$

Substituting Eqs. (3.34), (3.42), and (3.44) into Eq. (3.46), we have

$$\begin{aligned} \ddot{s} + 2\gamma \dot{s} + \Delta^2 s = 2\gamma [i\Delta d + i\Delta(r_1^* - r_1) \\ + 2\gamma(m + s + q) - 2\gamma r_2 + \dot{r}_1 + \dot{r}_1^*]. \end{aligned} \quad (3.47)$$

Furthermore, using Eqs. (3.43) and (3.47) to eliminate  $i\Delta d$ , we obtain

$$\begin{aligned} \ddot{s} + 4\gamma \dot{s} + \Delta^2 s = 2\gamma [2\gamma(r_1 + r_1^*) + \dot{r}_1 + \dot{r}_1^* \\ + i\Delta(r_1^* - r_1) - 2\gamma r_2]. \end{aligned} \quad (3.48)$$

As shown in the Appendix,  $r_1 = r_1(0)e^{-2\gamma t}$  and  $r_2 = r_2(0)e^{-2\gamma t}$ . Equation (3.48) then becomes

$$\ddot{s} + 4\gamma \dot{s} + \Delta^2 s = -4\gamma^2 r_2(0)e^{-2\gamma t}. \quad (3.49)$$

To simplify the above expression, we further introduce the dimensionless variables  $\phi = \gamma t$  and  $\lambda^2 = \Delta^2/\gamma^2$ . Equation (3.49) then takes the new form as

$$s''(\phi) + 4s'(\phi) + \lambda^2 s(\phi) = -4r_2(0)e^{-2\phi}, \quad (3.50)$$

where the primes refer to the derivatives with respect to  $\phi$ . The general solution of this equation is immediately given by

$$s(\phi) = K_1 e^{-\xi_+ \phi} + K_2 e^{-\xi_- \phi} + \frac{4}{4 - \lambda^2} r_2(0) e^{-2\phi}, \quad (3.51)$$

where  $\xi_{\pm} = 2 \pm (4 - \lambda^2)^{1/2}$ , except for the special case  $\lambda = 2$  which gives rise to a divergent solution.  $K_1$  and  $K_2$  are determined by the initial conditions  $s(0)$  and  $s'(0)$  and will be given later on for the different cases of  $\lambda$ .

Having obtained the solution for  $s(\phi)$ , we now proceed to solve the other equations. In terms of the new variable  $\phi$ , Eqs. (3.29), (3.34), and (3.38) become, respectively,

$$\Gamma(t) = -\gamma m'(\phi), \quad (3.52)$$

$$m'(\phi) + m(\phi) = -[s(\phi) + q(\phi)], \quad (3.53)$$

and

$$q'(\phi) + q(\phi) = -[m(\phi) + s(\phi) - 2r_2(0)e^{-2\phi}]. \quad (3.54)$$

Our goal is to find out  $m'(\phi)$ . For this purpose, we subtract Eq. (3.54) from Eq. (3.53) and obtain

$$\frac{d}{d\phi} [m(\phi) - q(\phi)] = -2r_2(0)e^{-2\phi}, \quad (3.55)$$

which gives the result

$$q(\phi) = m(\phi) - m(0) + q(0) + (1 - e^{-2\phi})r_2(0). \quad (3.56)$$

Substituting Eq. (3.56) into Eq. (3.53), we have

$$\begin{aligned} m'(\phi) + 2m(\phi) = -[s(\phi) - m(0) + q(0) \\ + (1 - e^{-2\phi})r_2(0)]. \end{aligned} \quad (3.57)$$

Thus, the solution of Eq. (3.57) can be obtained straightforwardly.

The properties of  $s(\phi)$  as expressed in Eq. (3.51) depend on the values of the parameter  $\lambda$ . Let us consider the following cases.

#### 1. $0 \leq \lambda < 2$

Writing  $\xi = (4 - \lambda^2)^{1/2}$ , we first determine  $K_1$  and  $K_2$  from the initial conditions  $s(0)$  and  $s'(0)$ :

$$K_1 = -\frac{1}{2\xi} \left[ \frac{4}{\xi} r_2(0) + (2 - \xi)s(0) + s'(0) \right] \quad (3.58)$$

and

$$K_2 = -\frac{1}{2\xi} \left[ \frac{4}{\xi} r_2(0) - (2 + \xi)s(0) - s'(0) \right], \quad (3.59)$$

where  $s'(0)$  can be obtained directly from Eq. (3.43),

$$s'(0) = -2m(0) - 2q(0) + 4r_1(0) - 2s(0) - i(4 - \xi^2)^{1/2}d(0). \quad (3.60)$$

Thus, Eq. (3.57) can be integrated with the expression (3.51) given for  $\Delta(\phi)$  and Eqs. (3.58)–(3.60). The desired quantity can then be obtained as

$$\begin{aligned} -m'(\phi) = e^{-2\phi} & \left\{ \left[ \left[ \frac{\xi^2 - 4}{\xi^2} \right] m(0) + \left[ \frac{\xi^2 - 4}{\xi^2} \right] q(0) + \frac{8}{\xi^2} r_1(0) - i \frac{2(4 - \xi^2)^{1/2}}{\xi^2} d(0) \right] + \left[ \frac{4}{\xi^2} - 2 \left[ \frac{4 - \xi^2}{\xi^2} \right] \phi \right] r_2(0) \right. \\ & + \left[ \frac{4}{\xi^2} m(0) + \frac{4}{\xi^2} q(0) - \frac{8}{\xi^2} r_1(0) - \frac{4}{\xi^2} r_2(0) + s(0) + i \frac{(4 - \xi^2)^{1/2}}{\xi^2} d(0) \right] \cosh(\xi\phi) \\ & \left. - \left[ \frac{2}{\xi} m(0) + \frac{2}{\xi} q(0) - \frac{4}{\xi} r_1(0) - \frac{8}{\xi^3} r_2(0) + \frac{2}{\xi} s(0) + i \frac{(4 - \xi^2)^{1/2}}{\xi} d(0) \right] \sinh(\xi\phi) \right\}. \quad (3.61) \end{aligned}$$

## 2. $\lambda > 2$

Writing  $\eta = (\lambda^2 - 4)^{1/2}$ , we can use the above results directly with  $\xi$  replaced by  $i\eta$ :

$$\begin{aligned} -m'(\phi) = e^{-2\phi} & \left\{ \left[ \left[ \frac{\eta^2 + 4}{\eta^2} \right] m(0) + \left[ \frac{\eta^2 + 4}{\eta^2} \right] q(0) - \frac{8}{\eta^2} r_1(0) - i \frac{2(4 + \eta^2)^{1/2}}{\eta^2} d(0) \right] + \left[ -\frac{4}{\eta^2} - 2 \left[ \frac{4 - \eta^2}{\eta^2} \right] \phi \right] r_2(0) \right. \\ & + \left[ -\frac{4}{\eta^2} m(0) - \frac{4}{\eta^2} q(0) + \frac{8}{\eta^2} r_1(0) + \frac{4}{\eta^2} r_2(0) + s(0) - i \frac{(4 + \eta^2)^{1/2}}{\eta^2} d(0) \right] \cos(\eta\phi) \\ & \left. - \left[ \frac{2}{\eta} m(0) + \frac{2}{\eta} q(0) - \frac{4}{\eta} r_1(0) + \frac{8}{\eta^3} r_2(0) + \frac{2}{\eta} s(0) - i \frac{(4 + \eta^2)^{1/2}}{\eta} d(0) \right] \sin(\eta\phi) \right\}. \quad (3.62) \end{aligned}$$

## 3. $\lambda = 2$

For this special case, we have to reconsider the solution  $s(\phi)$  for Eq. (3.50), where the characteristic polynomial has a root of double multiplicity and the inhomogeneous term has the same exponential of the homogeneous solution as well. Accordingly, the general solution is sought to be<sup>38</sup>

$$s(\phi) = (K_3 + K_4\phi)e^{-2\phi} - 2r_2(0)\phi^2 e^{-2\phi}, \quad (3.63)$$

where  $K_3$  and  $K_4$  are again to be determined by the initial conditions  $s(0)$  and  $s'(0)$ . Following the same procedures as before, we have

$$\begin{aligned} -m'(\phi) = e^{-2\phi} & \{ m(0) + q(0) + s(0) - 2[m(0) + q(0) + s(0) - 2r_1(0) - r_2(0) + id(0)]\phi \\ & + 2[m(0) + q(0) - 2r_1(0) - r_2(0) + id(0)]\phi^2 + \frac{4}{3}r_2(0)\phi^3 \}. \quad (3.64) \end{aligned}$$

The total radiation rate  $\Gamma(t)$  for the system is, as given by Eq. (3.52), proportional to the quantity  $-m'(\phi)$ , which is seen for every case to be determined by the initial values  $m(0)$ ,  $q(0)$ ,  $r_1(0)$ ,  $r_2(0)$ ,  $s(0)$ , and  $d(0)$ . These quantities, being the expectation values of atomic operators on the initial state of the whole system, depend on the initial states of the system and are considered for a variety of different initial states in the next section.

## IV. NUMERICAL RESULTS AND DISCUSSION

The following initial states for the system in which both atoms have transition dipoles oriented perpendicular to the metal surface are chosen for specific consideration:

$$(a) \quad |\Psi(0)\rangle = |e_1\rangle |e_2\rangle, \quad (4.1)$$

where both atoms are initially in their excited states, and

$$(b) \quad |\Psi(0)\rangle = |e_1\rangle |g_2\rangle, \quad (4.2)$$

where one atom is in its excited state while the other is in its ground state. Without loss of generality, we focus on the case in which the atom with resonance frequency  $\omega_1$  is in its excited state and the one with  $\omega_2$  is in its ground state. The solution will be identical to  $|\Psi(0)\rangle = |g_1\rangle |e_2\rangle$  since, as can be seen from Eqs. (3.61), (3.62), and (3.64), it depends only on  $\Delta^2$ . Next, we consider

$$(c) \quad |\Psi(0)\rangle = \frac{1}{\sqrt{2}}(|e_1\rangle |g_2\rangle + |e_2\rangle |g_1\rangle), \quad (4.3)$$

where only one atom is in its excited state while the whole system is in a symmetric combination. We have

$$(d) \quad |\Psi(0)\rangle = \frac{1}{\sqrt{2}}(|e_1\rangle|g_2\rangle - |e_2\rangle|g_1\rangle), \quad (4.4)$$

where only one atom is in its excited state while the whole system is in an antisymmetric combination. Then, we look at

$$(e) \quad |\Psi(0)\rangle = \left[ \frac{1}{\sqrt{2}}(|e_1\rangle + |g_1\rangle) \right] \times \left[ \frac{1}{\sqrt{2}}(|e_2\rangle + |g_2\rangle) \right], \quad (4.5)$$

where each atom of the system is in an in-phase superposition of states. Next,

$$(f) \quad |\Psi(0)\rangle = \left[ \frac{1}{\sqrt{2}}(|e_1\rangle + |g_1\rangle) \right] \times \left[ \frac{1}{\sqrt{2}}(|e_2\rangle - |g_2\rangle) \right], \quad (4.6)$$

where the atom with frequency  $\omega_1$  is in an in-phase superposition of the excited and ground levels while the one with frequency  $\omega_2$  is in an out-of-phase superposition of levels. For the same reason as in case (b), the solution is identical to the initial state,

$$|\Psi(0)\rangle = \left[ \frac{1}{\sqrt{2}}(|e_1\rangle - |g_1\rangle) \right] \left[ \frac{1}{\sqrt{2}}(|e_2\rangle + |g_2\rangle) \right].$$

Also,

$$(g) \quad |\Psi_\mu(0)\rangle = \frac{1}{\sqrt{2}}(|g_\mu\rangle + e^{i\delta_\mu}|e_\mu\rangle), \quad \mu=1,2 \quad (4.7)$$

where each atom is in an equal and random-phase superposition of levels. The density operator representing this situation is thus given by

$$\begin{aligned} \rho(0) &= \frac{1}{(2\pi)^2} \int_0^{2\pi} d\delta_1 \int_0^{2\pi} d\delta_2 [|\Psi_1(0)\rangle\langle\Psi_1(0)|] \otimes [|\Psi_2(0)\rangle\langle\Psi_2(0)|] \\ &= \left[ \frac{1}{2}|g_1\rangle\langle g_1| + \frac{1}{2}|e_1\rangle\langle e_1| \right] \otimes \left[ \frac{1}{2}|g_2\rangle\langle g_2| + \frac{1}{2}|e_2\rangle\langle e_2| \right]. \end{aligned} \quad (4.8)$$

In this case, the expectation value of an operator  $O$  is interpreted as

$$\langle O \rangle = \text{Tr}[\rho(0)O]. \quad (4.9)$$

We list in Table I all the expectation values necessary for evaluating the emission rate,  $\Gamma(t)$ . The normalized emission rate,  $\Gamma(t)/\gamma$ , for the different initial states of Eqs. (4.1)–(4.7), is plotted in Figs. 2–8 with respect to the normalized time,  $\phi = \gamma t$ , and the parameter characterizing the frequency difference,  $\lambda = \Delta/\gamma$ . The special case of two identical atoms is represented by the curves with  $\lambda = 0$ . These results can be roughly divided into two regions of different behavior for the time evolution of the emission rate. For  $\Delta < 2\gamma$  the emission rate exponentially decays towards zero, while for  $\Delta > 2\gamma$ , the rate shows oscillations before decaying to zero. We see that the dividing frequency occurs at  $\Delta = 2\gamma$ . The physical explanation

can be given as follows. For an excited atom with its transition dipole perpendicular to the metal surface, the half-width is doubly broadened as in the case of Dicke's superradiance,<sup>22</sup> since it is described by the state  $|e\rangle_+$  of Eq. (2.4) as a result of phase coherence between the atom and its image. Therefore, if  $\Delta < 2\gamma$ , the two atoms are hardly distinguishable from each other, and they undergo radiative emission as if there were two identical atoms. Only when  $\Delta > 2\gamma$  does the difference of the two resonance frequencies become pronounced, and beating takes place as manifested in the oscillation of  $\Gamma(t)$ .

Let us now examine closely the initial values of the emission rate,  $\Gamma(0)$ , which are used as a measure of the radiative intensity in the usual sense, in order to gain a better understanding of the problem. At  $t=0$ ,  $\Gamma(0)$  can be immediately written from Eqs. (3.29)–(3.34) in the form

TABLE I. Initial expectation values for  $m(0)$ ,  $q(0)$ ,  $r_1(0)$ ,  $r_2(0)$ ,  $s(0)$ , and  $d(0)$  for the initial states given in Eqs. (4.1)–(4.7).

$ \Psi(0)\rangle$	$m(0)$	$q(0)$	$r_1(0)$	$r_2(0)$	$s(0)$	$d(0)$
(a)	2	2	1	0	0	0
(b)	1	1	0	0	0	0
(c)	1	1	0	0	2	0
(d)	1	1	0	0	-2	0
(e)	1	1	$\frac{1}{4}$	0	1	0
(f)	1	1	$\frac{1}{4}$	0	-1	0
(g)	1	1	$\frac{1}{4}$	0	0	0



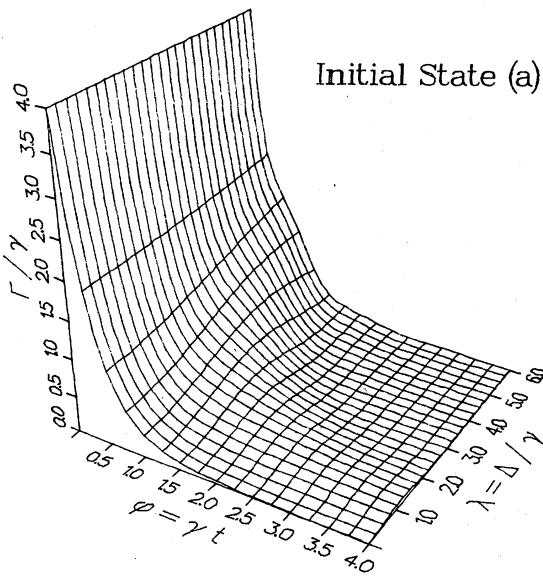


FIG. 2. Normalized emission rate for the initial state in Eq. (4.1).

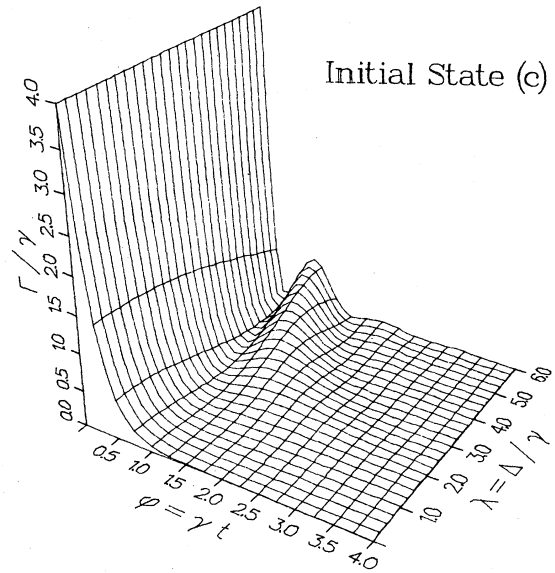


FIG. 4. Normalized emission rate for the initial state in Eq. (4.3).

$$\begin{aligned} \Gamma(0) &= \gamma \sum_{\mu} \langle c_{A\mu}^{\dagger}(0)c_{A\mu}(0) + c_{I\mu}^{\dagger}(0)c_{I\mu}(0) \rangle + \gamma \sum_{\mu \neq \nu} \langle [c_{A\mu}^{\dagger}(0) + c_{I\mu}^{\dagger}(0)][c_{A\nu}(0) + c_{I\nu}(0)] \rangle \\ &\quad + \gamma \langle c_{A1}^{\dagger}(0)c_{I1}(0) + c_{A2}^{\dagger}(0)c_{I2}(0) + c_{I1}^{\dagger}(0)c_{A1}(0) + c_{I2}^{\dagger}(0)c_{A2}(0) \rangle \\ &= \gamma [m(0) + q(0) + s(0)]. \end{aligned} \tag{4.10}$$

Clearly this equation contains only the initial excitation stored in the atomic system described by  $m(0)$ , and the induced dipole-dipole interaction among the atoms and images due to the energy fluctuation of the vacuum as specified by  $q(0)$  and  $s(0)$ . Therefore  $q(0)$  and  $s(0)$  to-

gether signify a certain degree of coherence in cooperative emission. It is, however, important to distinguish the difference of physical natures embedded in the correlations expressed by  $q(0)$  and  $s(0)$ .  $q(0)$ , as seen through the definition of its corresponding operator  $Q$  given in Eq.

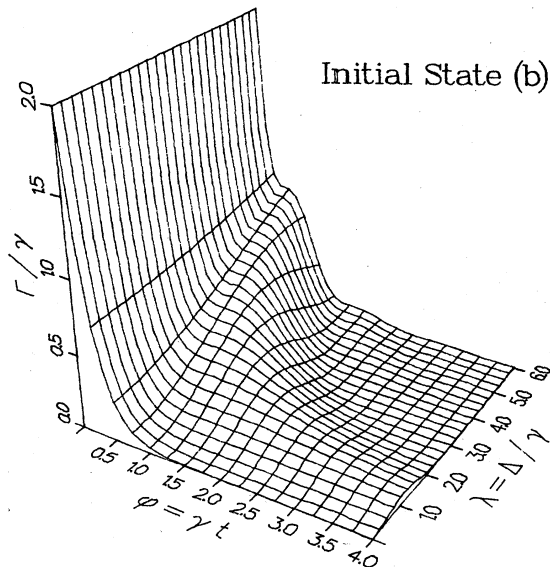


FIG. 3. Normalized emission rate for the initial state in Eq. (4.2).

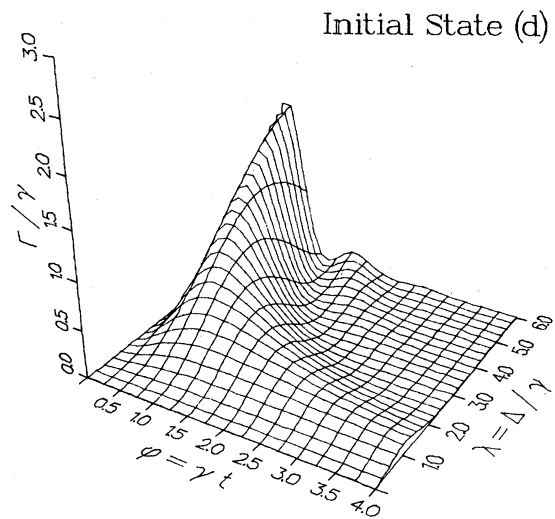


FIG. 5. Normalized emission rate for the initial state in Eq. (4.4).

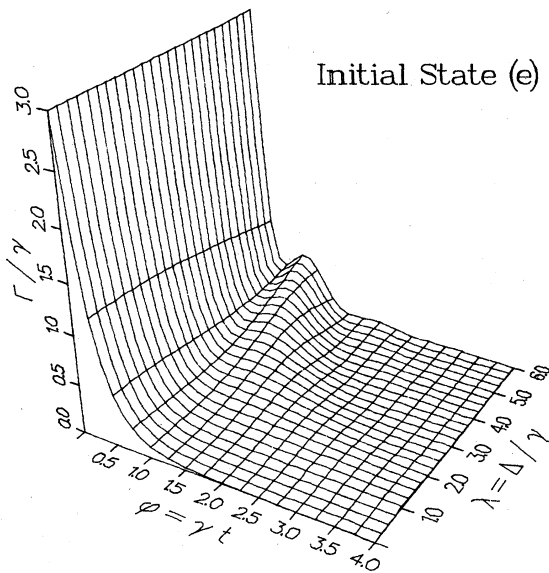


FIG. 6. Normalized emission rate for the initial state in Eq. (4.5).

(3.33), represents the correlation between an individual atom and its own image. Therefore,  $q(0)$  is solely due to the existence of the metal surface. In contrast,  $s(0)$  is attributed completely to the correlation brought about by the radiation fields. As time evolves, correlations of higher orders are literally needed to determine the emission rate. As we have seen in Sec. III C, the higher correlations are manifested in the appearance of operators such as  $R_1$  and  $R_2$ , in which more atomic operators are clustered together.

From Table I, we see that only the initial state (a) has two units of excitation stored in the atomic system, i.e.,  $m(0)=2$ , since this state has both atoms in their excited

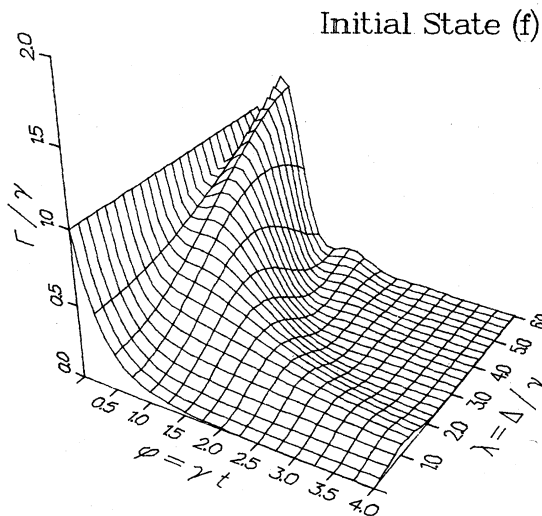


FIG. 7. Normalized emission rate for the initial state in Eq. (4.6).

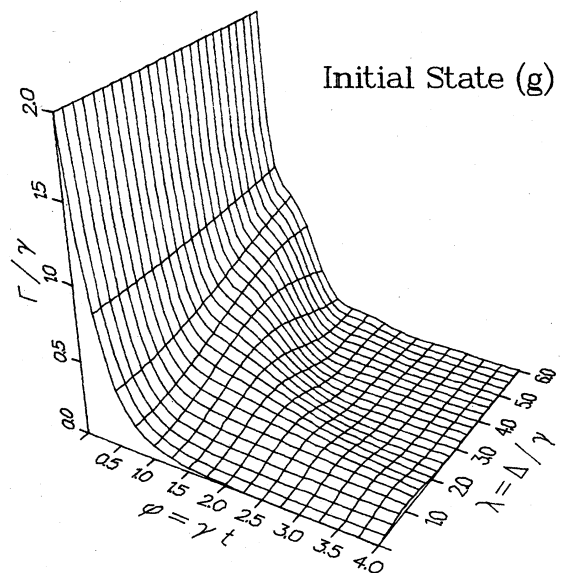


FIG. 8. Normalized emission rate for the initial state in Eq. (4.7).

states, while in each of the other cases, (b)–(g), only one atom is in effect in an excited state. Similarly, since the transition dipole can induce an image dipole, only the initial state (a) which contains two excited atoms has  $q(0)=2$ , but in cases (b)–(g) each has the value  $q(0)=1$ . We see that the enhancement in state (a) is solely due to the existence of the metal surface, because  $s(0)=0$  as shown in Table I. For the initial state (b), in which one atom is excited while the other one is in its ground state, there is an enhancement in the emission rate due to the existence of the metal surface as well, but with a reduced factor due to the fact that only the excited atom can induce an image to participate in the radiative process.

For the initial state (c), in which the two atoms are in an in-phase combination resembling the Dicke's superradiative state,<sup>22</sup> we have the highest enhancement factor in  $s(0)$ , and combined effect of  $q(0)$  and  $s(0)$  results in an initial emission rate  $\Gamma(0)$  as high as in the case of the initial state (a). In contrast, the initial state (d), in which the two atoms are in an out-of-phase combination, gives  $\Gamma(0)=0$ , as seen by summing  $m(0)$ ,  $q(0)$ , and  $s(0)$  in Table I. This state has the same feature of a photon-trapping state for the case of two atoms in free space.<sup>39</sup> For the initial state (e), each atom is itself in an in-phase combination of excited and ground atomic levels. On the average, each atom contributes a half unit of excitation; therefore there is one total unit of excitation,  $m(0)=1$ , and thus  $q(0)=0$ . The enhancement is not as strong as in case (c), owing to the fact that the coherent combination is not for the system but for the individual atoms only, giving  $s(0)=1$ . By the same token, the trapping effect of a photon is small as well, so that  $s(0)=-1$  for the initial state (f), in which one atom is in an in-phase and the other is in out-of-phase combination of the excited and ground atomic levels.

For the last case (g), each atom is in a random-phase combination of the excited and the ground atomic levels.

The excitation stored initially in the atomic system is the same as in cases (e) and (f), since the radiation energy has nothing to do with phase relation in the atomic states. However, there is no coherence resulting from the interaction with the radiation because of a null effect of random phases. In summary, in comparison with the corresponding cases of two different atoms in free space,<sup>32</sup> the emission rates of such an atomic pair with dipole transitions perpendicular to a metal surface are doubled in magnitude because of the induced images participating in phase with the emission from the source atoms, giving rise to a "superradiative" enhancement in the emission rate. This effect is completely due to the presence of the metal surface.

As a final remark, we note that the method of solution

presented in this paper is based on the assumption that the interaction of electromagnetic radiation and the atoms is confined to a frequency  $\omega_0$  between  $\omega_1$  and  $\omega_2$ , with a range roughly equal to  $\delta\omega$ . Therefore, the method is valid only under the condition  $\Delta \lesssim \delta\omega$ .

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#### APPENDIX

To see the time dependence of  $r_1(t) = \langle R_1(t) \rangle$  and  $r_2(t) = \langle R_2(t) \rangle$ , one has to examine the equations of motion for  $R_1(t)$  and  $R_2(t)$ . A typical term in  $R_1$  and  $R_2$ , as given by Eqs. (3.39) and (3.40), is  $c_i^\dagger c_i c_j^\dagger c_j$  with  $i \neq j$ , where  $i$  and  $j$  represent the double indices  $A\mu$  or  $I\mu$ . The equation of motion is

$$i\hbar \frac{d}{dt} (c_i^\dagger c_i c_j^\dagger c_j) = [c_i^\dagger c_i c_j^\dagger c_j, H_{\text{eff}}] \\ = -\hbar g \sum_{\alpha} \sum_{\beta} a_{\alpha\beta} [c_i^\dagger c_i, c_i] c_j^\dagger c_j \\ - \hbar g \sum_{\alpha} \sum_{\beta} [c_i^\dagger c_i, c_i^\dagger] a_{\alpha\beta} c_j^\dagger c_j - \hbar g \sum_{\alpha} \sum_{\beta} a_{\alpha\beta}^\dagger c_i^\dagger [c_j^\dagger c_j, c_j] - \hbar g \sum_{\alpha} \sum_{\beta} c_i^\dagger c_i [c_j^\dagger c_j, c_j^\dagger] a_{\alpha\beta}. \quad (\text{A1})$$

Making use of the anticommutation and commutation relations (2.1) and (2.2), we have

$$[c_i^\dagger c_i, c_i] = c_i^\dagger [c_i, c_i] + [c_i^\dagger, c_i] c_i = -c_i \quad (\text{A2})$$

and

$$[c_i^\dagger c_i, c_i^\dagger] = c_i^\dagger [c_i, c_i^\dagger] + [c_i^\dagger, c_i^\dagger] c_i = c_i^\dagger, \quad (\text{A3})$$

so that Eq. (A1) becomes

$$i\hbar \frac{d}{dt} (c_i^\dagger c_i c_j^\dagger c_j) = \hbar g \sum_{\alpha} \sum_{\beta} a_{\alpha\beta}^\dagger (c_i c_j^\dagger c_j + c_i^\dagger c_i c_j) \\ - \hbar g \sum_{\alpha} \sum_{\beta} (c_i^\dagger c_j^\dagger c_j + c_i^\dagger c_i c_j^\dagger) a_{\alpha\beta}. \quad (\text{A4})$$

Next, we substitute Eq. (3.16) and its Hermitian conjugate for  $a_{\alpha\beta}$  and  $a_{\alpha\beta}^\dagger$  in the above equation and take the expectation value to obtain

$$i\hbar \frac{d}{dt} \langle c_i^\dagger c_i c_j^\dagger c_j \rangle = -4i\pi\hbar g^2 \rho(\omega_0) \langle c_i^\dagger c_i c_j^\dagger c_j \rangle \quad (\text{A5})$$

or

$$\frac{d}{dt} \langle c_i^\dagger c_i c_j^\dagger c_j \rangle = -4\pi g^2 \rho(\omega_0) \langle c_i^\dagger c_i c_j^\dagger c_j \rangle \\ = -2\gamma \langle c_i^\dagger c_i c_j^\dagger c_j \rangle. \quad (\text{A6})$$

Thus, we conclude that

$$\dot{r}_1 = -2\gamma r_1 \quad (\text{A7})$$

and

$$\dot{r}_2 = -2\gamma r_2. \quad (\text{A8})$$

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