## Comments

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# Comment on "Electron mobility in modulation-doped heterostructures"

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The acoustic-mode-scattering mobility, as a component of the low-temperature Ohmic mobility, for electrons in a high-mobility GaAs single-interface heterolayer, is discussed. The numerical results in some recent publications, theoretical and experimental, are critically compared. The usefulness of the empirical values obtained from the linear relation between the reciprocal of the mobility and the temperature is pointed out. The calculation of the optical-mode-scattering component is briefly discussed.

A recent paper by Walukiewicz, Ruda, Lagowski, and Gatos[1] presents computed Ohmic mobilities $\mu$ versus temperature $T$ for electrons in a single-interface GaAs heterolayer. The present Comment discusses the contribution to $1/\mu$ from the scattering by acoustic-mode phonons, as given in that paper, in comparison with other recent experimental and theoretical values.[2-5] This contribution, $1/\mu_{ac}$, dominates the approach of $\mu(T)$ to the elastic-scattering limit $\mu(0)$, for high-mobility heterostructures. Because of the degeneracy of the heterolayer electrons at low temperatures, $\mu_{ac}(T)$ is normally proportional to $1/T$. This corresponds to the experimental finding[2] that, for high mobilities, $1/\mu(T)$ increases linearly with $T$, and the slope $\alpha \equiv d(1/\mu)/dT$ is a well-defined function of the electron sheet density $n$, so that $\alpha$ can be identified with $1/(T\mu_{ac})$. It was found[2] that this experimental $\alpha(n)$ agrees with the theoretical values that are calculated[3] on allowing for the screening of the electron-phonon interaction by the heterolayer electrons and taking the electron deformation potential (to be denoted here, as in Ref. 1, by $D$) to be 13–14 eV, about twice the 7 eV that is usually assumed.[6] Independently, Pfeffer, Gorczya, and Zawadzki[7] have obtained $D=15.7$ eV from analysis of free-carrier absorption in bulk GaAs. A somewhat larger value, 17.5 eV, can be inferred from calculations by Vergés, Glötzel, Cardona, and Andersen.[8] Analysis of the temperature dependence of mobility in $n$-type GaAs, such as had originally led to the 7 eV, more recently[9] has indicated $D=16.0\pm0.5$ eV. Thus there is independent support for an electron deformation potential comparable to or somewhat larger than the 13–14 eV arrived at in Ref. 2, and perhaps no good basis now for $D=7$ eV.

In Ref. 1, as in Ref. 3, the Fang-Howard-Stern model wave function[10] is used for the heterolayer electron states. The two calculations differ, however, in that the acoustic-mode scattering is taken to be *unscreened* in Ref. 1. In Fig. 1 here, the solid circles (●) are computed values in Ref. 1. They are taken from the straight dot-dash lines showing $\mu_{ac}$ in Figs. 2 and 3 there, which are log-log plots of mobilities against $T$. (The + and ○ points of Fig. 1 here were obtained in the same way from Refs. 4 and 5.) Therefore, they would be expected to be close to the upper curve. Instead, the point for $n=3\times10^{11}$ cm$^{-2}$ lies near the lower curve. A difference in the values of $\mu_{piez}$ (which does not appear to have been calculated in the same way as in Ref. 3) might account for this disparity, but at the same time would significantly lower the $2.2\times10^{11}$ cm$^{-2}$ point.[11]

The two curves of Fig. 1 are of computed values, from Ref. 3.[6] In addition, the lower curve agrees quite well with the $\alpha$ values obtained from the experimental data reported in Ref. 2 and also with the $\alpha$ values obtained there from the other cited experimental data [all shown plotted in Fig. 3(a) of Ref. 2]. In this respect, it is an empirical curve. To the extent that the five points in the present Fig. 1 could be

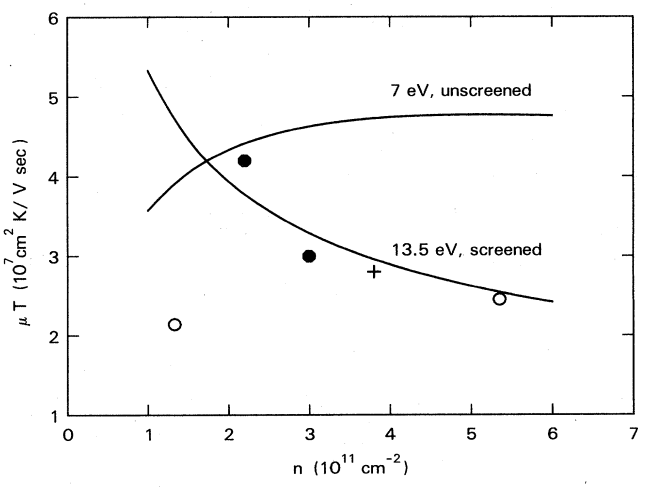


FIG. 1. Acoustic-mode mobilities times temperature, vs carrier sheet density. The curves are calculated, from Refs. 2 and 3. The ● points are from Ref. 1, the + from Ref. 4, and the ○ from Ref. 5.

 

taken as *experimental,* representing a fitting of the measured $\mu(T)$ to the various scattering components, four of them similarly agree satisfactorily with this "13.5 eV, screened" curve. The exception, the lower of the two points from Ref. 5, belongs to the low-mobility category of heterolayer [with $\mu(0) < 10^5$ cm$^2$/V sec], which has a different type of $\mu(T)$ curve,[12] as Fig. 2 of Ref. 5 shows. It would evidently be appropriate to make plots of $1/\mu$ against $T$ for all such high-mobility heterolayers, and consider the resulting $1/(T\alpha)$ as the $\mu_{ac}$ component in $\mu(T)$. One then has, as in Fig. 3(a) of Ref. 2, an empirically based upper limit for $\mu$ at low temperatures (above the Bloch-Grüneisen range[13]).

The + point and the ○ points of Fig. 1 are from the log-log plots of $\mu_{ac}$ against $T$ in Refs. 4 and 5, respectively. In Ref. 4, it appears that a formula for $\mu_{def}$ which is $\frac{1}{2}$ times that of Ref. 3 has been used with $D=7$ eV, while the formula for $\mu_{piez}$ and the value of the piezoelectric constant in it ($h_{14}$) are the same as in Ref. 3; and screening was not included in calculating these components of $\mu_{ac}$. In that case, the numerical value of $T\mu_{ac}$ to be expected is $T\mu_4 \equiv T\mu_{def}\mu_{piez}/(\mu_{def}+2\mu_{piez})$, where $\mu_{def}$ and $\mu_{piez}$ are the calculated values shown by the "def" and "piez" points, respectively, of Fig. 2 in Ref. 3. This quantity $T\mu_4$ varies slowly with the sheet density, being equal to $2.8\times10^7$ at $n=1\times10^{11}$ and at $n=6\times10^{11}$ and having a flat maximum of $3.0\times10^7$ near $n=2.5\times10^{11}$. Thus, on this basis (with the factor $\frac{1}{2}$ in $\mu_{def}$), the + point from Ref. 4 is consistent computationally with the calculated mobilities in Ref. 3 for the unscreened case. It also coincidentally is close to the "13.5 eV, screened" curve. This value of $1/(T\mu_{ac})$ is, as noted, compatible with the experimental curve of $\alpha(n)$ from other heterolayer data. In Ref. 5, it is stated that the formulas used there for the acoustic-mode mobilities are the same as in Ref. 4. We find, however, that the computed value of $T\mu_4$ is substantially higher than the ○ point for $n=1.33\times10^{11}$, and somewhat higher for $n=5.35\times10^{11}$. As noted above, the first of these two points represents a different category of heterolayer with a differing $\mu(T)$ dependence.

Polar optical-mode phonon scattering is included in the calculations of $\mu$ in Refs. 1, 4, and 5, but incorrectly in various respects.

(a) Although the mobility given by optical-mode scattering alone, $\mu_{op}$, numerically does not differ greatly from the ordinary "three-dimensional" equivalent, this has no fundamental basis. One should use the "two-dimensional" $\mu_{op}$, which will vary with heterolayer thickness or interface field, not the three-dimensional value.

(b) For two-dimensional heterolayer transport, $\mu_{op}$ must be obtained from a solution of the appropriate Boltzmann equation or the equivalent. It is not given, in particular, by setting the direct scattering rate weighted by a $(1-\cos\theta)$ factor equal to $1/\tau$ and then using $\tau$ in the textbook formula for mobility.

(c) Degeneracy must be properly taken into account in the linearized scattering-rate function used to calculate $\mu_{op}$.

(d) Matthiessen's rule does not apply to the combination of $\mu_{ac}$ and $\mu_{op}$ (because the polar optical-mode scattering is both anisotropic and inelastic). At optical-mode phonon frequencies, the screening effect can be expected to be small (and even, if anything, to enhance rather than reduce the scattering rate), and accordingly it is reasonable to disregard screening for the optical-mode scattering.

*Note added in proof.* An extensive experimental study [B. F. J. Lin, D. C. Tsui, and G. Weimann (unpublished)] has confirmed that $D=13.5$ eV together with screened scattering accounts for the behavior of the mobility versus temperature in a range of high-mobility samples.


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[6]The results shown in Figs. 1 and 2 of Ref. 3 are for $D=7$ eV. Values of $T\mu_{ac}$ (the "ac" curves of those figures) are easily derived for other values for both the screened (Fig. 1) and unscreened (Fig. 2) cases, since $\mu_{def}$ is inversely proportional to $D^2$ and since Matthiessen's rule applies to the combination of $\mu_{def}$ and $\mu_{piez}$. Reference 3 was written (except for the "Note added") before the more appropriate value of $D$ was arrived at as described in Ref. 2.

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[11]Figure 4 of Ref. 1 shows a $\mu_{def}$ comparable to the 7 eV unscreened $\mu_{def}$ of Ref. 3, but a $\mu_{piez}$ much lower than the unscreened $\mu_{piez}$ of Ref. 3 (at 5 K). These values from Fig. 4 of Ref. 1, however, when combined by Matthiessen's rule, give a $\mu_{ac}$ much less than that shown as "deformation potential + piezoelectric" in Fig. 3 of Ref. 1, and displayed in the present Fig. 1, for $n=2.2\times10^{11}$ cm$^{-2}$.

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