Low-temperature Peltier heat of an itinerant electron in a ferromagnetic semiconductor

Nai-Li H. Liu
Department of Physics, University of California at Riverside, Riverside, California 92521

David Emin
Sandia National Laboratories, Albuquerque, New Mexico 87185
(Received 21 January 1985)

The Peltier heat of a wide-band itinerant carrier in a ferromagnetic semiconductor has been calculated for temperatures below the Curie temperature. In this regime we treat the spin fluctuations within the spin-wave approximation. The coupling of the charge carrier to the local moments is via local intra-atomic (e.g., *s-f* or *s-d*) exchange. Taking the strength of the intra-atomic exchange interaction to be small compared with the carrier's electronic bandwidth, we treat the interaction between the carrier and the local moments perturbatively through second order. We use the perturbed energy to compute the free energy of the coupled electron-magnon system. From the carrier-induced change of the system's free energy we directly obtain the carrier's Peltier heat. The Peltier heat contains two terms of opposite sign which both increase in magnitude with increasing temperature. These two terms arise from the first- and second-order contributions to the energy of the coupled system. Except at very low temperatures, the first-order contribution dominates. Then the electron-magnon interaction provides a negative contribution to the Peltier heat of a ferromagnetic semiconductor. The magnitude of this contribution varies as $T^{3/2}$.

I. INTRODUCTION

The Peltier heat is the heat transported with a charge carrier during isothermal current flow.[1] The interaction of the electronic charge carrier with its environment generally significantly influences its Peltier heat. In a nonmagnetic semiconductor, contributions to the Peltier heat arise from the interactions of the charge carrier with the atomic vibrations.[2,3] In a magnetic semiconductor the distinctive interaction is that of the electronic carrier and the localized magnetic moments. We have recently calculated the magnetic contribution to the Peltier heat of a small polaron in a magnetic semiconductor.[4] The exchange interaction between the carrier and the local moments is found to significantly affect the carrier's Peltier heat. Here we address the low-temperature Peltier heat of an *itinerant* carrier in a ferromagnetic semiconductor.

In modeling the exchange interactions of a ferromagnetic semiconductor we restrict ourselves to low temperatures where the spin deviations of the host material are describable within the spin-wave approach. The interaction between the carrier and the local magnetic moments of the ferromagnetic semiconductor is via local (*s-f* or *s-d*) exchange. We presume, as is typically the case, that the intra-atomic exchange energy is very much less than the itinerant charge carrier's bandwidth. We then treat the intra-atomic exchange perturbatively in computing the system's energy levels. A similar procedure has been used in calculating the effect of the intra-atomic exchange on the resistivity,[5] optical properties,[6,7] and specific heat[8] of a magnetic semiconductor. Here we use these perturbed energy levels to determine the change of the free energy of the system upon introduction of a charge carrier. With this we readily obtain the low-temperature Peltier heat of a magnetic semiconductor.

II. FORMALISM

The Peltier heat generally consists of two contributions. The first is the heat associated with the placing of a carrier in a material. The second is the net energy flow associated with the process of moving the carrier. This latter contribution depends upon the details of the transport mechanism. For example, this term is associated with phonon drag in very pure semiconductors at low temperatures where the phonon lifetimes are long. However, in most commonly studied situations it is the first contribution that dominates. This term is calculated from equilibrium thermodynamics. Here we study the effect of the magnetic interactions on this thermodynamic contribution.

The thermodynamic portion of the Peltier heat associated with a carrier in state $\mathbf{k}$, $\Pi_{\mathbf{k}}$, is the product of the temperature T and the change of the entropy of the system upon adding a charge carrier to the kth state, $\Delta S_{\mathbf{k}}$. Specifically, we write

$$\Pi_{\mathbf{k}} = T\Delta S_{\mathbf{k}} = -T\frac{\partial}{\partial T}(F^{\mathbf{k}}_{n+1} - F_n) , \tag{1}$$

where F_n is the Helmholtz free energy of a system with n carriers and $F^{\mathbf{k}}_{n+1}$ is the Helmholtz free energy when a carrier is added into the kth electronic state. When the carrier density is sufficiently low, the carriers may be treated as noninteracting. Then the Peltier heat associated with a carrier introduced into the kth electronic state is given by[3,4]

$$\begin{aligned}\Pi_{\mathbf{k}} &= F^{\mathbf{k}}_1 - F_0 - T\frac{\partial}{\partial T}(F^{\mathbf{k}}_1 - F_0) - \mu \\ &= E^{\mathbf{k}}_Q - \mu ,\end{aligned} \tag{2}$$

 ©1985 The American Physical Society

where

$$E_Q^{\mathbf{k}} \equiv F_1^{\mathbf{k}} - F_0 - T\frac{\partial}{\partial T}(F_1^{\mathbf{k}} - F_0) . \tag{3}$$

Here $E_Q^{\mathbf{k}}$ is the change of the internal energy of the system upon introduction of a single charge carrier into the kth state (the quasiparticle energy) and μ is the chemical potential. For quasifree carriers the quasiparticle energy $E_Q^{\mathbf{k}}$ associated with the kth state simply reduces to the state's electronic energy, $E_{\mathbf{k}}$. The Peltier heat associated with the kth state is then given by the familiar expression $\Pi_{\mathbf{k}} = E_{\mathbf{k}} - \mu$. More generally, with an electron-lattice or electron-magnon interaction, $E_Q^{\mathbf{k}}$, the quasiparticle energy for the kth electronic state, depends upon these interactions. In this paper we calculate the effect of the interaction of an itinerant carrier with the local spins of a ferromagnetic semiconductor on the system's low-temperature Peltier heat.

The basic formalism follows that used to calculate the Peltier heat of an itinerant carrier in a (nonmagnetic) semiconductor in which the carrier interacts with the phonons.[3] Not surprisingly, we obtain several analogous results. Namely, the presence of an itinerant charge carrier in a coupled electron-lattice system produces infinitesimal shifts of the vibrational frequencies. Here, in a magnetic semiconductor at low temperatures, the presence of an itinerant carrier produces infinitesimal $(1/N)$ shifts of each of the N magnon frequencies. As a result of this electron-magnon interaction the *average* electronic energy becomes temperature dependent. Concomitantly, the presence of an electronic carrier produces a (generally) temperature-dependent change of the average energy of the magnon system. The net change of the energy of the system upon introducing an itinerant charge carrier, the quasiparticle energy, involves some cancellation of temperature-dependent contributions from the electronic and magnon energies. Nonetheless, the quasiparticle energy is typically temperature-dependent.

Although the present formalism is similar to that used to study an itinerant electron interaction with lattice vibrations,[3] there are significant differences. First, the dispersion relations of phonons and magnons differ from one another. This affects the temperature dependence of the quasiparticle energy at low temperatures. Second, despite analogous second-order contributions to the energy of the two coupled systems, the electron-magnon system contains a first-order contribution while the electron-lattice system does not. Prior treatments of the energy of the system comprising an itinerant electron coupled to magnons have ignored the first-order contribution.[8] Nonetheless, both the first- and second-order terms of the energy of the electron-magnon system generally contribute to the temperature dependence of quasiparticle energy. Indeed, we find that, except in the *limit* of very low temperatures, it is the first-order contribution that is dominant.

III. ENERGY OF THE ELECTRON-MAGNON SYSTEM

Here we consider the energy of the system comprising a single excess itinerant electron added to a ferromagnetic semiconductor. We address the low-temperature situation in which the spin deviations can be treated within the spin-wave approximation. In the absence of coupling between the electron and the magnetic moments of the magnetic semiconductor, the carrier is described as being free. That is, it has the wave function $|\mathbf{k}\rangle = V^{-1/2}\exp(i\mathbf{k}\cdot\mathbf{r})$, where V is the volume of the crystal. Its energy is denoted by $E_{\mathbf{k}}^0$. The spin state of the magnetic semiconductor is described by a set of spin-wave occupation numbers, the n_q's, where q is a magnon wave vector. The corresponding magnetic energy is represented by E_M^0. The state of the carrier and the spin-wave system in the absence of interaction is represented by $|k,\pm;\ldots,n_q,\ldots\rangle$. The corresponding energy is

$$E_{\mathbf{k}}^{(0)} + E_M^{(0)} = \hbar^2 k^2/2m + \sum_{\mathbf{q}} n_{\mathbf{q}}\hbar\omega_{\mathbf{q}} , \tag{4}$$

where $\omega_{\mathbf{q}}$ and $n_{\mathbf{q}}$ are, respectively, the frequency and occupation number of a magnon of wave vector $\mathbf{q}$.

The interaction between a charge carrier of spin s and the local spins of the host lattice is written as[9]

$$H_{\text{ex}} = N^{-1}\sum_{\mathbf{g}} I(\mathbf{r}-\mathbf{g})\mathbf{s}\cdot\mathbf{S}_{\mathbf{g}} , \tag{5}$$

where $\mathbf{S}_{\mathbf{g}}$ is spin at $\mathbf{g}$, the position vector for one of the N spin sites of the solid. This interaction between the carrier and spin deviations is now treated as a perturbation.

To first order the perturbed energy is simply the expectation value of the perturbation in a noninteracting eigenstate of the system:

$$E_{\mathbf{k},+}^{(1)} = \left\langle \mathbf{k},\pm;\ldots,n_{\mathbf{q}},\ldots \left| N^{-1}\sum_{\mathbf{g}} I(\mathbf{r}-\mathbf{g}) s^z S_{\mathbf{g}}^z \right| \mathbf{k},\pm;\ldots,n_{\mathbf{q}},\ldots \right\rangle . \tag{6}$$

Here the $+$ and $-$ signs refer to the alignment of the carrier spin relative to the direction of the saturation magnetization, the z direction. We note that only the z component of the interaction enters in first order. This is because the x and y components of the interaction are associated with spin flips. These do not survive in the diagonal matrix element. In terms of the magnon creation and annihilation operators, $b_{\mathbf{q}}^\dagger$ and $b_{\mathbf{q}}$, the z component of the host spin at site $\mathbf{g}$ can be expressed as

$$S_{\mathbf{g}}^z = S - N^{-1}\sum_{\mathbf{q},\mathbf{q}'} e^{i(\mathbf{q}-\mathbf{q}')\cdot\mathbf{g}} b_{\mathbf{q}}^\dagger b_{\mathbf{q}'} , \tag{7}$$

where S is the magnitude of a host spin. Straightforward evaluation of the matrix element of Eq. (6) yields

$$E_{\mathbf{k},\pm}^{(1)} = \pm I(0)\left[S - \sum_{\mathbf{q}} n_{\mathbf{q}}/N\right]\Big/2 , \tag{8}$$

where $I(0)$ is the zero-momentum component of the

Fourier transform of the local-spin-carrier interaction, $I(\mathbf{r}-\mathbf{g})$.

Below the Curie temperature, in the ferromagnetic regime, the interaction of the carrier with the local moments lifts the spin degeneracy of the carrier's state. For simplicity, we shall henceforth ignore the state of energetically unfavorable alignment. This is because the magnitude of the energy splitting between the aligned and antialigned states (of the order of 0.1 eV) is typically much greater than the thermal energy at the low temperatures which are of interest to us. We observe from Eq. (8) that the presence of a charge carrier induces a shift of the magnon energies. Namely, each magnon energy is lowered by $I(0)/2N$. This, due to the lowering's proportionality to $1/N$, produces an infinitesimal shift for each of the N magnon modes. Nonetheless, the change of magnon energy for the totality of the N magnon modes is finite. It should be noted that the presence of this first-order contribution to the perturbed energy is a feature of the electron-magnon interaction which is absent for the electron-phonon interaction. In particular, the electron-phonon interaction is nondiagonal in the phonon occupation numbers, while the electron-magnon interaction contains the diagonal component displayed in Eq. (8).

We now consider the second-order contribution to the energy of the system comprising a carrier that has an energetically favorable alignment with the net moment of the host spins. This requires us to compute the square of the matrix element connecting the favorably aligned carrier state with the unfavorably aligned carrier state. This is straightforwardly found to be

$$|\langle \mathbf{k}',-;\ldots,n_{\mathbf{q}}-1,\ldots|H_{\mathrm{ex}}|\mathbf{k},+;\ldots,n_{\mathbf{q}},\ldots\rangle|^2=(S/2N)\sum_{\mathbf{q}}n_{\mathbf{q}}I(\mathbf{k}-\mathbf{k}')^2\sum_{\mathbf{G}}\delta_{\mathbf{G},\mathbf{k}-\mathbf{k}'+\mathbf{q}}\ , \tag{9}$$

where $\mathbf{G}$ is the reciprocal-lattice vector and

$$I(\mathbf{k}-\mathbf{k}')\equiv V^{-1}\int d\mathbf{r}\,e^{i(\mathbf{k}-\mathbf{k}')\cdot(\mathbf{r}-\mathbf{g})}I(\mathbf{r}-\mathbf{g})\ . \tag{10}$$

At this point, for simplicity, we specify a contact interaction between the carrier and the spins of the host. Namely, we take $I(\mathbf{r}-\mathbf{g})$ to have the form $IV\delta(\mathbf{r}-\mathbf{g})$, where I is the interaction constant. With this simplification, $I(0)$ and $I(\mathbf{k}-\mathbf{k}')$ both equal I. As an additional simplification we consider only normal processes, ignoring umklapp processes, i.e., we take $\mathbf{G}=\mathbf{0}$ in Eq. (9). The second-order correction to the energy of the system when the carrier spin is aligned parallel to the material's magnetization direction is then given by

$$E^{(2)}_{\mathbf{k},+}=(SI^2/2N)\sum_{\mathbf{q}}n_{\mathbf{q}}[-(\hbar^2/2m)(2\mathbf{k}\cdot\mathbf{q}+q^2)+\hbar\omega_{\mathbf{q}}]^{-1}\ . \tag{11}$$

Our second-order contribution to the energy of the system is essentially identical to that obtained previously with Green's-function techniques by Woolsey and White.[8] Here, in Eq. (11), as in Eq. (8), the energy shift is proportional to a sum of terms which are each proportional to n_q. As a result, the second-order contribution to the energy also produces a shift of the magnon frequencies. However, the second-order term produces a shift which is dependent on magnon wave vector.

Finally, we combine Eqs. (6), (8), and (11) and write the energy of the coupled system comprising the charge carrier and the magnetic semiconductor's local moments. We have, through second-order perturbation theory, that

$$E_{\mathbf{k}}+E_M=\hbar^2k^2/2m+IS/2+\sum_{\mathbf{q}}n_{\mathbf{q}}\hbar\omega'_{\mathbf{q}}\ , \tag{12}$$

where $\omega'_{\mathbf{q}}=\omega_{\mathbf{q}}+\delta\omega_{\mathbf{q}}$ is the shifted magnon frequency for the qth magnon mode. The carrier-induced shift of the qth magnon mode is given by

$$\hbar\delta\omega_{\mathbf{q}}=-I/2N+(SI^2/2N)\times[-(\hbar^2/2m)(2\mathbf{k}\cdot\mathbf{q}+q^2)+\hbar\omega_{\mathbf{q}}]^{-1}\ . \tag{13}$$

IV. CARRIER-INDUCED CHANGE OF THE SYSTEM'S FREE ENERGY

With the addition of a single charge carrier to the magnetic semiconductor, the energy levels of the system are altered from $E_M^{(0)}$ to $E_{\mathbf{k}}+E_M$. Concomitantly, the free energy of the system is changed. The carrier-induced change of the free energy of the system is given by

$$F_1-F_0=-k_BT\ln\left[\frac{\sum_{\mathbf{k}}\sum_{\ldots,n_{\mathbf{q}},\ldots}\exp[-\beta(E_{\mathbf{k}}+E_M)]}{\sum_{\ldots,n_{\mathbf{q}},\ldots}\exp(-\beta E_M^{(0)})}\right]\ , \tag{14}$$

where $\beta=1/k_BT$ and k_B is the Boltzmann constant. We can cast this free-energy expression into an analogous form to that for a noninteracting carrier by defining an "effective" electronic energy.[3] Specifically, we write the free energy as

$$F_1-F_0=-k_BT\ln\sum_{\mathbf{k}}\exp(-\beta E'_{\mathbf{k}})\ , \tag{15}$$

where the effective electronic energy is defined by

$$E'_{\mathbf{k}}=-k_BT\ln\left[\frac{\sum_{\ldots,n_{\mathbf{q}},\ldots}\exp[-\beta(E_{\mathbf{k}}+E_M)]}{\sum_{\ldots,n_{\mathbf{q}},\ldots}\exp(-\beta E_M^{(0)})}\right]\ . \tag{16}$$

The contribution to the free energy associated with the kth state is then simply $E'_{\mathbf{k}}$. Similarly, using Eq. (3), we have that the quasiparticle energy associated with a carrier in the kth state is

$$E^{\mathbf{k}}_Q\equiv E'_{\mathbf{k}}-T(\partial E'_{\mathbf{k}}/\partial T)\ . \tag{17}$$

Carrying out the standard procedure of summing over boson occupation numbers, we readily find that Eq. (16) becomes

$$E'_{\mathbf{k}}=\hbar^2k^2/2m+IS/2+k_BT\sum_{\mathbf{q}}\ln\left[\frac{1-e^{-i\beta\hbar\omega'_{\mathbf{q}}}}{1-e^{-\beta\hbar\omega_{\mathbf{q}}}}\right]. \quad (18)$$

The quasiparticle energy associated with the kth state is then readily found, using Eq. (17), to be

$$E_Q^{\mathbf{k}}\equiv E'_{\mathbf{k}}+\delta E_M=\hbar^2k^2/2m+IS/2+\sum_{\mathbf{q}}(\bar{n}'_{\mathbf{q}}\hbar\omega'_{\mathbf{q}}-\bar{n}_{\mathbf{q}}\hbar\omega_{\mathbf{q}}), \quad (19)$$

where $\bar{n}_{\mathbf{q}}=[\exp(\beta\hbar\omega_{\mathbf{q}})-1]^{-1}$ is the equilibrium number of magnons of energy $\hbar\omega_{\mathbf{q}}$ at temperature T. The first three terms of Eq. (19) are just the energy of the system comprising a carrier and the local magnetic moments. The final term is simply the negative of the energy of the carrier-free magnon system. Thus, $E_Q^{\mathbf{k}}$ is the change of the energy of the system when a charge carrier is added to it, i.e., the quasiparticle energy.

We now express the effective electronic energy and the quasiparticle energy in terms of the carrier-induced magnon frequency shifts. In doing this we take cognizance, as in Ref. 3, of the infinitesimal nature of these carrier-induced shifts of the magnon frequencies. We find

$$E'_{\mathbf{k}}=\hbar^2k^2/2m+IS/2+\sum_{\mathbf{q}}\bar{n}_{\mathbf{q}}\hbar\delta\omega_{\mathbf{q}} \quad (20)$$

and

$$E_Q^{\mathbf{k}}=\hbar^2k^2/2m+IS/2+\sum_{\mathbf{q}}(\bar{n}_{\mathbf{q}}\hbar\delta\omega_{\mathbf{q}}-\beta\bar{n}_{\mathbf{q}}^2\hbar^2\omega_{\mathbf{q}}\delta\omega_{\mathbf{q}}e^{\beta\hbar\omega_{\mathbf{q}}}). \quad (21)$$

We now proceed to evaluate the $\mathbf{q}$ summation of Eq. (21). To accomplish this task we must first specify the magnon dispersion relation. We take the magnon dispersion relation to be that for a cubic ferromagnet,

$$\hbar\omega_{\mathbf{q}}=Dq^2, \quad (22)$$

with $D=2JSa^2$, where J is the nearest-neighbor intersite exchange integral in the cubic direction and a is the lattice constant. We now separately evaluate the terms associated with the first- and second-order perturbative contributions. Thus, for the first-order term we write

$$E_Q^{\mathbf{k}(1)}=E'^{(1)}_{\mathbf{k}}+\delta E_M^{(1)}. \quad (23)$$

Converting the $\mathbf{q}$ summations to integrals and evaluating them, we obtain

$$E'^{(1)}_{\mathbf{k}}=\hbar^2k^2/2m+I\left[S-\sum_{\mathbf{q}}\bar{n}_{\mathbf{q}}/N\right]/2$$
$$=\hbar^2k^2/2m+IS/2-(I/16N_0)(k_BT/\pi D)^{3/2}\zeta(\tfrac{3}{2}) \quad (24)$$

and

$$\delta E_M^{(1)}=(3I/32N_0)(k_BT/\pi D)^{3/2}\zeta(\tfrac{3}{2}), \quad (25)$$

where $N_0=N/V$ is the number of host spins in a unit volume and $\zeta(\frac{3}{2})$ is the Riemann zeta function of argument $\frac{3}{2}$; it equals 2.612. The $T^{3/2}$ dependence of the first-order term just reflects the well-known $T^{3/2}$ deviation of the magnetic semiconductor's magnetization from its saturation value in the spin-wave regime. We also note that there is a partial cancellation between the effective electronic energy and the change of the magnetic energy.

The second-order contribution to the quasiparticle energy is written in strict analogy with the first-order contribution, Eq. (23):

$$E_Q^{\mathbf{k}(2)}\equiv E'^{(2)}_{\mathbf{k}}+\delta E_M^{(2)}. \quad (26)$$

Here $E'^{(2)}_{\mathbf{k}}$ and $\delta E_M^{(2)}$ are obtained by replacing $\hbar\delta\omega_{\mathbf{q}}$ in Eq. (21) by the I^2 term of Eq. (13). This explicitly yields

$$E'^{(2)}_{\mathbf{k}}=(SI^2/2N)\sum_{\mathbf{q}}(e^{\beta\hbar\omega_{\mathbf{q}}}-1)^{-1}\times[-(\hbar^2/2m)(2\mathbf{k}\cdot\mathbf{q}+q^2)+\hbar\omega_{\mathbf{q}}]^{-1} \quad (27)$$

and

$$\delta E_M^{(2)}=-(SI^2/2N)\sum_{\mathbf{q}}\beta\hbar\omega_{\mathbf{q}}e^{\beta\hbar\omega_{\mathbf{q}}}(e^{\beta\hbar\omega_{\mathbf{q}}}-1)^{-2}\times[-(\hbar^2/2m)(2\mathbf{k}\cdot\mathbf{q}+q^2)+\hbar\omega_{\mathbf{q}}]^{-1}. \quad (28)$$

We now convert the above $\mathbf{q}$ summations into integrals over q space with the azimuthal axis chosen parallel to $\mathbf{k}$. The angular integrations are then performed, yielding

$$E'^{(2)}_{\mathbf{k}}=-\frac{SI^2m}{8\pi^2N_0\hbar^2k}\int_0^{q_m}dq\,q(e^{\beta Dq^2}-1)^{-1}\times\ln\left|\frac{q(D-\hbar^2/2m)-\hbar^2k/m}{q(D-\hbar^2/2m)+\hbar^2k/m}\right| \quad (29)$$

and

$$\delta E_M^{(2)}=\frac{SI^2m}{8\pi^2N_0\hbar^2kk_BT}\int_0^{q_m}dq\,Dq^3e^{\beta Dq^2}(e^{\beta Dq^2}-1)^{-2}\times\ln\left|\frac{q(D-\hbar^2/2m)-\hbar^2k/m}{q(D-\hbar^2/2m)+\hbar^2k/m}\right|, \quad (30)$$

where q_m is the maximum magnon wave vector (π/a). It is clear that there is some cancellation between the two contributions to the second-order portion of the quasiparticle energy. In fact, it is expeditious to combine these two contributions, the integrals of Eqs. (29) and (30). We then (1) change the integration variable from q to $y=(D\beta)^{1/2}q$, and (2) neglect D in comparison with $\hbar^2/2m$ in the integrand. The latter step is motivated by the observation that $\hbar^2/2m$ typically is very much larger than D (e.g., $2mD/\hbar^2\simeq10^{-3}$ in EuO). We then have

$$E_Q^{\mathbf{k}(2)}=\frac{SI^2mk_BT}{8\pi^2N_0\hbar^2kD}\int_0^{y_m}dy\,f(y)g(y),$$

with (31)

$$f(y)\equiv[e^{y^2}(y^2-1)+1](e^{y^2}-1)^{-2}\ ,$$

$$g(y)\equiv\ln\left|\frac{y+2k\sqrt{D\beta}}{y-2k\sqrt{D\beta}}\right|^{y}$$

and

$$y_m\equiv\sqrt{D\beta}q_m\ .$$

We direct our attention to the circumstance in which charge carriers are in thermal equilibrium, possessing energies $\hbar^2k^2/2m$, which are not much greater than the thermal energy k_BT. In this situation the parameter $k(D\beta)^{1/2}$ is of the order of $(2mD/\hbar^2)^{1/2}$, a value very much less than unity. For $y>1$ [$\gg k(D\beta)^{1/2}$], we approximate $g(y)$ by $4k(D\beta)^{1/2}$ and $f(y)$ by $y^2\exp(-y^2)$. For small values of y, $y<1$, we just replace $f(y)$ by $(1-y^2/3)/2$. Finally, since we are interested in the situation at low temperatures, we extend the upper limit of the y integration in Eq. (31), y_m, to infinity. Incorporating these steps, we have

$$\int_0^{y_m}dy\,f(y)g(y)\cong\tfrac{1}{2}\int_0^1 dy(1-y^2/3)\times\ln\left|\frac{y+2k\sqrt{D\beta}}{y-2k\sqrt{D\beta}}\right|^{y}+4k\sqrt{D\beta}\int_1^\infty dy\,y^2e^{-y^2}\ .$$

To the lowest order in our smallness parameter, $k(D\beta)^{1/2}$, this integral simply becomes $1.3k(D\beta)^{1/2}$. Inserting this result into Eq. (31), we find that the second-order contribution to the quasiparticle energy is

$$E_Q^{\mathbf{k}(2)}=\frac{1.3SI^2m}{8\pi^2\hbar^2N_0}\left[\frac{k_BT}{D}\right]^{1/2}\ . \tag{32}$$

We can now combine the results of Eqs. (24), (25), and (32) and write the quasiparticle energy:

$$E_Q^k=\hbar^2k^2/2m+IS/2+(1.3I/16N_0)(k_BT/\pi D)^{3/2}\times\left[1+\frac{2mDSI}{\sqrt{\pi}\hbar^2k_BT}\right]\ . \tag{33}$$

Several features are noteworthy. First, a numerical estimate for an example (EuO: $S=7/2$, $I=-0.1$ eV, and $2mD/\hbar^2=0.001$) indicates that the temperature dependence is dominated by that arising from the first-order contribution, the $T^{3/2}$ dependence, at temperatures above a few K. It is only at the lowest of temperatures that the second-order contribution dominates. Second, the first-order contribution to the quasiparticle energy is dependent on the sign of I, while the second-order contribution to the quasiparticle energy is not. Thus, the temperature dependence of the quasiparticle energy is dependent on whether or not the carrier finds it energetically favorable to be aligned parallel ($I<0$) or antiparallel ($I>0$) to the local spins of the magnetic semiconductor.

Having obtained an expression for the quasiparticle energy E_Q^k, Eq. (33), the Pelter heat is readily obtained. To begin, we recall that the observed Peltier heat Π is the average of the contributions associated with each electronic state Π_k weighted by its partial conductivity σ_k:[2-4]

$$\Pi=\sum_k\sigma_k\Pi_k\Big/\sum_k\sigma_k\ . \tag{34}$$

Inserting expressions for Π_k, Eq. (2), and then E_Q^k, Eq. (33), into Eq. (34), the expression for the Peltier heat becomes

$$\Pi=\sum_k(E_k^{(0)}-\mu)\sigma_k\Big/\sum_k\sigma_k+(1.3I/16N_0)(k_BT)^{3/2}(1+2mDSI/\hbar^2k_BT\sqrt{\pi})\ . \tag{35}$$

The first term, containing the summations over k, does not explicitly contain the electron-magnon interaction. It exists even in the limit of vanishing I. Furthermore, the contribution which explicitly arises from the electron-magnon interaction, the remainder of Eq. (35), is unaffected by the averaging over the electronic states. Thus, the averaging over the electronic states is carried out as in the case of noninteracting carriers.

Making the standard simplifying assumption that the mobility of the itinerant carrier is independent of k, the k dependences of the partial electronic conductivities, the σ_k's, arise solely from the Fermi factor governing the occupation of the kth electronic state. Then, $\sigma_k\propto\exp[-(E_k^{(0)}-\mu)/k_BT]$ for nondegenerate transport, i.e., when the chemical potential lies outside of the electronic band of in the itinerant carriers. In this case the k summations yield

$$\Pi=(3k_BT/2)-\mu+(1.3I/16N_0)(k_BT)^{3/2}\times(1+2mDSI/\hbar^2k_BT\sqrt{\pi})\ . \tag{36}$$

For the complementary case of a strongly degenerate semiconductor, where the chemical potential lies deep within the band of itinerant carriers ($\mu\gg k_BT$), only states in the vicinity of the Fermi level contribute to conduction. Here,

$$\sigma_k\propto\exp\{-[(E_k^{(0)}-\mu)/2k_BT]^2\}$$

and

$$\Pi=(k_BT)^2/\mu+(1.3I/16N_0)(k_BT)^{3/2}\times(1+2mDSI/\hbar^2k_BT\sqrt{\pi})\ . \tag{37}$$

Thus, we see that the magnitude and the temperature dependence of the Peltier heat are generally significantly affected by the electron-magnon interaction.

V. DISCUSSION AND SUMMARY

The standard contribution to the Peltier heat associated with a given electronic state is the product of the temperature and the change of the entropy of the system when that electronic state is occupied by a charge carrier. The Peltier heat associated with a given electronic state $\mathbf{k}$ can be written as the difference between the quasiparticle energy associated with that state and the chemical potential: $E_Q^k-\mu$. When the carrier interacts with some other sys-

tem of the solid, e.g., magnons or phonons, the quasiparticle energy can garner a temperature dependence.

We have calculated the standard contribution to the Peltier heat of an itinerant carrier in a broadband ferromagnetic semiconductor at low temperatures. Restricting our attention to low temperatures, we have been able to employ the spin-wave approximation. Thus, spin deviations are represented by the excitations of magnons. The interaction of the itinerant carrier with the spin deviations leads to carrier-induced shifts of the magnon energies. These carrier-induced shifts of the magnon energies lead to temperature-dependent quasiparticle energies.

We treat the interaction between the charge carrier and the local spins of a magnetic semiconductor perturbatively through second order. Correspondingly, the quasiparticle energy is written as the sum of terms associated with the first- and second-order perturbative contributions to the energy of the coupled system. Except at very low temperatures, the first-order contribution dominates the magnitude and temperature dependence of the quasiparticle's energy. Then, the quasiparticle energy varies with temperature as $T^{3/2}$.

The quasiparticle energy can be viewed as the sum of two terms. The first is the energy of the electronic carrier. The second is the change of the energy of the remainder of the coupled system, e.g., the magnons, as a result of introducing a charge carrier to the material. In our case these two terms partially cancel. For an itinerant carrier coupled to phonons, the analogous contributions completely cancel above the phonon temperature,[3] but not below it. Extending our studies of the electron-magnon system beyond their regime of validity to temperatures above the maximum magnon energy, we also find the complete cancellation of the temperature-dependent contributions to the quasiparticle energy. Below the maximum magnon temperature the electron-phonon and electron-magnon calculations are not strictly analogous to one another. This is due to nonanalogous aspects of the electron-phonon and electron-magnon interactions and to the differences between the phonon and magnon dispersion relations.

It is common practice to treat the Peltier heat of a charge carrier in a solid as if the charge carrier were free. Then the quasiparticle energy is simply an electronic energy eigenvalue, a constant. Hence, any temperature dependence of the Peltier heat is attributed to the chemical potential. The present calculation shows that the quasiparticle energy which enters into the Peltier heat for an itinerant carrier in a ferromagnetic semiconductor is, however, generally temperature dependent. We have calculated this temperature dependence. With this result one may better determine any temperature dependence of the chemical potential, the other term entering into the Peltier heat.

ACKNOWLEDGMENT

One of us (D.E.) accomplished this work at Sandia National Laboratories with the support of the U. S. Department of Energy under Contract No. DE-AC04-76-DP00789.

[1]H. B. Callen, *Thermodynamics* (Wiley, New York, 1961), p. 299.
[2]D. Emin, Solid State Commun. **22**, 409 (1977).
[3]D. Emin. Phys. Rev. B **30**, 5766 (1984).
[4]N. L. Liu and D. Emin, Phys. Rev. B **30**, 3250 (1984).
[5]C. Hass, Phys. Rev. **168**, 531 (1968).
[6]F. Rhys, J. Helman, and W. Baltensperger, Phys. Kondens. Mater. **6**, 105 (1967).
[7]R. M. White, Phys. Rev. Lett. **23**, 858 (1969).
[8]R. B. Wooley and P. M. White, Phys. Rev. B **1**, 4474 (1970).
[9]T. Kasuya, in *Magnetism*, edited by G. T. Rado and H. Suhl (Academic, New York, 1966), Vol. III B, p. 215.