PHYSICAL REVIEW B VOLUME 32, NUMBER 3 1 AUGUST 1985

Small normal-metal loop coupled to an electron reservoir

M. Büttiker
IBM Thomas J. Watson Research Center, Yorktown Heights, New York 10598
(Received 26 April 1985)

A conceptually simple approach is proposed to introduce dissipation into a normal-metal loop penetrated by a flux. The loop is coupled via a single current lead to an electron reservoir. Scattering processes in the loop are elastic. Inelastic processes occur only in the reservoir and are the source of dissipation. We investigate the effect of this reservoir on the persistent currents in the loop and the absorption of power in the presence of a sinusoidally modulated flux.

Flux sensitive effects with period $\Phi_0 = hc/e$ have been the subject of several recent theoretical and experimental investigations. References 1 and 2 treated a closed ring without leads and found persistent currents with period Φ_0 in presence of a time-independent flux and an oscillating current if the flux is increased linearly with time. References 3–5 considered such a loop connected to two current leads and pointed out that Aharonov-Bohm–like interference effects lead to oscillations in the magnetoconductance with period Φ_0. These oscillations have been observed[6] in Au rings.

This paper examines the model in Fig. 1. A loop of normal metal penetrated by a flux is connected to a single wire, which in turn connects to a reservoir. The reservoir emits carriers with the Fermi distribution

$$f(E) = [\exp(E - E_F)/kT + 1]^{-1}$$

into the lead and absorbs carriers coming from the lead irrespective of their energy. In the reservoir the carriers are scattered inelastically and there is, therefore, no phase relationship between carriers absorbed by the reservoir and carriers emitted by the reservoir. Carriers emitted by the reservoir travel along the lead to the junction with the loop, where they are reflected or else enter the loop. In the loop all scattering processes are elastic. After some time a carrier in the loop will eventually escape through the junction into the lead and reach the reservoir. Thus, the coupling of the loop to the reservoir gives rise to electron states in the loop with a finite lifetime.

In this model we have a complete *spatial* separation between elastic processes in the loop and the inelastic processes in the reservoir. Such a spatial separation is characteristic of Landauer's discussion[5,7] of the resistance of an obstacle in an otherwise perfect wire. Inelastic processes are essential to obtain a resistance, but owing to the spatial separation the conductance can be expressed in terms of elastic scattering properties of the sample alone.

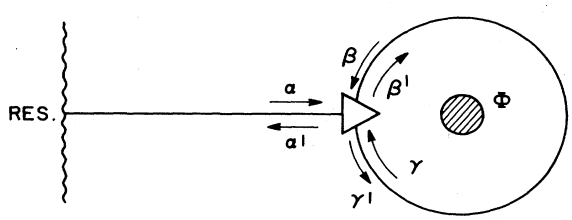

FIG. 1. Loop coupled via an ideal conductor to a dissipative electron reservoir.

Our first objective is to study the persistent currents in the loop of Fig. 1. For simplicity we take both the lead and the loop to be strictly one-dimensional ideal wires. Elastic scattering arises only from the junction between the lead and the loop. The junction is described by an $\underline{S}$ matrix which yields the amplitudes $\boldsymbol{\alpha}' = (\alpha', \beta', \gamma')$ of the outgoing waves in terms of the incoming waves $\boldsymbol{\alpha} = (\alpha, \beta, \gamma)$. Here, we chose for the $\underline{S}$ matrix[4]

$$\underline{S} = \begin{pmatrix} -(a+b) & \epsilon^{1/2} & \epsilon^{1/2} \\ \epsilon^{1/2} & a & b \\ \epsilon^{1/2} & b & a \end{pmatrix} , \tag{1}$$

with $a = \frac{1}{2}(\sqrt{1-2\epsilon} - 1)$, $b = \frac{1}{2}(\sqrt{1-2\epsilon} + 1)$. ϵ is a coupling parameter. $\epsilon = \frac{1}{2}$ is the maximum coupling. For $\epsilon = 0$, the loop and the lead are decoupled; we have an ideal closed loop described by the Hamiltonian

$$H = (1/2m)(p - eA/c)^2 , \tag{2}$$

subject to periodic boundary conditions. The vector potential A is related to the flux by $A = \Phi/L$, where L is the circumference of the loop. The spectrum of such a perfect loop[8] is shown in Fig. 2. (Additional scattering in the loop opens gaps[1,2,4] at $\Phi = 0$ and $\Phi = \pm\Phi_0/2$.) To each flux Φ there corresponds a ladder of states $E_n(\Phi)$. To find the wave functions for $\epsilon \neq 0$, we have to match a solution of Eq. (2) to the wave function in the lead with the help of Eq. (1). For particles emitted by the reservoir with energy $E = \hbar^2 k^2/2m$, the wave function in the loop is of the form[8]

$$\psi = e^{2i\theta y/L}(Ae^{iky} + Be^{-iky}) , \tag{3}$$

where $\theta = \pi\Phi/\Phi_0$ and y is the coordinate along the loop. At $y = 0$, Eq. (3) determines the amplitudes β and β' and at $y = L$ the amplitudes γ and γ' (see Fig. 1). The wave function in the lead which describes carriers emitted from the reservoir and carriers traveling towards the reservoir is

$$\psi = \mathcal{N}^{1/2}(e^{ikx} + Ce^{-ikx}) . \tag{4}$$

We can take the junction to be at $x = 0$ and Eq. (4) determines α and α'. The normalization factor is determined in the following way: In a small energy interval dE the current injected by the reservoir into the lead is $dj_{\text{in}} = ev(dn/dE)f(E)dE$. Here, $dn/dE = 1/2\pi\hbar v$ is the density of states in the perfect wire, $v = \hbar k/m$. The wave given in Eq. (4) yields the correct incident current if we put $\mathcal{N} = ef(E)dE/2\pi\hbar v$.

 ©1985 The American Physical Society

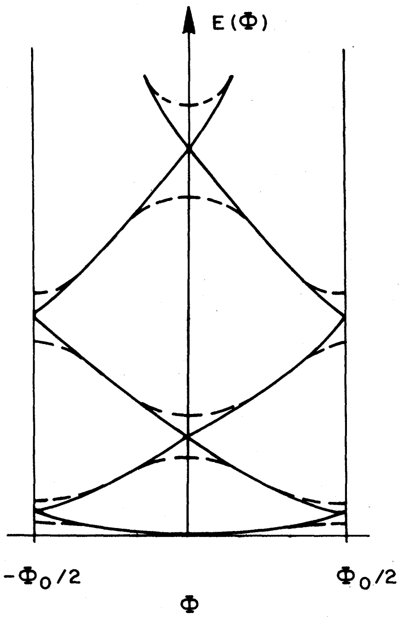

FIG. 2. Electron energies of the decoupled loop as a function of flux. The ideal loop is represented by full lines, the loop with additional elastic scattering by broken lines.

Using Eqs. (3), (4), and (1) we calculate the coefficients A, B, and C. In the lead we find no net current flow, $|C|^2=1$. The current in the loop in the energy interval dE is $dj=v(|A|^2-|B|^2)$. The total circulating current is obtained by integrating over the energy and this yields

$$j=-\frac{e}{2\pi\hbar}\int_0^\infty dE\, f(E)g(E,\theta)\sin\phi\sin 2\theta \ . \tag{5}$$

Here, we have introduced the abbreviation $\phi=kL=L\times(2mE/\hbar^2)^{1/2}$. The function $g(E,\theta)$ is given by

$$g(E,\theta)=\epsilon/[b^2(\cos\phi-\cos 2\theta)^2+a^2\sin^2\phi] \ . \tag{6}$$

Figure 3(a) shows the persistent currents, Eq. (5), for different coupling parameters ϵ at $T=0$ as a function of flux for the Fermi energies $E_F=\pi^2(2n+1)^2E_0$. Here, we have expressed E_F, which we treat as a parameter, in units of $E_0=\hbar^2/2mL^2$, which sets the energy scale of Fig. 2. The ratio j/j_0, where $j_0=ev_F/L$, with v_F the Fermi velocity, is independent of n. [The same is true for $E_F=\pi^2(2n)^2E_0$.] With increasing coupling strength ϵ the maximum amplitude of the persistent currents decreases. This is due to the broadening of the energy states in the loop. The density of particles in the loop in an energy interval dE is $dn=|A|^2+|B|^2$. Using this expression our calculation yields a density of states in the loop given by

$$dn/dk=(1/2\pi)g(E,\theta)[1-\cos(2\theta)\cos\phi] \ . \tag{7}$$

In the weak-coupling limit, for a flux away from the center and the boundary of the Brillouin zone (Fig. 2), Eq. (7) can be approximated by a sum of Lorentzians $2\Gamma_n^2/\epsilon\pi\times\{[E-E_n(\theta)]^2+\Gamma_n^2\}$, where $E_n(\theta)$ is the energy of an eigenstate of the uncoupled loop and $\Gamma_n=\epsilon[E_0E_n(\theta)]^{1/2}$ is the width of the energy level. Γ_n is, therefore, proportional to the coupling parameter and also proportional to the velocity $v=\hbar k_n/m$ of the carriers in the nth eigenstate. As ϵ is increased the levels broaden further. Near $\theta=0$ and $\theta=\pm\pi/2$ the behavior of the density of states is not as simple and will be discussed elsewhere. Figure 3(b) shows the decrease of the persistent currents with increasing temperature for two different coupling strengths. In this model calculation the Fermi energy is $E_F=25\pi^2E_0$; i.e., at $T=0$ five states are occupied. For small temperature the currents decrease proportional to T^2.

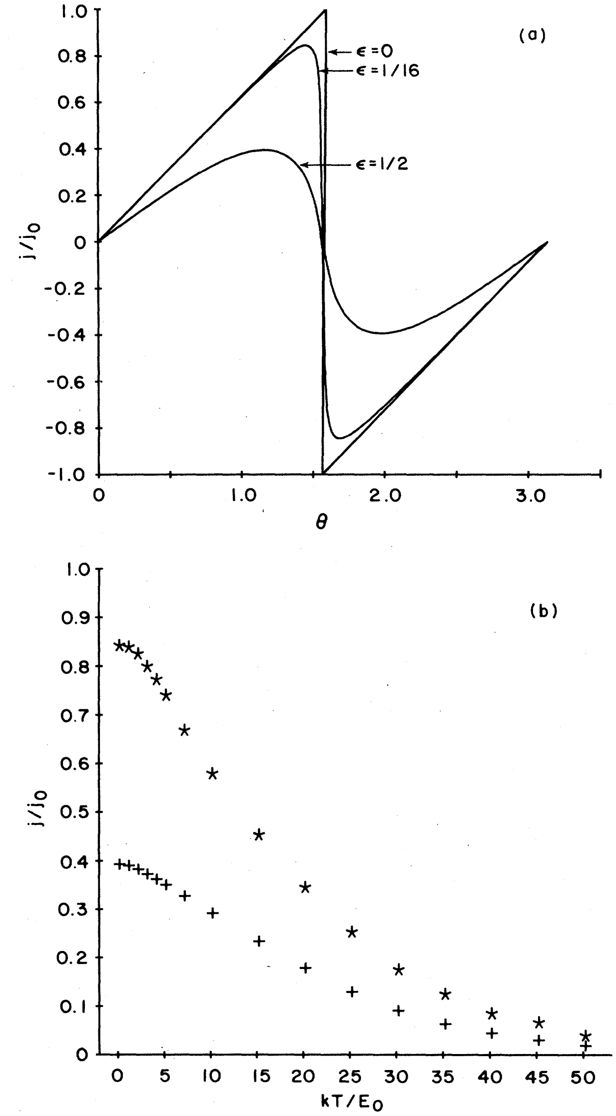

FIG. 3. Presistent currents in the loop in units $j_0=ev_F/L$. (a) For $T=0$ as a function of flux $\theta=\pi\Phi/\Phi_0$. (b) Temperature dependence of the maximum amplitude for $\epsilon=\frac{1}{2}$ (+) and $\epsilon=\frac{1}{16}$ (*). $E_0=\hbar^2/2mL^2$.

Dissipation of power occurs if we consider a time-dependent flux. Reference 2 considered a flux $\Phi=-cUt$ and showed that the Josephson currents induced by the voltage U have a dc component due to inelastic scattering. Here, we consider the simpler case of a small sinusoidal flux superimposed on a time-independent flux, $\Phi=\Phi_1+\Phi_2\cos(\omega t)$, and calculate the dissipated power. This problem was brought to our attention by Shiren and Imry.[9] Consider Eq. (2) with a vector potential $A=[\Phi_1+\Phi_2\cos(\omega t)]/L$ and extended along the whole y axis. To first order in Φ_2/Φ_0 the time-dependent Eq. (2) has the solution[10,11]

$$\psi(\pm k,\theta)=e^{2i\theta y/L}e^{\pm iky}e^{-iEt/\hbar}\times[1\pm 2(EV_0/\hbar^2\omega^2)^{1/2}\sin(\omega t)+O(V_0)] \ , \quad (8)$$

with $\theta=\pi\Phi_1/\Phi_0$, $E=\hbar^2k^2/2m$, and

$$V_0=(\hbar^2/2m)(2\pi/L)^2(\Phi_2/\Phi_0)^2 \ .$$

The sinusoidal modulation of the flux gives rise to side bands at the frequencies $(E/\hbar)+\omega$ and $(E/\hbar)-\omega$. To obtain the wave function[11] for our system (Fig. 1), we take within the loop a superposition of the two solutions, Eq. (8), for each of the three energies $E, E\pm\hbar\omega$. The solution in the lead consists of a wave incident at the energy E and reflected waves at the three energies E and $E\pm\hbar\omega$. The junction Eq. (1) connects the solutions in the loop and the lead at each instant of time. We can, therefore, use Eq. (1) for each frequency component separately. For $E/\hbar$, the calculation is the same as for the case of a static flux. The matching at $E/\hbar\pm\omega$ determines the probability $R_\pm(E)$ that an electron incident with energy E is reflected back to the reservoir with energy $E_\pm=E\pm\hbar\omega$. We find

$$R_\pm(E)=\frac{4V_0(EE_\pm)^{1/2}}{\hbar^2\omega^2}g(E,\theta)g(E\pm\hbar\omega,\theta)\times\sin^2 2\theta\sin^2\left(\frac{\phi-\phi_\pm}{2}\right)\sin^2\left(\frac{\phi+\phi_\pm}{2}\right) \ , \quad (9)$$

where $\phi_\pm=k_\pm L=L(2mE_\pm/\hbar^2)^{1/2}$ and $g(E,\theta)$ is given by Eq. (6). The numerator of Eq. (9) is correct to order ϵ^2 only. For $E<\hbar\omega$, $R_-(E)$ is zero.

The current in the lead now has a component oscillating with the modulation frequency ω. The time-average current, however, is zero and we can use Eq. (9) to find the probability $R_0(E)$ that electrons are reflected from the loop with their energy unchanged. Current conservation[12] yields $R_0(E)=1-R_+(E)-R_-(E)$. To calculate the dissipated power, we consider the energy flow in the lead. We have a time-averaged energy stream into the reservoir given by

$$dW=(E_+R_++E_-R_-+ER_0-E)(dj_{\text{in}}/e) \ . \quad (10)$$

Here, dj_{in} is the current injected by the reservoir in an energy interval dE. Using the expression for R_0 derived above, we find $dW=\hbar\omega(R_+-R_-)(dj_{\text{in}}/e)$. To obtain the total energy flow W we have to integrate this expression over all energies. Next, we relate the dissipated power W to the voltage induced in the loop: $U=-(1/c)d\Phi/dt=(\Phi_2/c)\omega\sin\omega t$. The reflection probabilities $R_\pm$ are proportional to

$$V_0=(\hbar^2/2m)(2\pi/L)^2(\Phi_2/\Phi_0)^2 \ .$$

Since we are considering the time-average energy flow we relate Φ_2 to the time-average voltage $U^2=1/2(\Phi_2/c)^2\omega^2$ and thus $V_0=2E_0e^2U^2/\hbar^2\omega^2$ with $E_0=\hbar^2/2mL^2$. We can now characterize the absorption of power by a voltage-independent coefficient $\alpha(\omega)=W/U^2$. Using Eq. (10) and noticing that Eq. (9) has the symmetry $R_-(E)=R_+(E-\hbar\omega)$, we find

$$\alpha(\omega)=\frac{e^2}{2\pi\hbar}\int_0^\infty dE[f(E)-f(E+\hbar\omega)]\frac{8E_0(EE_+)^{1/2}}{\hbar^3\omega^3}g(E,\theta)g(E+\hbar\omega,\theta)\sin^2(2\theta)\sin^2\left(\frac{\phi-\phi_+}{2}\right)\sin^2\left(\frac{\phi+\phi_+}{2}\right) \ . \quad (11)$$

On purpose, we avoid calling α a conductance, even though it has that dimension. The low-frequency limit of α is not the conductance of a closed loop in the presence of a linearly increasing flux (i.e., the conductance due to a time-independent voltage) studied in Ref. 2. In this case, the carriers are driven through the whole "Brillouin zone" of Fig. 2. In contrast, Eq. (11) describes only small excursions of the carriers away for the states $E_n(\Phi_1)$. Furthermore, α is also unrelated to the conductance of carriers through a loop connected to two current leads.[3-6] Equation (11) is simply a measure of the power absorbed in the presence of a microwave field.[13] Power is dissipated in the loop due to two processes: For large modulation frequencies we have transitions *between* the broadened energy levels $E_n(\Phi)$ of Fig. 2; for small modulation frequencies, Eq. (11) describes transitions *within* the broadened energy levels $E_n(\Phi)$. It is these *intraband* transitions which dominate the low-frequency behavior. Figure 4 shows the low-frequency limit of α as a function of flux for $\epsilon=\frac{1}{16}$. The Fermi energy is $E_F=\frac{49}{4}\pi^2E_0$ and intersects the level $E_3(\theta)$ of the decoupled loop (Fig. 2) at $\theta=\pm\pi/4$. The coupling to the reservoir gives rise to a width $\Gamma_3=(7\pi/32)E_0$ of this level. The temperature in Fig. 4 is $kT=E_0\approx\Gamma_3$. The absorption of power

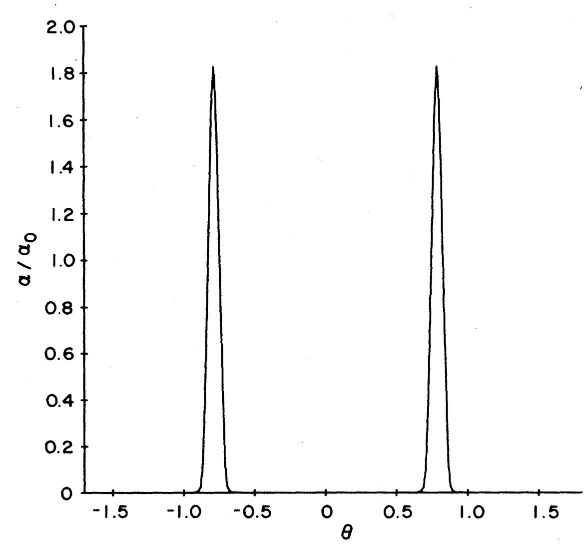

FIG. 4. Low-frequency limit of the absorption of power characterized by $\alpha(\omega)$, Eq. (11), due to transitions within a single broadened energy level of the loop as a function of flux. $\alpha_0=128e^2/\pi\hbar$.

is thus due to transitions within this single broadened energy level. Equation (11) contains a wealth of other interesting information which will be discussed elsewhere.

To summarize, we have proposed a conceptually simple approach to introduce dissipative effects into a small metallic loop and have shown its usefulness by presenting a model calculation. The concept introduced here can be extended and refined in many ways.

[1]M. Büttiker, Y. Imry, and R. Landauer, Phys. Lett. **96A**, 365 (1983).

[2]R. Landauer and M. Büttiker, Phys. Rev. Lett. **54**, 2049 (1985).

[3]Y. Gefen, Y. Imry, and M. Ya. Azbel, Phys. Rev. Lett. **52**, 139 (1984).

[4]M. Büttiker, Y. Imry, and M. Ya. Azbel, Phys. Rev. A **30**, 1982 (1984).

[5]M. Büttiker, Y. Imry, R. Landauer, and S. Pinhas, Phys. Rev. B **31**, 6207 (1985).

[6]R. A. Webb, S. Washburn, C. P. Umbach, and R. B. Laibowitz, Phys. Rev. Lett. **54**, 2696 (1985).

[7]R. Landauer, Philos, Mag. **21**, 863 (1970); in *Localization, Interaction and Transport Phenomena in Impure Metals,* edited by G. Bergmann, Y. Bruynseraede, and B. Kramer (Springer, Heidelberg, New York, 1985), p. 38.

[8]N. Byers and C. N. Yang, Phys. Rev. Lett. **7**, 46 (1961).

[9]N. Shiren and Y. Imry invoke the approach of G. Czycholl and B. Kramer, Solid State Commun. **32**, 945 (1979) for the closed loop.

[10]P. K. Tien and J. P. Gordon, Phys. Rev. **129**, 647 (1963).

[11]M. Büttiker and R. Landauer, Phys. Rev. Lett. **49**, 1739 (1982); Phys. Scr. (to be published).

[12]D. L. Haarig and R. Reifenberger, Phys. Rev. B **26**, 6408 (1982).

[13]M. Büttiker, in *Proceedings of the First International Conference on Superconducting Quantum Devices, Berlin, 1985,* edited by H. D. Hahlbohm and H. Lüddig (de Gruyter, Berlin, in press). This paper discusses the absorption of power in the presence of a microwave field using the approach of Ref. 2.