Supercurrent in ^{3}He-B

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The $T=0$ supercurrent in the B phase of superfluid ^{3}He in a weakly inhomogeneous magnetic field is calculated by a method developed by Mermin and Muzikar and is found to agree with our earlier result.

I. INTRODUCTION

There is presently a controversy over the expression of the hydrodynamical mass-current density in ^{3}He-B. By an expansion of Gor'kov's equations and making use of a gauge transformation to follow the rotation of the order parameter, Combescot and Dombre[1] showed that the supercurrent at zero temperature is given by

$$g_i = \rho v_i^s + m_\alpha v_{i\alpha}^s - \frac{\hbar}{4m}\epsilon_{ijk}\partial_j(R_{\alpha k} m_\alpha) , \tag{1}$$

where $\mathbf{v}^s$ is the superfluid velocity, $\mathbf{v}_\alpha^s$ the spin superfluid velocity, ρ the mass density, m_α the magnetization density (divided by the ^{3}He magnetic moment), and $R_{\alpha i}$ the spin-orbit B-phase rotation. There is a rather simple physical explanation of the last two terms of Eq. (1): The first one merely corresponds to a convection mass current due to a difference between, say, spin-up and -down superfluid velocity (resulting in a spin superfluid velocity) with unequal spin-up and -down populations. This term is also present in the A phase. The second term is much more typical of the B-phase structure. It may be linked to the appearance of an intrinsic angular momentum

$$L_i = -\tfrac{1}{2} R_{\alpha i} m_\alpha \tag{2}$$

in the presence of a nonzero spin density, due to the quantum rigidity of the Cooper pairs, which in ^{3}He-B are formed in a state of zero total angular momentum, after which the spins are denoted by R:

$$\hat{J} = \hat{L} + R^{-1}\hat{S} \equiv 0 . \tag{3}$$

Recently, however, Mineev and Volovik[2] have described another calculation of the zero-temperature mass current based on a gradient expansion of Gor'kov's equations for the B phase in a magnetic field (also see Muzikar[3] for similar results) and have obtained a rather different result. In particular, they do not find any pure curl term as in (1), but rather a contribution of the form

$$-K^s \epsilon_{ijk} R_{\alpha k} \partial_j m_\alpha - K^L \epsilon_{ijk} m_\alpha \partial_j R_{\alpha k} , \tag{4}$$

with $K^L \ll K^S$. They argued that this is due to the smallness of the intrinsic angular momentum carried by the Cooper pairs, which prevents the existence of a well-defined angular momentum as given by Eq. (2). We note that the two calculations[4] are expansions of Gor'kov's equations. The only difference is that in Ref. 1 the magnetization is dynamically induced by the order-parameter motion, while in Ref. 2 it is produced by a magnetic field. However, the expression for the current should not depend on the physical origin of the magnetization and the two calculations should reasonably lead to the same result.

We shall not discuss the general question of intrinsic angular momentum, which is a very delicate one. We find it more important to try to resolve the discrepancies concerning a quantity of primary physical interest such as the ground-state mass current in the B phase. In order to do this we present here a third calculation, using an approach previously introduced in a beautiful paper by Mermin and Muzikar[5] to answer similar problems in the A phase. The spirit of the method is to expand to first order in gradients an equation relating directly the one-particle density matrix to the order parameter. This equation may be shown to result from Gor'kov's equations, but it leads to a computation much more compact and with very few hypotheses. We take the same situation as Mineev and Volovik and consider the B phase in a weakly spatially varying magnetic field $\mathbf{H}(\mathbf{r})$ and an arbitrary spatial configuration of the order parameter. We obtain the same results as Ref. 1 for the expression of the mass and the spin currents to first order in $\mathbf{H}$ and in gradients. This provides an unambiguous and independent proof of the validity of our expression (1) for the supercurrent.

II. CALCULATION OF THE CURRENTS

Our starting point will be the following relation between the one-particle density matrix $\bar{\rho}$ and the order parameter ψ as given in Ref. 5:

$$\bar{\rho} - \bar{\rho}\cdot\bar{\rho} = \psi\cdot\psi^\dagger , \tag{5}$$

where the functional product notation $f\cdot g$ means

$$f\cdot g(\mathbf{r}_1,\mathbf{r}_2) = \int d\mathbf{r}_3 f(\mathbf{r}_1,\mathbf{r}_3) g(\mathbf{r}_3,\mathbf{r}_2)$$

and (6)

$$f^\dagger(\mathbf{r}_1,\mathbf{r}_2) = f^*(\mathbf{r}_2,\mathbf{r}_1) .$$

For a justification of Eq. (5) and further details on some technical aspects of this method, the interested reader should refer to the paper of Mermin and Muzikar. We shall work for both ρ and ψ in a mixed representation where the variables are the center-of-mass variable $\mathbf{r} = \frac{1}{2}(\mathbf{r}_1 + \mathbf{r}_2)$ and the momentum $\mathbf{p}$ relative to the difference variable $\mathbf{r}_1-\mathbf{r}_2$. In this representation, when ρ and ψ are slowly varying functions of $\mathbf{r}$, we can use the gradient expansion[5] (correct up to first order):

$$f\cdot g=f(\mathbf{r},\mathbf{p})g(\mathbf{r},\mathbf{p})+\tfrac{1}{2}i[f(\mathbf{r},\mathbf{p}),g(\mathbf{r},\mathbf{p})]\ , \tag{7}$$

where f,g are 2×2 (spin) matrix functions and $[f,g]$ is the Poisson bracket defined in spin space by

$$[f,g]_{s_1s_2}=\sum_{s_3,i}\left[\frac{\partial}{\partial r_i}f_{s_1s_3}\frac{\partial}{\partial p_i}g_{s_3s_2}-\frac{\partial}{\partial p_i}f_{s_1s_3}\frac{\partial}{\partial r_i}g_{s_3s_2}\right]. \tag{8}$$

Let us now recall the structure of the order parameter in the homogeneous B phase: ψ may be written as

$$\psi(\mathbf{p})=i\boldsymbol{\sigma}\cdot\hat{d}(\mathbf{p})b(\mathbf{p})\sigma_y\ , \tag{9}$$

where the spin vector $\hat{d}$ is defined by

$$\hat{d}_\alpha(\mathbf{p})=R_{\alpha i}\hat{p}_i\ , \tag{10}$$

and $b(\mathbf{p})$ is a real function which is isotropic.

We shall describe an inhomogeneous B phase in a nonuniform magnetic field $\mathbf{H}(\mathbf{r})$ by the following order parameter:

$$\psi(\mathbf{r},\mathbf{p})=[\boldsymbol{\sigma}\cdot\hat{d}(\mathbf{r},\mathbf{p})b(\mathbf{r},\mathbf{p})+i\boldsymbol{\sigma}\cdot\hat{d}(\mathbf{r},\mathbf{p})\times\mathbf{H}(\mathbf{r})c(\mathbf{r},\mathbf{p})+S(\mathbf{r},\mathbf{p})]i\sigma_y\ , \tag{11}$$

where

$$\hat{d}_\alpha(\mathbf{r},\mathbf{p})=R_{\alpha i}(\mathbf{r})\hat{p}_i\ . \tag{12}$$

$b(\mathbf{r},\mathbf{p})$ and $c(\mathbf{r},\mathbf{p})$ are two real functions for which we do not need to assume any relation. As we will see, it is necessary to introduce the above nonunitary first-order correction to the order parameter induced by $\mathbf{H}$, in order to achieve a nonzero polarization in the liquid. This correction has necessarily the above form if one requires the triplet part of ψ to stay in the p-wave manifold: one must build from $\mathbf{H}$ and $\hat{d}$ a spin vector first order in $\mathbf{H}$ and linear in $\mathbf{p}$. Clearly, the only choice is $\hat{d}\times\mathbf{H}$. The function $c(\mathbf{r},\mathbf{p})$ must be real if the triplet part has the same transformation law under time reversal as in the homogeneous case. The form of the density matrix we find with this term is in agreement with the result of other methods. It can be checked that an imaginary part in $c(\mathbf{r},\mathbf{p})$ would not affect the mass current to first order in $\mathbf{H}$, but the spin current would be modified.

The singlet correction $S(\mathbf{r},\mathbf{p})$ is required in order to avoid singularities in the density matrix at the Fermi surface. It is first order in gradients and is already present in zero magnetic field. Therefore it is not directly related to our problem. We will see that it plays actually no role and we keep it for consistency only. Finally, by taking $b(\mathbf{r},\mathbf{p})$ real we discard the possibility of local phase changes, which are of no interest to us. It would be easy to handle them and they would lead to the $\rho\mathbf{v}^S$ term in the mass current and the $m_\alpha\mathbf{v}^S$ term in the spin current, both of which result directly from Galilean invariance.

We now solve Eq. (5) to first order in $\mathbf{H}$ and in gradients with an order parameter given by Eq. (11). We set

$$\bar{\rho}=\bar{\rho}_0+\bar{\rho}_1\ , \tag{13}$$

where ρ_0 is zeroth order and ρ_1 first order in gradients. To zeroth order in gradients, we obtain

$$\bar{\rho}_0-\bar{\rho}_0^2=\psi_0\psi_0^\dagger\ , \tag{14}$$

which gives, to first order in $\mathbf{H}$,[6]

$$\rho_0-\rho_0^2=b^2\ , \tag{15}$$

$$\mathbf{M}_0(1-2\rho_0)=2bc\hat{d}\times(\hat{d}\times\mathbf{H})\equiv 2\mathbf{V}\ . \tag{16}$$

We have expressed the matrix $\bar{\rho}$ in terms of the Pauli matrices ($\sigma_1,\sigma_2,\sigma_3$ or $\sigma_x,\sigma_y,\sigma_z$) and the 2×2 identity matrix σ_0 as

$$\bar{\rho}=\sigma_0\rho+\boldsymbol{\sigma}\cdot\mathbf{M}\ , \tag{17}$$

and dropped explicit dependence on $\mathbf{r},\mathbf{p}$ for simplicity. Note here the need of a nonunitary order parameter to obtain $\mathbf{M}_0\neq\mathbf{0}$. To first order in gradients, Eq. (5) gives

$$\bar{\rho}_1-\bar{\rho}_0\bar{\rho}_1-\bar{\rho}_1\bar{\rho}_0-\tfrac{1}{2}i[\bar{\rho}_0,\bar{\rho}_0]=\tfrac{1}{2}i[\psi_0,\psi_0^+]+\psi_0\psi_1^++\psi_1\psi_0^+\ . \tag{18}$$

By expressing $\bar{\rho}_0$ as in Eq. (17), it is easily seen that only $[\mathbf{M}_0\cdot\boldsymbol{\sigma},\ \mathbf{M}_0\cdot\boldsymbol{\sigma}]$ contributes to $[\bar{\rho}_0,\bar{\rho}_0]$, but this is second order in H and therefore $[\bar{\rho}_0,\bar{\rho}_0]$ drops out. Defining

$$\tfrac{1}{2}i[\psi_0,\psi_0^+]=\sigma_0A+\boldsymbol{\sigma}\cdot\mathbf{B}\ , \tag{19}$$

we have, to first order in $\mathbf{H}$,

$$A=[b\hat{d}_\alpha,\ c(\hat{d}\times\mathbf{H})_\alpha]\ ,B_\alpha=-\tfrac{1}{2}\epsilon_{\alpha\beta\gamma}[b\hat{d}_\beta,b\hat{d}_\gamma]\ . \tag{20}$$

With these definitions, Eq. (18) becomes

$$\begin{aligned}&\rho_1(1-2\rho_0)-2\mathbf{M}_1\cdot\mathbf{M}_0=A\ ,\\&\mathbf{M}_1(1-2\rho_0)-2\rho_1\mathbf{M}_0=\mathbf{B}+2b\hat{d}\,\mathrm{Re}S+2c\hat{d}\times\mathbf{H}\,\mathrm{Im}S\ .\end{aligned} \tag{21}$$

Since A and $\mathbf{M}_0$ are first order in $\mathbf{H}$, so is ρ_1 from the first equation. This makes the term $\rho_1\mathbf{M}_0$ second order in the second equation and we can neglect it. From Eq. (21) we have

$$\begin{aligned}&\hat{d}\cdot\mathbf{M}_1(1-2\rho_0)=\hat{d}\cdot\mathbf{B}+2b\,\mathrm{Re}S\ ,\\&\hat{d}\cdot\mathbf{B}=-\tfrac{1}{2}\epsilon_{\alpha\beta\gamma}\hat{d}_\alpha[b\hat{d}_\beta,b\hat{d}_\gamma]=-\tfrac{1}{2}\epsilon_{\alpha\beta\gamma}b^2d_\alpha[\hat{d}_\beta,\hat{d}_\gamma]\ .\end{aligned} \tag{22}$$

At the Fermi surface, $\rho_0=\frac{1}{2}$ and $\hat{d}\cdot\mathbf{B}\neq 0$, and $\hat{d}\cdot\mathbf{M}_1$ would diverge if we had not the compensating term $2b\,\mathrm{Re}S$. Now if we follow Mermin and Muzikar[7] and assume that the pair wave function retains the B-phase form, we have[8]

$$S=\tfrac{1}{2}\rho_0b\epsilon_{\alpha\beta\gamma}\hat{d}_\alpha[\hat{d}_\beta,\hat{d}_\gamma]\ . \tag{23}$$

This makes $\mathrm{Im}S=0$ and gives

$$\hat{d}\cdot\mathbf{M}_1=-\tfrac{1}{2}b^2\epsilon_{\alpha\beta\gamma}\hat{d}_\alpha[\hat{d}_\beta,\hat{d}_\gamma]\ . \tag{24}$$

This term can be seen to give in the spin current a correction of order T_c/E_F to terms with the same symmetry, and therefore we will omit it from now on. Note that this term is slightly different from the analogous one in kinetic theory, where the gap is assumed to retain the B-phase form. This one gives a $(T_c/E_F)^2$ correction.

We are left with the component $\mathbf{M}_1^\perp$ of $\mathbf{M}_1$ perpendicular to $\hat{d}$, and, since $\mathbf{M}_0$ is also perpendicular to $\hat{d}$, Eq. (21) simplifies into

$$\rho_1(1-2\rho_0)-2\mathbf{M}_1^\perp\cdot\mathbf{M}_0=A\ ,\ \mathbf{M}_1^\perp(1-2\rho_0)=\mathbf{B}^\perp\ . \tag{25}$$

We now define two Hermitian operators D and C as

$$D=b\hat{d}\cdot\boldsymbol{\sigma}\ ,\quad C=c(\hat{d}\times\mathbf{H})\cdot\boldsymbol{\sigma}\ . \tag{26}$$

They are related to ψ_0 by

$$\psi_0=(D+iC)i\sigma_y\ . \tag{27}$$

By comparison with Eq. (19), we have

$$\sigma_0 A=\tfrac{1}{2}([D,C]+\text{H.c.}),\quad \boldsymbol{\sigma}\cdot\mathbf{B}=\tfrac{1}{2}i[D,D]\ . \tag{28}$$

On the other hand, Eqs. (15) and (16) read, in terms of D and C,

$$\begin{gathered}\rho_0-\rho_0^2=D^2\ ,\\ \boldsymbol{\sigma}\cdot\mathbf{M}_0(1-2\rho_0)=-2iDC=2iCD=2\boldsymbol{\sigma}\cdot\mathbf{V}\ ,\end{gathered} \tag{29}$$

which, after a derivation ∂ with respect to $\mathbf{r}$ or $\mathbf{p}$, gives

$$\partial\rho_0(1-2\rho_0)=\partial D^2,\quad \partial\mathbf{M}_0(1-2\rho_0)-2\partial\rho_0\mathbf{M}_0=2\partial\mathbf{V}\ . \tag{30}$$

Bearing in mind these relations, we now try to introduce in the Poisson brackets, Eq. (28), the operators D^2 and CD. This strategy will reveal a nice solution of Eq. (25). To do this we use the standard identity between Poisson brackets,

$$[fg,h]-[f,gh]=f[g,h]-[f,g]h\ . \tag{31}$$

Applying it a first time for $f=g=h=D$, we obtain

$$[D^2,D]-[D,D^2]=D[D,D]-[D,D]D\ ,$$

which, in view of the scalar character of the operator D^2, simplifies into

$$2[D^2,\hat{D}]=\hat{D}[D,D]-[D,D]\hat{D}\ , \tag{32}$$

where $\hat{D}$ is the unitary operator defined by

$$\hat{D}=\boldsymbol{\sigma}\cdot\hat{d}\quad(\text{with }\hat{D}\hat{D}^\dagger=\sigma_0)\ . \tag{33}$$

This relation is equivalent to

$$[D^2,\hat{d}]=2\hat{d}\times\mathbf{B} \tag{34}$$

or

$$2B_\alpha^\perp=\epsilon_{\alpha\beta\gamma}\hat{d}_\beta[\hat{d}_\gamma,D^2]\ . \tag{35}$$

Then we apply a second time the identity Eq. (31) for $f=h=D$ and $g=C$ and obtain

$$[DC,D]-[D,CD]=D[C,D]-[D,C]D\ , \tag{36}$$

or, equivalently,

$$([DC,D]+\text{H.c.})=(D[C,D]+\text{H.c.})\ . \tag{37}$$

Setting $\text{Herm}(A)=\frac{1}{2}(A+\text{H.c})$, we deduce from Eq. (36) the relation

$$\text{Herm}(D\,\text{Herm}([DC,D]))=D^2\text{Herm}([C,D])\ , \tag{38}$$

where we use the fact that D^2 and $\text{Herm}([C,D])$ are scalars.

This is equivalent to

$$b^2A=\epsilon_{\alpha\beta\gamma}b\hat{d}_\alpha[b\hat{d}_\beta,V_\gamma]=\epsilon_{\alpha\beta\gamma}b^2d_\alpha[\hat{d}_\beta,V_\gamma] \tag{39}$$

(where in the last step we have made use of elementary Poisson-bracket algebra).

Making use of Eq. (30), we obtain

$$\begin{gathered}2A=(1-2\rho_0)\epsilon_{\alpha\beta\gamma}\hat{d}_\alpha[\hat{d}_\beta,M_{0\gamma}]-2M_{0\gamma}\epsilon_{\alpha\beta\gamma}\hat{d}_\alpha[\hat{d}_\beta,\rho_0]\ ,\\ 2B_\alpha=(1-2\rho_0)\epsilon_{\alpha\beta\gamma}\hat{d}_\beta[\hat{d}_\gamma,\rho_0]\ .\end{gathered} \tag{40}$$

Comparing with Eq. (25), we obtain, by inspection, that

$$\rho_1=\tfrac{1}{2}\epsilon_{\alpha\beta\gamma}\hat{d}_\alpha[\hat{d}_\beta,M_{0\gamma}],\quad M_{1\alpha}^\perp=\tfrac{1}{2}\epsilon_{\alpha\beta\gamma}\hat{d}_\beta[\hat{d}_\gamma,\rho_0]\ , \tag{41}$$

which can be written as

$$\bar{\rho}_1=-\tfrac{1}{2}\text{Herm}(i\hat{D}[\hat{D},\bar{\rho}_0])\ . \tag{42}$$

It is interesting to note at this stage that the density part ρ_0 does not appear anymore in ρ_1. A consequence of this striking feature is that the supercurrent will be expressible in terms of the magnetization, its gradients, and the gradients of the rotation matrix $R_{\alpha i}$ only.

In order to compute the currents, we write explicitly the Poisson brackets in Eq. (41). The resulting expression can be rewritten as

$$\bar{\rho}_1=-\tfrac{1}{2}\text{Herm}\left\{i\hat{D}\left[\frac{\partial}{\partial p_j}\left[\frac{\partial\hat{D}}{\partial r_j}\bar{\rho}_0\right]-\frac{\partial}{\partial r_j}\left[\frac{\partial\hat{D}}{\partial p_j}\bar{\rho}_0\right]\right]\right\}\ . \tag{43}$$

When we insert these expressions in the formulas for the currents [here, $\int$ is for $2\int d^3p/(2\pi)^3$ and we set $m=1$],

$$\bar{g}_i=\int p_i\bar{\rho}_1\ , \tag{44}$$

and integrate by parts, we obtain

$$\bar{g}_i=\tfrac{1}{2}\text{Herm}\left[i\left[\frac{\partial}{\partial r_j}\int p_i\hat{D}\frac{\partial\hat{D}}{\partial p_j}\bar{\rho}_0+\int\hat{D}\frac{\partial\hat{D}}{\partial r_i}\bar{\rho}_0-\int p_i[\hat{D},\hat{D}]\bar{\rho}_0\right]\right]\ . \tag{45}$$

In order to calculate these various contributions explicitly, it is convenient to use

$$p_i=pR_{\alpha i}\hat{d}_\alpha,\quad \frac{\partial\hat{d}_\alpha}{\partial p_j}=\frac{1}{p}(\delta_{\alpha\beta}-\hat{d}_\alpha\hat{d}_\beta)R_{\beta j}\equiv\frac{\delta_{\alpha\beta}^\perp R_{\beta j}}{p}\ . \tag{46}$$

Also, ρ_0 depends isotropically on $\mathbf{p}$, while, from Eq. (16),

$$M_{0\alpha}=(\delta_{\alpha\beta}-\hat{d}_\alpha\hat{d}_\beta)h_\beta-\delta^\perp_{\alpha\beta}h_\beta \ , \tag{47}$$

where h_β is isotropic in $\mathbf{p}$. Performing the angular average, we have, for the mass density ρ and the magnetization density m_α,

$$\rho=\int\rho_0, \quad m_\alpha=\tfrac{2}{3}\int h_\alpha \ . \tag{48}$$

The first term in Eq. (45) gives to the mass current a contribution

$$\tfrac{1}{2}\epsilon_{\alpha\beta\gamma}\frac{\partial}{\partial r_j}\int p_i M_{0\alpha}\frac{\partial\hat{d}_\beta}{\partial p_j}\hat{d}_\gamma=\tfrac{1}{2}\epsilon_{\alpha\beta\gamma}\frac{\partial}{\partial r_j}R_{\lambda_i}R_{\mu j}\int\hat{d}_\lambda\hat{d}_\gamma\delta^\perp_{\beta\mu}\delta^\perp_{\alpha\nu}h_\nu$$

$$=\tfrac{1}{2}\epsilon_{\alpha\beta\gamma}\frac{\partial}{\partial r_j}R_{\lambda i}R_{\beta j}\int\hat{d}_\lambda\hat{d}_\gamma h_\alpha=-\tfrac{1}{4}\epsilon_{ijk}\frac{\partial}{\partial r_j}(R_{\alpha k}m_\alpha) \ , \tag{49}$$

where we have made use in the second step of the antisymmetry of $\epsilon_{\alpha\beta\gamma}$ and taken advantage in the last step of the identity

$$\epsilon_{\alpha\beta\gamma}R_{\alpha i}R_{\beta j}R_{\gamma k}=\epsilon_{ijk} \ . \tag{50}$$

The second term in Eq. (45) contributes to the mass current as

$$\tfrac{1}{2}\epsilon_{\alpha\beta\gamma}\int M_{0\alpha}\frac{\partial\hat{d}_\beta}{\partial r_i}\hat{d}_\gamma=\tfrac{1}{2}\epsilon_{\alpha\beta\gamma}\frac{\partial R_{\beta j}}{\partial r_i}R_{\lambda j}\int\hat{d}_\lambda\hat{d}_\gamma\delta^\perp_{\alpha\mu}h_\mu=v^S_{i\alpha}m_\alpha \ , \tag{51}$$

where the spin superfluid velocity is defined by

$$v^S_{i\alpha}=\tfrac{1}{4}\epsilon_{\alpha\beta\gamma}\frac{\partial R_{\beta j}}{\partial r_i}R_{\gamma j} \ . \tag{52}$$

Finally, the last term in Eq. (45) gives a contribution

$$\epsilon_{\alpha\beta\gamma}\int p_i M_{0\alpha}\frac{\partial\hat{d}_\beta}{\partial r_j}\frac{\partial\hat{d}_\gamma}{\partial p_j} \ , \tag{53}$$

which is zero because the three vectors $\mathbf{M}$, $\partial\hat{d}/\partial r_j$, and $\partial\hat{d}/\partial p_j$ are perpendicular to $\hat{d}$, which makes their mixed product vanish. The two terms (49) and (51) for the mass current give a result identical to our former expression, Eq. (1).

The spin current is obtained along the same lines. The first term in Eq. (45) gives, similar to Eq. (49),

$$\tfrac{1}{2}\epsilon_{\alpha\beta\gamma}\frac{\partial}{\partial r_j}\int p_i\rho_0\frac{\partial\hat{d}_\beta}{\partial p_j}\hat{d}_\gamma=-\tfrac{1}{6}\epsilon_{ijk}\frac{\partial}{\partial r_j}(\rho R_{\alpha k})=-\tfrac{1}{6}\epsilon_{ijk}\left[R_{\alpha k}\frac{\partial\rho}{\partial r_j}+\rho\frac{\partial R_{\alpha k}}{\partial r_j}\right] \ . \tag{54}$$

The result for the second term is completely analogous to Eq. (51):

$$\tfrac{1}{2}\epsilon_{\alpha\beta\gamma}\int\rho_0\frac{\partial\hat{d}_\beta}{\partial r_i}\hat{d}_\gamma=\tfrac{2}{3}\rho v^S_{i\alpha} \ . \tag{55}$$

Finally, the last term leads to

$$\epsilon_{\alpha\beta\gamma}\int p_i\rho_0\frac{\partial\hat{d}_\beta}{\partial r_j}\frac{\partial\hat{d}_\gamma}{\partial p_j}=\epsilon_{\alpha\beta\gamma}R_{\lambda i}R_{\nu j}\frac{\partial R_{\beta k}}{\partial r_j}R_{\mu k}\int\hat{d}_\lambda\hat{d}_\mu\delta^\perp_{\gamma\nu}\rho_0$$

$$=2\rho\epsilon_{\alpha\beta\gamma}\epsilon_{\sigma\beta\mu}R_{\lambda i}R_{\nu j}v^S_{j\sigma}[\tfrac{4}{15}\delta_{\lambda\mu}\delta_{\gamma\nu}-\tfrac{1}{15}(\delta_{\gamma\lambda}\delta_{\mu\nu}+\delta_{\lambda\nu}\delta_{\gamma\mu})]$$

$$=\frac{2\rho}{15}v^S_{i\alpha}+\frac{2\rho}{15}(R_{\alpha j}R_{\beta i}-4R_{\alpha i}R_{\beta j})v^S_{\beta j} \ . \tag{56}$$

The second term in Eq. (54) can be rewritten as

$$-\frac{\rho}{6}\epsilon_{ijk}\frac{\partial R_{\alpha k}}{\partial r_j}=\frac{\rho}{3}(R_{\alpha i}R_{\beta j}-R_{\beta i}R_{\alpha j})v^S_{j\beta} \tag{57}$$

by making use of Eqs. (50) and (54). When we combine Eqs. (54)—(57), we obtain, for the spin current,

$$g_{i\alpha}=\frac{4\rho}{5}v^S_i-\frac{\rho}{5}(R_{\alpha i}R_{\beta j}+R_{\beta i}R_{\alpha j})v^S_{j\beta}-\tfrac{1}{6}\epsilon_{ijk}R_{\alpha k}\partial_j\rho \ , \tag{58}$$

in agreement with our earlier results

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[1]R. Combescot and T. Dombre, Phys. Lett. **76A**, 293 (1980).

[2]V. P. Mineev and G. E. Volovik, in Proceedings of the 17th International Conference on Low Temperature Physics [Physica (1984)].

[3]P. Muzikar, in Proceedings of the 17th International Conference on Low Temperature Physics [Physica (1984)].

[4]See R. Combescot [J. Phys. C **14**, 4765 (1981)] for the details of our calculation.

[5]N. D. Mermin and P. Muzikar, Phys. Rev. B **21**, 980 (1980).

[6]Here we make the natural assumption that the density matrix in a BCS phase is a smooth function at zero temperature, even in the presence of a magnetic field, as long as the energy associated with this field is much smaller than the magnitude of the gap. This means, in particular, that M_0 should be of first order in H, from which Eqs. (15) and (16) are easily deduced. Note that the results, which are obtained in this way to zeroth order in gradients, are in agreement with those given by more standard calculations.

[7]See especially Appendix C, Eq. (C7) [of N. D. Mermin and P. Muzikar, Phys. Rev. B **21**, 980 (1980)] [a factor i is missing and the sign should be changed in Eq. (C6)].

[8]This kind of term may also be revealed by an expansion of the Gor'kov's equations where one assumes a pure *B*-phase structure for the gap and then solves for the order parameter in terms of the gap to first order in gradients. For this purpose it is essential to take into account terms coming from the **p** dependence of the gap (see, for example, Ref. 4). The precise value of the coefficient b in Eq. (23) depends, however, on the hypothesis which is made, whether this is the gap or the pair wave function which retains the *B*-phase form.