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Ferromagnetic and transmission resonance in magnetic metal superlattices

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We have developed a mathematical formulation by which surface-impedance calculations of magnetic layered structures are significantly simplified and, hence, calculable. A transfer-function matrix is introduced in order to relate the electric and magnetic fields at one surface of a magnetic metal layer to fields at the other surface. The surface impedance of a layered structure is then expressed in terms of the four transfer-function matrix elements corresponding to surface fields of the layered structure instead of internal microwave fields corresponding to *all* the layers within the layered structure. The ferromagnetic resonance fields and line shapes are calculated for layered structures containing alternating dielectric and iron magnetic layers. In addition, spin-pinning boundary conditions are assumed at the surfaces of each iron layer. The calculations are of sufficient generality so that the insulating dielectric layer may be replaced by a conductive metal layer. We find that standing spin-wave resonance may be excited in the layered structure, even for zero spin pinning at the iron-layer surfaces. From calculations of internal microwave fields within each layer, we deduce that the spin-wave resonance is due to an asymmetrical magnetic field excitation of each iron layer, although the layered structure as a whole is symmetrically excited. For iron-layer thicknesses of ≥ 1500 Å there is no discernible difference between the ferromagnetic resonance fields of the main line and the first-spin-wave—resonance mode—an accidental degeneracy. If nonzero spin pinning is assumed at each iron-layer surface, the accidental degeneracy is removed.

I. INTRODUCTION

With the advent of the so-called magnetic superlattice it may now be possible to study properties of artificial, periodic layered material structures whose separation between magnetic layers may be on the order of intra-atomic distances. Magnetic superlattices are defined as periodic layered structures with alternating layers having different magnetic and/or electrical properties. In order to maintain periodicity in the layered structure the composition and thickness of each layer is fixed. Precise periodic layered structures have been constructed[1] and this could only be realized with recent improvements in metalization or material-evaporation techniques.

Magnetic properties of layered structures are expected to be different from properties of a single magnetic layer. This difference may be accounted for by the fact that neighboring layers couple to each other via the magnetostatic[2] and exchange[3] interactions. In a microwave environment as we are considering, the internal microwave fields at a point in the layered structure are the result of many electromagnetic interference effects caused by all the layers within the layered structure and not simply by interference from a particular layer containing the point of interest. Hence, the microwave or dynamic properties of a single layer are expected to be different from the properties of layered structures.

Of the many analytical techniques by which one may characterize the microwave properties of magnetic layered structures, the technique of ferromagnetic resonance (FMR) still remains a powerful analytical tool in exploring new microwave properties of such structures. It is the intent of this paper to develop a FMR analysis appropriate to periodic layered magnetic structures. Basically there are two mathematical approaches by which FMR fields and line shapes may be calculated. The first[4] approach extends previous magnetic-metal-film calculations[5] to the magnetic layered problem. This approach[5] matches the number of boundary-surface condition equations to the number of internal-field amplitudes. The surface impedance, Z_s, may then be expressed in terms of internal-field amplitudes. In these calculations the surface impedance is calculated,[5] since this quantity is measured directly in a FMR experiment.

In this paper we develop an alternative approach which is more suitable to the calculation of Z_s for layered structures. We introduce a transfer-function matrix which relates the microwave fields at the two surfaces of a given magnetic layer. Heaviside introduced the concept of transfer function to be associated with linear systems containing one allowable electromagnetic wave-propagation constant. We extend this concept by defining a transfer-function matrix for linear systems which can allow many modes of electromagnetic wave propagation as in a magnetic medium,[6] for example. Clearly, as in Heaviside theorem the *total* transfer-function matrix of a layered structure is the product of all the single-layer transfer-function matrices. The *total* transfer-function matrix is a (2×2) matrix, since the surface fields are two dimensional, electric and magnetic microwave fields. Z_s is expressed in terms of the four matrix elements corresponding to the *total* transfer-function matrix of the layered structure.

Specifically, we consider the following problem: We assume normal incidence of an electromagnetic wave upon a symmetric periodic magnetic layered structure composed

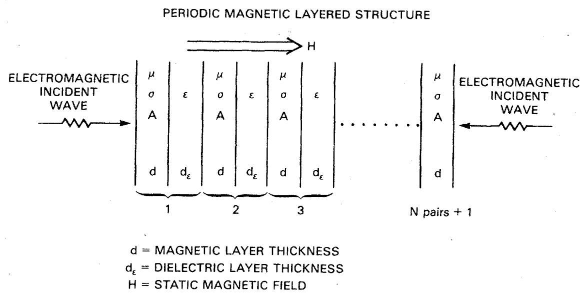

FIG. 1. Periodic magnetic layered structure is composed of alternating magnetic and dielectric layers. The magnetic layer is characterized by conductivity σ, exchange stiffness constant A, and permeability μ. The dielectric layer is characterized by the dielectric constant ϵ.

of iron/dielectric/iron/dielectric/iron layers. The external static magnetic field, H, is applied normal to the plane of the layered structure, see Fig. 1. Each iron layer was assumed to be 1000 Å thick. It is further assumed that the dielectric layer is characterized by a scalar dielectric constant rather than a tensor quantity. This means that the calculations are valid at relatively low microwave frequencies ($\lesssim 100$ GHz). The dielectric thickness was varied between 100 and 10 000 Å. For these separation distances the exchange coupling between iron layers is very small in comparison to surface magnetic anisotropy energies.[7] The magnetostatic energy coupling between layers is also ignored, since there is no microwave magnetization component normal to the plane of the layered structure. The magnetization M, exchange-stiffness constant A, g value, and the conductivity σ of iron are well known in the literature[8] and we assume those values. Spin-pinning boundary conditions at the surfaces of each iron layer are also included in this analysis.

We predict the excitation of spin-wave spectra, even for zero spin pinning of surface spins at each iron layer. The spin-wave excitation-mode intensity increased as the number of iron layers was increased. Internal microwave field solutions reveal that the microwave component of the magnetization, m, was asymmetrically excited within each iron layer, so that a net microwave absorption was possible at magnetic fields below the resonant field of the main FMR mode. In addition, we have calculated a transmission maximum at fields below the FMR field of the main line. This is a consequence of the fact that the imaginary part of the characteristic propagation constant of the layered structure exhibited a minimum (when plotted as a function H) at transmission resonance. In Sec. II the mathematical formalism is developed. In Secs. III and IV computer numerical results and discussion of the calculations are presented.

II. THEORETICAL FORMALISM

We require a transformation which expresses the internal fields of each layer in terms of the microwave electric and magnetic fields at the two surfaces. It is then mathematically convenient to define a matrix transfer function which relates the microwave surface fields at the two surfaces. The relationship between the microwave surface fields and internal fields for H normal to a magnetic-metal-film plane are given below. For detailed derivation of these relationships see Ref. 5. They are as follows:

$$e_{s1}=\sum_{n=1}^{2} Z_n(h_n^+-h_n^-)\,, \tag{1a}$$

$$h_{s1}=\sum_{n=1}^{2} (h_n^+ + h_n^-)\,, \tag{1b}$$

$$e_{s2}=\sum_{n=1}^{2} Z_n(h_n^+ e^{-jk_n d}-h_n^- e^{+jk_n d})\,, \tag{1c}$$

$$h_{s2}=\sum_{n=1}^{2} (h_n^+ e^{-jk_n d}+h_n^- e^{+jk_n d})\,, \tag{1d}$$

$$0=\sum_{n=1}^{2} (P_n h_n^+ + R_n h^-)\,, \tag{1e}$$

$$0=\sum_{n=1}^{2} (R_n h_n^+ e^{-jk_n d}+P_n h_n^- e^{+jk_n d})\,. \tag{1f}$$

The four surface fields of the metallic magnetic film or layer are e_{s1}, h_{s1}, e_{s2}, and h_{s2}, where the subscript 1 denotes one surface and 2 the other. The film thickness is d. Equations (1a)–(1d) represent the electromagnetic field continuity equations at the two surfaces. Equations (1e) and (1f) represent the spin-pinning boundary[8] conditions. Equations (1e) and (1f) may be derived from the pinning-boundary condition:[8]

$$A\frac{\partial m}{\partial y}+K_s m=0\,, \tag{2}$$

where K_s is the surface uniaxial magnetic anisotropy energy parameter, m is the microwave component of the magnetization, and y is the coordinate normal to the film plane. It can be shown[5] from the application of Eq. (2) that

$$P_n=Q_n(K_s+jk_n A)$$

and

$$R_n=Q_n(K_s-jk_n A)\,,$$

where[5]

$$Q_n=\frac{1}{4\pi}\left[1-\frac{j}{2}\delta_0^2 k_n^2\right]$$

and

$$\delta_0^2=\frac{c^2}{2\pi\sigma\omega}\,.$$

The Gaussian system of units is used.

By coupling Maxwell's equations and the Landau-Lifshitz equation of motion for the magnetization there results a secular equation which is quartic[5] in k^2, where k is the propagation constant. It can be shown that for H normal to a film plane there correspond only two[5] magnetically resonant modes or allowable electromagnetic wave-propagation constants, k_n. For each k_n (where $n=1,2$) mode there corresponds a forward (+) and a negatively ($-$) traveling electromagnetic wave and two internal magnetic fields ($h_n^{\pm}$). Hence, there are four internal-field amplitudes. The two allowed propagation constants k_n are related to σ, H, M, A, ... via the secular equation (see Ref. 5). Also, for each k_n there corresponds a characteristic impedance, Z_n,

$$Z_n = j\frac{k_n}{4\pi\sigma/c} ,$$

where c is the velocity of light and $j=\sqrt{-1}$. The internal microwave fields as well as the surface fields are circularly polarized.[5]

Using Eqs. (1c)—(1d) $h_n^{\pm}$ may be solved in terms of e_{s2} and h_{s2}. Thus, we may write

$$h_n^{\pm} = \alpha_n^{\pm} e_{s2} + \beta_n^{\pm} h_{s2} , \tag{3}$$

where $n=1,2$ and $\alpha_n^{\pm}$ and $\beta_n^{\pm}$ may be expressed in terms of magnetic parameters P_n, R_n, Q_n, etc. (see the Appendix). By substituting the solutions for $h_n^{\pm}$ into Eqs. (1a) and (1b), e_{s1} and h_{s1} may then be related to e_{s2} and h_{s2}. Finally, we may write

$$\begin{bmatrix} e_{s1} \\ h_{s1} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} e_{s2} \\ h_{s2} \end{bmatrix} \equiv \underline{a} \begin{bmatrix} e_{s2} \\ h_{s2} \end{bmatrix} ,$$

where

$$a_{11} = \sum_{n=1}^{2} Z_n(\alpha_n^{+} - \alpha_n^{-}) ,$$

$$a_{12} = \sum_{n=1}^{2} Z_n(\beta_n^{+} - \beta_n^{-}) ,$$

$$a_{21} = \sum_{n=1}^{2} (\alpha_n^{+} + \alpha_n^{-}) ,$$

and

$$a_{22} = \sum_{n=1}^{2} (\beta_n^{+} + \beta_n^{-}) .$$

The matrix $\underline{a}$ is defined as the transfer-function matrix of a single magnetic metal layer. Similarly, one may write

$$\begin{bmatrix} e_{s2} \\ h_{s2} \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} e_{s1} \\ h_{s1} \end{bmatrix} \equiv \underline{b} \begin{bmatrix} e_{s1} \\ h_{s1} \end{bmatrix} .$$

The matrices $\underline{a}$ and $\underline{b}$ obey the following algebraic properties:

$$\underline{a}\,\underline{b} = \underline{I} ,$$

where $\underline{I}$ is the unit matrix. Thus, $\underline{b}=\underline{a}^{-1}$. Making use of the identities $a_{11}a_{22}-a_{12}a_{21}=1$ and $b_{11}b_{22}-b_{12}b_{21}=1$ which are special properties of linear passive systems, it can be shown that

$$b_{11} = a_{22} ,$$

$$b_{22} = a_{11} ,$$

$$b_{12} = -a_{12} ,$$

and

$$b_{21} = -a_{21} .$$

For the dielectric layer only one electromagnetic wave-propagation constant is allowed, k_ϵ, so that one may make use of classical electromagnetic theory results for the derivation of the transfer-function matrix of the dielectric layer:

$$\underline{a}' = \begin{bmatrix} \cos(k_\epsilon d_\epsilon) & jZ_\epsilon \sin(k_\epsilon d_\epsilon) \\ \dfrac{j\sin(k_\epsilon d_\epsilon)}{Z_\epsilon} & \cos(k_\epsilon d_\epsilon) \end{bmatrix} ,$$

where $k_\epsilon = (\omega/c)\sqrt{\epsilon}$, $Z_\epsilon = \sqrt{1/\epsilon}$, and the dielectric layer thickness is d_ϵ. The above relations hold for linear as well as circularly polarized internal fields.

Let us now define the "pair" transfer-function matrix $\underline{C}$ of the metallic magnetic and dielectric layers as the product of their respective transfer-function matrices. Thus, we write

$$\underline{C} \equiv \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} = \underline{a}\,\underline{a}' .$$

A. Transfer-function matrix of periodic layered structure

Assuming the thickness of each magnetic and dielectric layer and their respective composition to be constant, the total transfer-function matrix may be written as

$$\underline{A} = \underline{C}^N , \tag{4}$$

since the *total* transfer-function matrix $\underline{A}$ is obtained by multiplying the "pair" matrix $\underline{C}$ N times, where N is the total number of magneto-dielectric pair layers. Equation (4) occurs quite often in the field of solid-state physics. Standard mathematical operations to Eq. (4) have been applied in order to reduce the number of algebraic steps. We briefly outline the procedure here.

A unitary matrix transformation $\underline{U}$ is introduced so that Eq. (4) may be rewritten as follows:

$$\underline{A} = U(U^{-1}CUU^{-1}CUU^{-1}C \cdots CU)U^{-1}$$
$$= U(D^N)U^{-1} ,$$

where

$$\underline{I} = UU^{-1}$$

and

$$\underline{D} = U^{-1}CU .$$

$\underline{D}$ is a diagonal matrix with eigenvalues $\lambda_{\pm}$.

$$\lambda_{\pm} = \frac{c_{11}+c_{22}}{2} \pm \tfrac{1}{2}[(c_{11}-c_{22})^2 + 4c_{12}c_{21}]^{1/2} .$$

The unitary matrices which diagonalize the pair matrix $\underline{C}$ are of the form (assuming no degeneracy)

$$\underline{U} = \begin{bmatrix} -c_{12} & -c_{12} \\ (\lambda_{-} - c_{22}) & (\lambda_{+} - c_{22}) \end{bmatrix} ,$$

$$\underline{U}^{-1} = \frac{-1}{(\lambda_{+}-\lambda_{-})c_{12}} \begin{bmatrix} (\lambda_{+}-c_{22}) & c_{12} \\ -(\lambda_{-}-c_{22}) & -c_{12} \end{bmatrix} .$$

Thus, $\underline{A}$ may be reduced to a single (2×2) matrix or simply

$$\underline{A}=\underline{U}\begin{bmatrix}\lambda_+^N & 0\\ 0 & \lambda_-^N\end{bmatrix}(\underline{U}^{-1}) . \tag{5}$$

Clearly, the $\underline{A}$ matrix relates the microwave surface electric e_{s1} and magnetic h_{s1} fields of the first iron layer to the surface electric (e_{sN}) and magnetic (h_{sN}) fields of the last dielectric layer. In order that we may consider a symmetrical layered structure, where the first and last layers are the same, the total transfer-function matrix is simply the product of $\underline{A}$ and $\underline{a}$ or

$$\underline{A}_T=\underline{A}\times\underline{a}=\begin{bmatrix}A_{11} & A_{12}\\ A_{21} & a_{22}\end{bmatrix} , \tag{6a}$$

where $\underline{a}$ is the (2×2) transfer-function matrix of a single iron layer which was previously defined. The subscript T is to denote a symmetric layered structure or the total transfer-function matrix. The inverse matrix $\underline{B}_T$ may simply be obtained from

$$\underline{B}_T=\underline{A}_T^{-1}=\begin{bmatrix}B_{11} & B_{12}\\ B_{21} & B_{22}\end{bmatrix} . \tag{6b}$$

$\underline{A}_T$ and $\underline{B}_T$ obey the same algebraic properties discussed previously for the single-layer case.

B. Connection to surface impedance

Assuming symmetrical electromagnetic wave excitation of the periodic layered structure, the surface fields may be expressed in terms of the matrix elements of $\underline{A}_T$ and $\underline{B}_T$. Applying the superposition principle to surface microwave fields, we have

$$e_{s1}=(1+\rho_1)h_{\mathrm{inc}(+)}+T_1h_{\mathrm{inc}(-)} , \tag{7a}$$

$$h_{s1}=(1-\rho_1)h_{\mathrm{inc}(+)}+T_2h_{\mathrm{inc}(-)} , \tag{7b}$$

$$e_{sL}=-(1+\rho_L)h_{\mathrm{inc}(-)}+T_3h_{\mathrm{inc}(+)} , \tag{7c}$$

$$h_{sL}=(1-\rho_L)h_{\mathrm{inc}(-)}+T_4h_{\mathrm{inc}(+)} . \tag{7d}$$

In the above expressions the surface fields are expressed in terms of the incident electromagnetic magnetic fields $h_{\mathrm{inc}(\pm)}$, where the subscripts $(\pm)$ indicate forward $(+)$ and negatively $(-)$ traveling incident waves. The incident electric field is related to $h_{\mathrm{inc}(\pm)}$ by $\pm Z_0$, where $Z_0=1$ for free space (in Gaussian units). The reflection coefficients at the first (1) and last (L) iron layers are defined as ρ_1 and ρ_L, respectively. T_1 and T_2 are the transmission coefficients due only to a negatively traveling incident wave and T_3 and T_4 due only to a forward incident traveling wave. The surface electric field on either surface is composed of the incident and reflected electric field components from one incident wave and transmitted electric field component due to the oppositively traveling incident wave.

In calculating ρ_1, T_3, and T_4 we normalize $h_{\mathrm{inc}(+)}$ to 1 and $h_{\mathrm{inc}(-)}$ to 0. The opposite is assumed, when ρ_L, T_1, and T_2 are calculated. After few algebraic operations the reflection and transmission coefficients may be expressed in terms of the matrix elements of $\underline{A}_T$ and $\underline{B}_T$. Hence,

$$\rho_1=[Z_{\mathrm{in}}(1)-1]/(Z_{\mathrm{in}}(1)+1) ,$$

$$\rho_L=[Z_{\mathrm{in}}(L)-1]/(Z_{\mathrm{in}}(L)+1) ,$$

$$T_1=(1+\rho_L)[B_{22}+B_{12}/Z_{\mathrm{in}}(L)] ,$$

$$T_2=(1+\rho_L)[B_{11}+B_{21}/Z_{\mathrm{in}}(L)] ,$$

$$T_3=(1+\rho_1)[A_{22}-A_{12}/Z_{\mathrm{in}}(1)] ,$$

$$T_4=(1+\rho_1)[A_{11}-A_{21}/Z_{\mathrm{in}}(1)] ,$$

where

$$Z_{\mathrm{in}}(1)=(A_{11}+A_{12})/(A_{21}+A_{22}) ,$$

$$Z_{\mathrm{in}}(L)=(B_{11}-B_{12})/(B_{22}-B_{21}) .$$

Finally, the surface impedance at the two surfaces of the layered structure are obtained form the ratio e_{s1}/h_{s1} and e_{sL}/h_{sL} or

$$Z_s(1)=\frac{(1+\rho_1)-(1+\rho_L)[B_{22}+B_{12}/Z_{\mathrm{in}}(L)]}{(1-\rho_1)+(1-\rho_L)[B_{11}+B_{21}/Z_{\mathrm{in}}(L)]} , \tag{8}$$

$$Z_s(L)=\frac{(1+\rho_L)-(1+\rho_1)[A_{22}-A_{12}/Z_{\mathrm{in}}(1)]}{(1-\rho_L)+(1-\rho_1)[A_{11}-A_{21}/Z_{\mathrm{in}}(1)]} . \tag{9}$$

A quick inspection of the above surface impedance expressions reveals that

$$Z_s(1)=-Z_s(L) ,$$

as it should be for the case of symmetrical excitation.

III. CALCULATIONAL RESULTS

Quantitative determination of Z_s require computer aid, since the complex algebra involved in the computations of Z_s can be tedious and laborious. One advantage of our mathematical formalism is that we have reduced the mathematical complexities of the periodic layered problem to manageable proportions. For example, we were able to numerically calculate Z_s for a layered structure containing 100 layers of iron and dielectric materials in very rapid computer execution time.

The following values of metallic iron parameters were assumed in the calculations: $M=1700$ G, $A=1.9\times10^{-6}$ ergs/cm, $g=2.09$, and $\sigma=1.42\times10^5$ mhos/cm, and the magnetic damping parameter,[8] $\lambda=10^7$ Hz. The surface uniaxial magnetic anisotropy parameter, K_s, was varied in the calculations, since it is strongly dependent on surface crystal symmetry as distorted by the dielectric layer. For the dielectric layer, $\epsilon=10-0.02j$. In FMR experiments usually the frequency is fixed and H is varied or swept through the resonance. Hence Z_s is calculated as a function of H, as in Fig. 2. The Lorentzian-shaped curve for $\mathrm{Re}(Z_s)$ is typical of a FMR line shape. The value of H at which $\mathrm{Re}(Z_s)$ is maximum is defined as the FMR field. The peak value itself is related to the resonance-mode intensity strength or simply related[5] to m. In Fig. 2, in which we take $K_s=0$, $\mathrm{Re}(Z_s)$ is plotted as a function of $(H_i-\omega/\gamma)$ is the operating frequency and is fixed at 35 GHz in this calculation. $H_i=\omega/\gamma$ or $H=4\pi M+\omega/\gamma$ is recognized as the resonance field condition for uniform spin precession.

For $N=1$ (the single iron layer) only one FMR mode is excited and the exchange-conductivity[5] shifts from

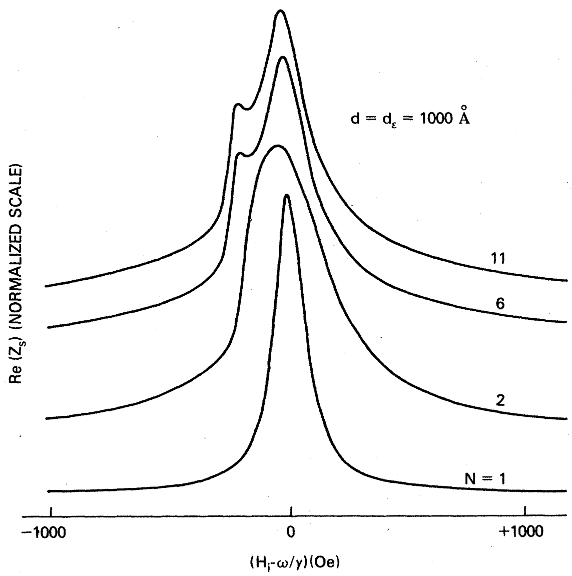

FIG. 2. Real part of the surface impedance Re(Z_s) is plotted as a function of $H_i - \omega/\gamma$. The vertical scale is conveniently normalized so that each curve can be overlayed with respect to each other for comparison purpose. $H_i = \omega/\gamma$ is the FMR field for uniform precession of magnetic moments. N represents the total number of iron dielectric layer pairs, $K_s = 0$.

$H_i = \omega/\gamma$ are negligible. As N is increased a subsidiary FMR excitation occurs for $H_i < \omega/\gamma$. The intensity of this subsidiary line or the peak value of Re(Z_s) increases with N. For $N \geq 11$, where N is the number of magnetodielectric layer pairs, the subsidiary resonance line intensity remains constant. This is understandable in view of the fact that the microwave fields attenuate considerably after about 5 iron layers.

In Fig. 3 the thickness of the iron layer was varied between 600 and 1500 Å in order to establish a dependence of the FMR field of the subsidiary line upon d. The dielectric thickness was again fixed at 1000 Å, $K_s = 0$, and $N = 11$. The calculated FMR line shapes as exhibited by Re(Z_s) are shown for various values of d. In order to compare the line shapes directly, Re(Z_s) was normalized for each calculation so that they could be superimposed. The separation (δH) between the subsidiary and the main FMR fields appears to be inversely proportional to d. In Fig. 4 δH is plotted as a function of d. We see that δH scales approximately with d^{-2} like the separation between the first two spin-wave modes excited in a thin film[5] ($N = 1$ case), with $K_s \neq 0$. Here we have taken $K_s = 1.0$ ergs/cm^2 at one surface and $K_s = 0$ at the other. However, there is a basic difference between the spin-wave structure excited in the layered case here and that of a thin film.[5] Whereas for the thin film case[5] there are n standing spin-wave resonance (SWR) modes, where n is the spin-wave order number, there are two and only two spin-wave modes excited in the layered case: the main FMR and subsidiary spin-wave modes.

The effect of alternating iron layers with dielectric layers is to give the appearance of an asymmetrical spin-pinning condition. This is not surprising in view of the fact that the microwave fields are considerably attenuated in each iron layer. This means that each individual iron layer is asymmetrically excited by the magnetic microwave field, although the layered structure is symmetrically excited externally, see Fig. 5. It is well known[5] in thin-film results that asymmetrical excitation gives rise to spin-wave excitation. In fact, the half-wavelength of m is approximately equal to the iron-layer thickness. There are no higher-order spin-wave-mode excitations as reflected in the plot of Re(Z_s), since the internal microwave

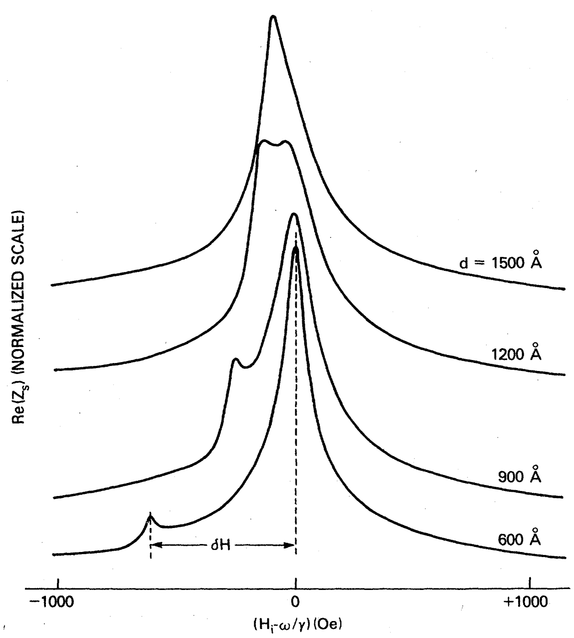

FIG. 3. Re(Z_s) is plotted as a function of $H_i - \omega/\gamma$ as in Fig. 2. In this plot $N = 11$ and the iron-layer thickness is varied from 600 to 1500 Å.

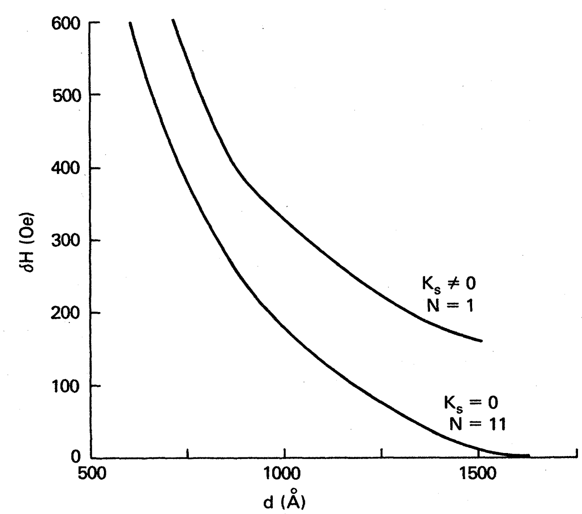

FIG. 4. FMR field expressed in $H_i - \omega/\gamma$ is plotted as a function of the iron-layer thickness for the first two mode excitations. $N = 1$ represents the thin-film calculation. For the single layer, $K_s = 1.0$ ergs/cm^2 at one surface and 0 at the other.

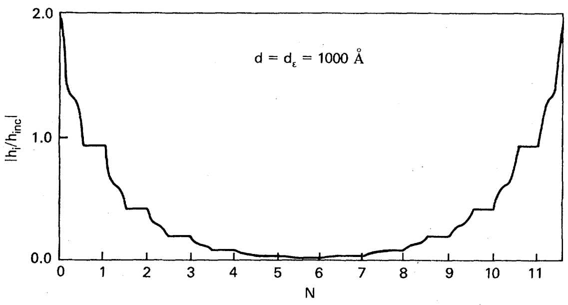

FIG. 5. Amplitude of the internal microwave magnetic field is plotted as a function of N. The incident microwave field is normalized to 1. The total thickness of the layered structure is $N(d+d_\epsilon)+d$ where N, d, and d_ϵ have been defined in the text.

field is not efficiently coupled to high-order spin-wave modes (see Fig. 5).

Another feature of the spin-wave excitation in the layered structure which contrasts with thin-film[5] SWR excitations is that for $d \geq 1500$ Å the FMR fields of the main and first spin-wave modes occur at the same value—an accidental degeneracy. Normally, for SWR excitation in a thin film,[5] the SWR fields of the first two modes are well separated (even up to thicknesses of 2—3000 Å). We find that this degeneracy is removed, if we assume nonzero spin pinning at the surfaces of each iron layer within the layered structure. For example, by assuming symmetrical spin-pinning condition with a typical K_s value of 1.0 ergs/cm^2 at both surfaces the difference in FMR fields between the main and first spin-wave mode is about 120 Oe for $d=1500$ Å.

We have also calculated transmission resonance for the same layered structure configuration as above. In the calculation for electromagnetic transmission efficiency we calculate the ratio of transmitted, h_t, magnetic field to the incident, H_{inc}, magnetic field, which is exactly the definition for either T_2 or T_4. The relation for T_2 was used in obtaining Fig. 6. For this calculation we assume the same parameter values as before. In particular, in Fig. 6 the transmitted amplitude of the microwave magnetic field is plotted as a function of $(H_i-\omega/\gamma)$. As we would expect the transmitted amplitudes are small $(\sim 10^{-6})$, since transmission is through many iron layers. For our particular example, $N=11$. Transmission resonance occurs for static magnetic fields below FMR of the main line.

This is a consequence of the fact that the $\text{Im}(K_c)$ is a minimum at transmission resonance (see Fig. 7), where K_c is the characteristic electromagnetic propagation constant of the layered structure and $\text{Im}(K_c)$ is a measure of propagation loss in the system. For example, $\text{Im}(K_c)$ is maximum at FMR. Thus, at FMR the amplitude of the transmitted electromagnetic magnetic field reaches a minimum. K_c is calculated by setting $A_{11}=A_{22}=\cos[K_c(2N+1)d]$. The off-diagonal elements of $\underline{A}_T$ may be related to the characteristic impedance, Z_c, as in the definition of $\underline{a}'$. Z_c is analogous to the Thevinin equivalent impedance which is often used to represent all of the source impedances in electrical circuits. In our case Z_c represents all of the natural characteristic impedances associated with each allowable electromagnetic wave-propagation constants in all the layers. There are no noticeable transmission resonances at static magnetic fields, where the so-called "subsidiary" spin-wave mode is excited, since $\text{Im}(K_c)$ exhibits no special structure at these field values. It is interesting to note that the transmission resonance linewidth is about 50 Oe which is the intrinsic limit value for the FMR linewidth of iron at 35 GHz (excluding exchange-conductivity effects[5]). It is interesting to point out that there are no conductivity effects in the transmission resonance linewidth, although the signal is attenuated because of conductivity.

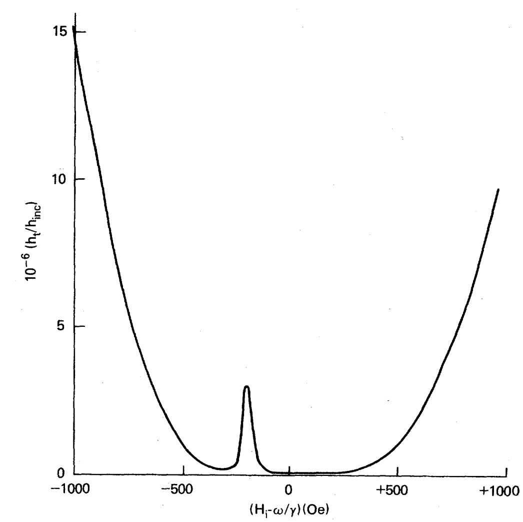

FIG. 6. Amplitude of the transmitted microwave magnetic field is plotted as a function of $H_i-\omega/\gamma$. $H_i=\omega/\gamma$ is the FMR internal field for uniform magnetic resonance precession. For this calculation $N=11$ and $d=d_\epsilon=1000$ Å.

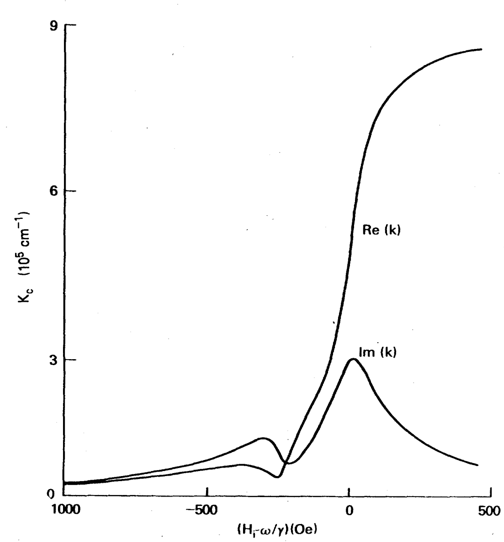

FIG. 7. The characteristic propagation constant K_c of the layered structure is plotted as a function of $H_i-\omega/\gamma$. For this calculation $N=11$ and $d=d_\epsilon=1000$ Å.

IV. DISCUSSION

The application of the transfer-function matrix to magnetic layered structures which contain metallic layers is very useful. It allows for quick and exact quantitative determinations of surface impedances of such structures. Although we have applied this technique to a layered structure in which the magnetic layer is characterized by two allowable electromagnetic wave-propagation constants, it will prove more useful to complicated magnetic layer systems with many[6] normal modes of wave propagations. For example, in a magnetoelastic conductive media there are as many as seven allowable[6] propagation constants. It is clear that FMR techniques can be utilized to study the magnetic properties of layered structures. The surface uniaxial magnetic anisotropy parameter exhibits a strong influence on FMR. K_s and exchange coupling between layers "play" a similar role in the FMR of layered structures as expressed in the boundary equation below for a given magnetic layer:

$$A\frac{\partial m}{\partial y}+K_s m + A_{12}\Delta m = 0 . \tag{10}$$

The first two terms represent[7] the spin-pinning surface boundary condition of a single layer with no exchange coupling between layers. The last term introduced here includes the effect of exchange coupling[3] between magnetic layers on the spin-boundary condition. The exchange coupling[9] between surface magnetic moments is proportional to A_{12}, where A_{12} is in units of ergs/cm^2, and Δm is the difference in the surface microwave magnetic moments at two adjacent nearest-neighbor layers. The magnetic layers are assumed to be similar with respect to their static field magnetic properties.

In the limit that the separation distance between magnetic layers is exactly zero, A_{12} is maximum and equal to A but $\Delta m \rightarrow 0$. Hence, the third term in Eq. (10) vanishes. In this limit simple layer film calculations[5] are appropriate. In the opposite limit (large separation between layers) $A_{12}\rightarrow 0$, but $\Delta m \neq 0$ (the case considered in this paper). Again the exchange interaction term in Eq. (10) must vanish. Clearly, there is a separation distance for which the exchange interaction term may be maximized. This separation distance should be in the order of intra-atomic distances. In this limit it is appropriate to designate the transfer-function matrix of each magnetic-dielectric layer pair as $\underline{C}_{ni}$, since the transfer function matrix of the magnetic layer $\underline{a}_{ni}$ is different for each magnetic layer, where ni indicates the particular layer in the layered structure. This is due to the fact that the surface magnetic moments are different at each magnetic layer surface. Clearly, in this situation the overall transfer-function matrix is the product of all the pair transfer-function matrices and not simply $\underline{C}^N$. An iteration procedure[9] has been used in obtaining a consistent solution for the surface magnetic moments and for Z_s in this case. We believe that this is an interesting regime to explore in order to study fundamental effects of exchange interaction in layered magnetic structures.

APPENDIX

In this section we solve the expressions which relate the internal magnetic fields $h_n^{\pm}$ to the surface electric e_{s2} and magnetic h_{s2} fields for one magnetic layer. The subscript n designates the number of allowable[5] propagation constants in the magnetic layer. In this case $n=2$. In general we may write this relationship as follows:

$$h_n^{\pm}=\alpha_n^{\pm}e_{s2}+\beta_n^{\pm}h_{s2} . \tag{A1}$$

This expression is the same as Eq. (3). The superscript $\pm$ indicates positive $(+)$ and negative $(-)$ propagation direction of the internal electromagnetic field components, $h_n^{\pm}$. Our purpose here is to solve $\alpha_n^{\pm}$ and $\beta_n^{\pm}$ in terms of parameters defined in Eqs. (1c)—(1f). Using Eqs. (1c)—(1f), we find

$$\Delta\alpha_1^{+}=P_2(P_1-P_2)F(k_1+k_2)+R_1(P_2-R_2)$$
$$+R_2(R_2-P_1)F(k_1-k_2)$$

and

$$\Delta\beta_1^{+}=Z_2[P_1P_2F(k_1+k_2)-R_1(R_2+P_2)$$
$$+P_1R_2F(k_1-k_2)]$$
$$+Z_1[R_2^2F(k_1-k_2)-P_2^2F(k_1+k_2)] .$$

Similarly,

$$\alpha_2^{\pm}=\alpha_1^{\pm}(1\rightarrow 2) ,$$
$$\beta_2^{\pm}=\beta_1^{+}(1\rightarrow 2) ,$$
$$\alpha_1^{-}=\alpha_1^{+}(P\rightarrow R ; j\rightarrow -j) ,$$
$$\beta_1^{-}=\beta_1^{+}(P\rightarrow R ; j\rightarrow -j) ,$$

where

$$\Delta=\det\begin{vmatrix} P_1 & P_2 & R_1 & R_2 \\ R_1F_1^{-} & R_2F_2^{-} & P_1F_1^{+} & P_2F_2^{+} \\ Z_1F_1^{-} & Z_2F_2^{-} & -Z_1F_1^{+} & -Z_2F_2^{+} \\ F_1^{-} & F_2^{-} & F_1^{+} & F_2^{+} \end{vmatrix} ,$$

$$F_1^{\pm}=e^{\pm jk_1d} ,$$
$$F_2^{\pm}=e^{\pm jk_2d} .$$

The propagation constants k_1 and k_2 and the corresponding characteristic impedance Z_1 and Z_2 are defined in the text. The parameters P_n and R_n are defined in the application of the spin-pinning boundary condition; see Eq. (2) and thereafter. The symbolic operation $(1\rightarrow 2)$ requires changing the subscripts of a particular function from 1 to 2 in order to obtain a new function. For example, $\alpha_2^{+}=\alpha_1^{+}(1\rightarrow 2)$ means that if we change the subscripts in the expression for α_1^{+} from 1 to 2, we obtain α_2^{+}.

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