Breakdown of self-similar scaling in the two-dimensional random-field Ising model: A Monte Carlo study

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We have simulated the two-dimensional ferromagnetic Ising model in a random magnetic field with spin-flip dynamics. After the system is deeply quenched into the unstable region of the phase diagram, we observe novel dynamical behavior, during an early-to-intermediate time regime, for the average size of the growing domains, $\overline{R}$. We find that self-similar scaling for the structure factor [that is, scaling to a single time-dependent length such as $\overline{R}(t)$] breaks down for long times. Our results may have relevance to the problem of island-growth kinetics in some chemisorbed systems.

I. INTRODUCTION

Impurities can affect the equilibrium and nonequilibrium behavior of pure substances in unexpected ways. As they are inherent in many physical systems, their consequences (random interactions and random fields) have been the subject of much study. In particular, for the last decade, considerable effort has been devoted to the analysis of a simplified model of such impurities: the Ising model in a random magnetic field.[1–5] Only recently has a consensus been reached on the dimensionality above which there is long-range order.[3] The lower critical dimension is now thought to be $d_l=2$.

Most of the work on the random-field Ising model has considered the system either at, or close to, equilibrium (some of the work on dynamical properties will be briefly discussed below). The behavior of systems far from equilibrium is, however, an active area of basic research in its own right.[6–11] In this paper we will study the growth of unstable domains far from equilibrium in the random-field Ising model.[12]

We consider a rapid quench from a high-temperature disordered state to a low-temperature state. In the absence of a random field this is a typical first-order phase transition in which the standard scenario for phase separation is as follows.[6,7] The evolution involves different time regimes, such as early and late, for which different theories can be developed. In an early time regime, following the quench into the unstable state, highly interconnected domains of ordered phase form. These domains evolve during an early-to-intermediate time regime so as to reduce their curvature, and thus their surface free energy. In an order-disorder transition in a two-state degenerate system (such as a binary alloy, where the scalar order parameter is nonconserved) this growth process is relatively well understood:[8] the average size $\overline{R}$ of a domain grows in time t via

$$\overline{R}^{\,2}(t) \propto t \ . \tag{1}$$

This has been observed in metallurgical systems,[9] chemisorbed systems,[10] and computer simulations of lattice-gas models.[11] It is also found in this time regime that all lengths scale to $\overline{R}(t)$. In particular, the structure factor (which gives the nonequilibrium elastic scattering intensity) is found to be

$$S(k,t)=\overline{R}^{\,d}(t)F(k\overline{R}(t)) \ , \tag{2}$$

where k is the wave number, and $F(x)$ is the scaling function.[13] Late-time theories describe the approach to equilibrium.

On a two-dimensional substrate, such as we are considering, impurities enter through, for example, steps, terraces, or vacancies. In the most simple case, these can be modeled by a random field and random interactions (we will not consider random interactions here). Villain[3] and Imry[2] have given very explicit discussions of the way random fields enter these systems, which we will now paraphrase. A chemisorbed monolayer experiences a potential energy due to its coupling to the three-dimensional host crystal, that is, the substrate. This acts as an external field, since in the most simple case the potential couples linearly to the two-dimensional order parameter of the chemisorbed system. Therefore, when the substrate is not perfect the potential becomes random, and so the chemisorbed system experiences a random field. One specific example of this, which is discussed by Villain, is Xe chemisorbed on the (110) face of Cu in the presence of random impurities.[14,15]

Theories of various aspects of domain growth in the presence of a random field have recently been proposed by two of us,[16] Villain,[17] and Grinstein and Fernandez.[18,19] The former theory deals with the earlier stages of evolution, while the latter two deal with the later stages. These will be discussed below. We know of two other Monte Carlo studies of the dynamics of the random-field Ising model, those by Stauffer *et al.*[20] and Pytte and Fernandez.[21] Both of these primarily address the equilibration problems which have plagued experimental studies of dilute antiferromagnets in a uniform magnetic field[22] (which have been argued to be in the same equilibrium universality class as the random-field Ising model[23]). Thus, they have considered much larger field strengths, as

well as a variety of different quenching procedures (including the quenching procedure we will consider herein). We believe these studies are complementary to our work.

The format of the remainder of this paper is as follows. In Sec. II we define our model and the Monte Carlo algorithm used to simulate the dynamics. A short discussion of relevant theory is also given. In Sec. III we present our main results: the time dependence of the average size of domains, and the nonequilibrium structure factor. We find that the random field has a dramatic effect on the well-known Allen-Cahn growth law [Eq. (1)]: the evolution is considerably slowed down, and the zero field $\overline{R}^2 \propto t$ behavior breaks down. It is impossible to precisely determine the functional form of $\overline{R}(h,t)$ at present. This is not surprising since very few Monte Carlo studies of domain growth have been able to determine such precise growth laws (see, for example, the well-known work of Lebowitz and co-workers[24]). Nevertheless, an analysis in terms of existing theory (which, however, does not constitute a definitive test) is given below. We find that self-similar scaling of the structure factor breaks down for long times. This is due to the absence of long-range order in two dimensions: although domains grow, they only become ordered over some "small" length scale determined by the random-field strength. The late-time form of the structure factor reflects the morphology of the system as it approaches this limiting behavior. Finally, in Sec. IV we conclude our paper with a discussion of our results.

II. DYNAMICAL MODEL

The Hamiltonian for the two-dimensional random-field Ising model is

$$H=-J\sum_{\langle i,j\rangle}\sigma_i\sigma_j-\sum_{i=1}^{N}h_i\sigma_i\ , \tag{3}$$

where the interaction sum runs over nearest-neighbor pairs on the square lattice, and the N spins can take the values $\sigma=\pm 1$. We have considered the case where the random magnetic field h is given by a Gaussian probability distribution,

$$P_i=\frac{1}{\sqrt{2\pi}h}\exp(-h_i^2/2h^2)\ , \tag{4}$$

so that

$$\langle h_i\rangle=0 \tag{5a}$$

and

$$\langle h_i h_j\rangle=h^2\delta_{ij}\ . \tag{5b}$$

Theoretical studies of the random-field Ising model often make use of this distribution. We have found this distribution to give less noisy data than a double-peaked distribution, which has been studied by other authors.[20,21]

The dynamics involves the following. After an instantaneous quench from temperature $T/J=\infty$ to $T/J=1$ (where Boltzmann's constant is set equal to unity) the system evolves via the standard Metropolis spin-flip procedure. To be explicit, a spin σ_i flips if

$$W_i=\begin{cases}\exp(-\Delta E_i/T), & \Delta E_i\geq 0\\ 1, & \Delta E_i\leq 0\end{cases} \tag{6}$$

exceeds a random number between zero and one, where ΔE_i is the resultant change in energy on flipping the spin. During a Monte Carlo step (our fundamental unit of time) each spin on the lattice is updated in this manner. We do this through $N/15$ attempts to flip randomly chosen groups of 15 widely separated spins. (The factor of 15 involves the multispin coding algorithm used to store and update the lattice. Note that other authors sometimes call N attempts to update spins a Monte Carlo step *per spin*, rather than simply a Monte Carlo step.) Since the order parameter (the magnetization) is not conserved in this algorithm, the simulation corresponds to "model A" in field theory.

Following the quench, small highly interconnected domains form, which grow as time evolves, as we have discussed above, and is evident in Figs. 1–3. We have monitored the evolving orientational order through several measures of the "domain size" which are standard in the literature.[11,24–26] In the presence of self-similar scaling, these are all equivalent to the inverse perimeter density $\overline{R}_p$. The perimeter density, in units of the thermal correlation length, is half the number of broken bonds per spin. Although we have calculated $\overline{R}_p$ for several times during the simulation, it is difficult to evaluate it for all times of interest. The other measures we have considered are the following. From the circularly-averaged structure factor

$$S(k,t)\equiv\frac{1}{2\pi}\int_0^{2\pi}d\theta\, s(\mathbf{k},t)\ , \tag{7a}$$

with

$$s(\mathbf{k},t)=\frac{1}{N}\left\langle\left|\sum_{i=1}^{N}\sigma_i e^{i\mathbf{k}\cdot\mathbf{r}_i}\right|^2\right\rangle \tag{7b}$$

(where $\mathbf{r}_i$ denotes positions on the lattice, $k=2\pi j/N$ with $j=0,1,2,\ldots,10$, and $\tan\theta=k_y/k_x$), one can define

$$\overline{R}_M^{\,2}(t)=S(0,t)\ , \tag{8}$$

following Sadiq and Binder,[25] and

$$\overline{R}_k^{\,2}(t)\equiv\left[\frac{\sum_k k^2 S(k,t)}{\sum_k S(k,t)}\right]^{-1}\ . \tag{9}$$

Both of these follow from self-similar scaling [Eq. (2)]. The quantity $\overline{R}_M$ is relatively easy to obtain, while $\overline{R}_k$ is not because of the Fourier sums which are involved. $\overline{R}_k$ is also somewhat sensitive to the choice of the large k cutoff (which corresponds to $j=10$ here). We have also considered the quantity

$$\overline{R}_E(t)\equiv\frac{2}{2+E(t)/J}\ , \tag{10}$$

where $E(t)$ is the average nonequilibrium energy per spin. In the absence of a random field $\overline{R}_E(t)\equiv\overline{R}_p(t)$. This will be discussed further in the following section. Our results for the growth laws thus involve the four measures $\overline{R}_M$, $\overline{R}_E$, $\overline{R}_p$, and R_k.

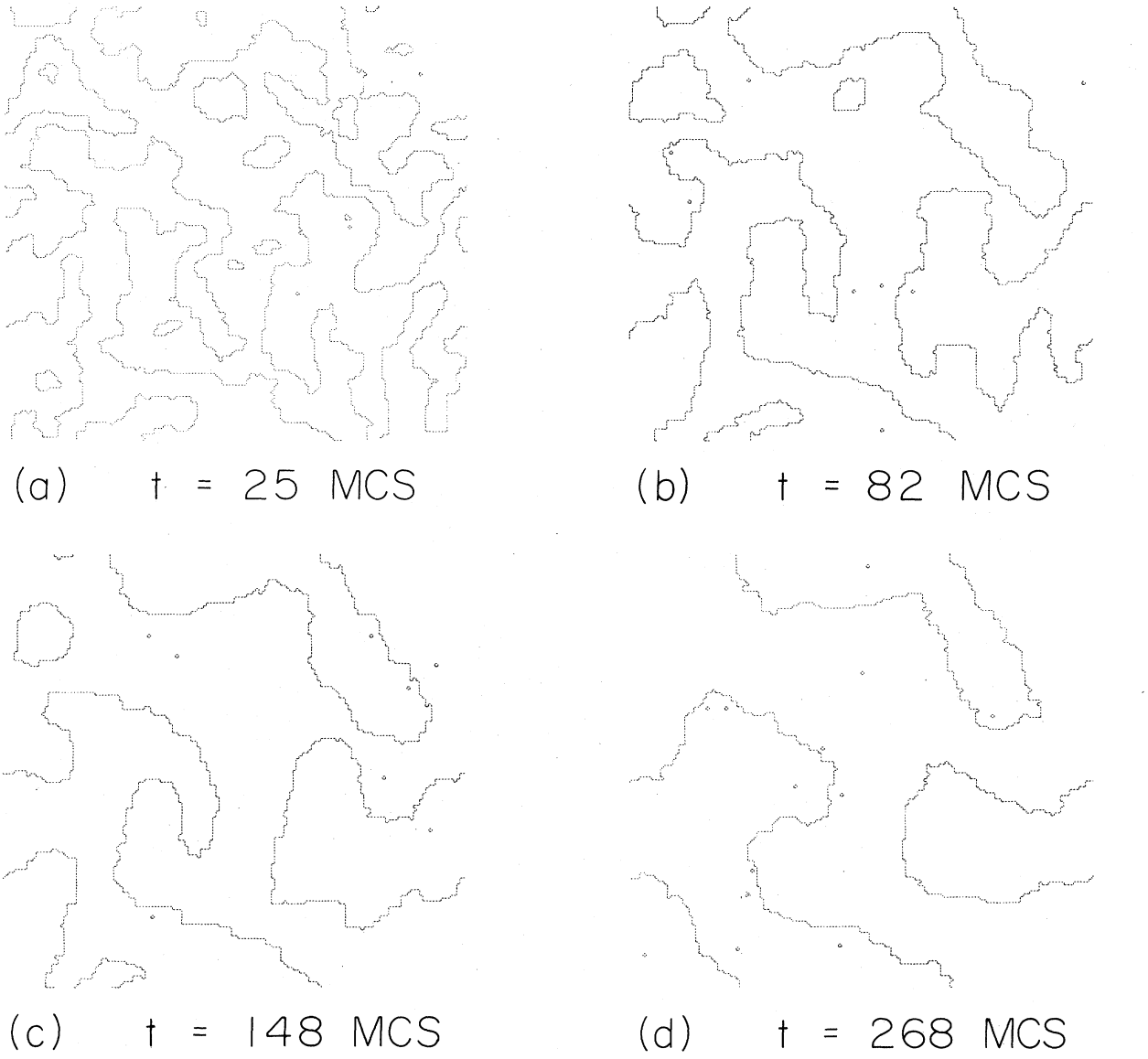

FIG. 1. Spin configurations are shown for $h=0$. Times in Monte Carlo steps (MCS) as indicated. System size is 150^2. Note that growth acts to decrease interfacial curvature.

In Monte Carlo simulations of domain growth one must balance a number of requirements.[26] Three which are particularly relevant for this study are the size of the system, the duration in Monte Carlo steps of the runs, and the number of quenches. We have done this in two ways, one which is appropriate for the study of phenomena at small field strengths, and another which is appropriate for a larger field strength.

For small field strengths, namely $h=0.311$, 0.415, and 0.518, we have chosen to do a large number of runs (352 runs for $h=0$, and 450 runs for each small nonzero h) on an $N=75^2$ lattice. This improves the quality of our data (error bars are approximately 7%); test runs on larger and smaller lattices indicate that our results are not dependent upon lattice size. The random field roughens the interfaces separating domains, which is a short-range effect during the growth process. Thus, doing a large number of runs on a relatively small system is a sensible approach to study the formation and growth of domains in the early-to-intermediate stages of evolution. The runs on small field strengths were terminated after 500 Monte Carlo steps since percolation effects occur at this limit. (These effects typically become important for[25,26] $\overline{R}\sim 0.4\sqrt{N}$.) The usefulness of studying the small field strengths, and thus this time regime, can be summarized as follows. Firstly, these times involve the fundamental nonlinear problem of unstable domain growth. Secondly, we can test a previous dynamical theory, which is discussed below. Thirdly, our results should be related to domain growth in real systems with random impurities, as we

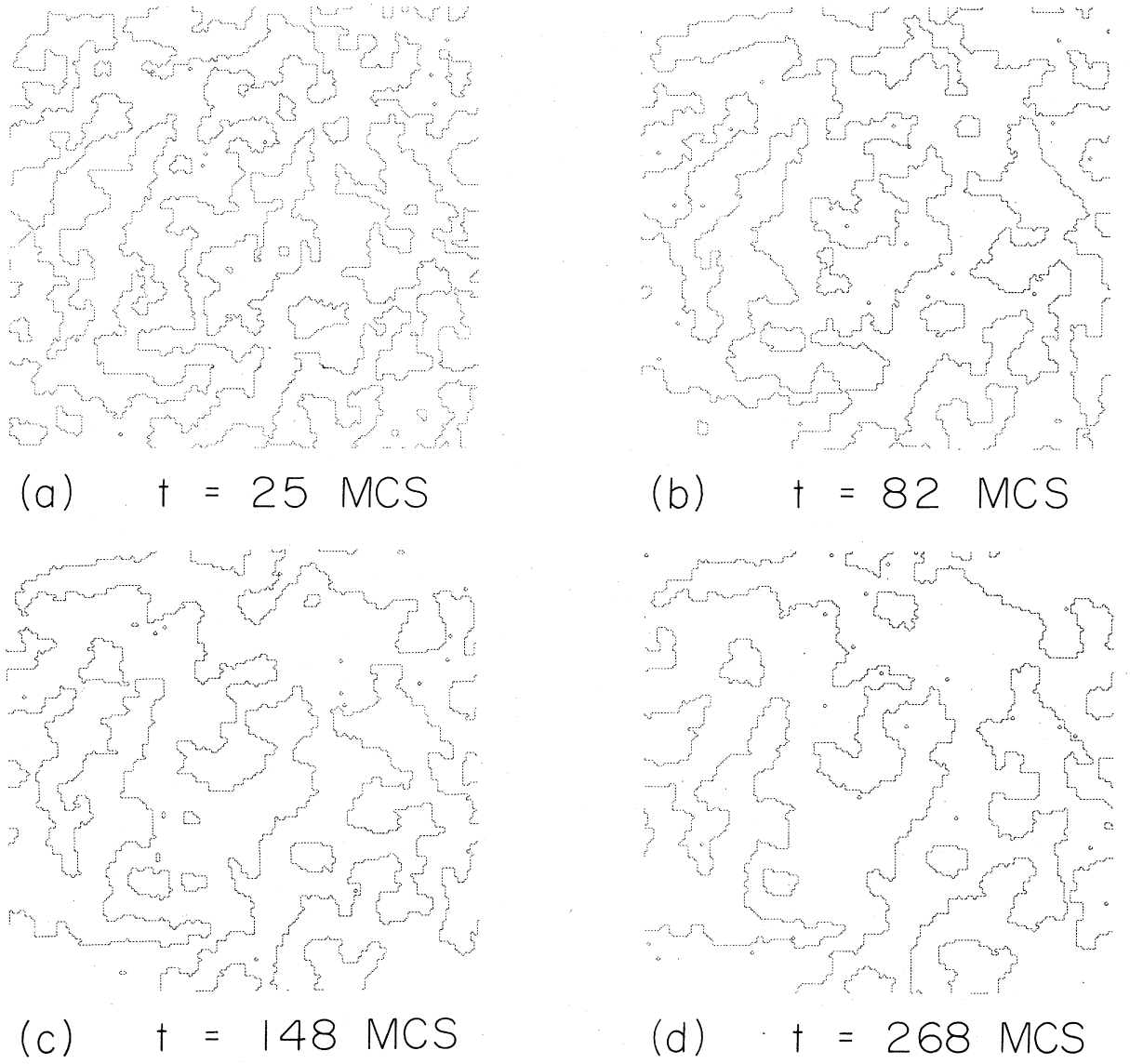

FIG. 2. Spin configurations are shown for $h=0.829$. Times as indicated. System size is 150^2. Note that growth is much slower than in Fig. 1, and that the simulations are primarily over the regime before large isolated droplets form.

have mentioned above.

We have also considered a large field strength, $h=0.829$, for an $N=240^2$ system over 5000 Monte Carlo steps. The price we pay for doing this, however, is that we have only done 44 runs, and so the data are substantially more noisy (error bars are approximately 10%). Nevertheless, by considering a larger field strength, and hence a longer time regime, we can investigate the subtle breakdown of long-range order in the random-field Ising model. These complementary approaches for small and large field strengths thus enable us to study different aspects of the dynamics.

Before presenting our results we will briefly discuss existing theory. The rather interconnected structure in Figs. 1—3 is typical of the initial stages of an unstable state's evolution. Previously, two of us have presented a theory for such domain growth in the random-field Ising model.[16] In two dimensions the growth law was predicted to be

$$\overline{R}^{\,2}(h,t)=\overline{R}^{\,2}(h=0,t)[1-h^2a\ln(t/b)]\ , \tag{11}$$

where $\overline{R}^{\,2}(h=0,t)$ is proportional to t. An estimate from theory gives[27] $a\sim\ln b\sim 1$ for $T/J=1$. The theory is consistent with $d_1=2$,[28] and predicts the breakdown of self-similar scaling in two dimensions. Note that, for $h^2b\ll 1$, this will be numerically similar to a power law. It is difficult to determine the range of validity of this theory since the approximations used to derive it are, to some extent, uncontrolled. (In fact, a test of this result

also serves to test the physically appealing approximation scheme of Ohta, Jasnow, and Kawasaki[8] which is used to derive it.) For late times, however, as domains become compact it is clear that the theory will break down because it is based on an assumption of isotropy. It is also worth mentioning that the field dependence of Eq. (11) may be strictly true only for a continuum model, and could be different on a lattice.[29]

Recently, other dynamical theories for the random-field Ising model have been presented.[17–19] These address different issues than that of the theory discussed above. "Growth" in those theories is limitied by the rate at which large fluctuations can occur, in analogy to the theory for nucleation rates from metastable states. Since these theories do not include the well-established Allen-Cahn curvature-driven growth mechanism, one would expect that they could only be correct for late times when domains are almost compact (but, in two dimensions, before domains have reached their maximum size). By compact, we mean the domains are close to their local equilibrium configurations (circles for $h=0$). While those configurations involve roughened domain walls, they are certainly much more compact than the interconnected structure which forms following the quench. It is clear from Figs. 1–3 that our simulations have principally studied the motion of highly convoluted domain walls rather than that of large, isolated droplets. (Note that the convoluted domain walls are *not* due to roughening effects, which those theories incorporate.) A Monte Carlo study of Villain's[17] and Grinstein-Fernandez's[18] theories has recently been given by Pytte and Fernandez,[21] in which the equilibration time of large, isolated droplets is considered explicitly (as well as quenching procedures somewhat analogous to that studied here). They find their data to be consistent with those theories.

III. RESULTS

A. Small field strengths

Our results for the growth law (that is, for $\overline{R}_M$, $\overline{R}_E$, $\overline{R}_p$, and $\overline{R}_k$) are shown in Figs. 3–7 for the field strengths $h=0$, 0.311, 0.415, and 0.518. We believe that the physics of this process corresponds to the following simple picture. The small domains which form after the quench have large curvatures, so that the system initially reduces its surface free energy by flattening interfaces. Later, as the interfaces flatten and domains grow, it becomes advantageous to roughen interfaces and slow down the growth. Thus, immediately after the quench the deviation from the zero-field ($\overline{R}^2 \propto t$) growth law is small, while as time increases, deviations become more pronounced.

Clearly, from all the measures of length, a dramatic time-dependent slow down in growth is observed in the presence of the random field. As we have discussed, this should be related to domain growth in experimental systems with random impurities (although we have not considered the effect of random interactions which we would expect to be present in addition to random fields). In fact, experimental studies have observed a slow down of

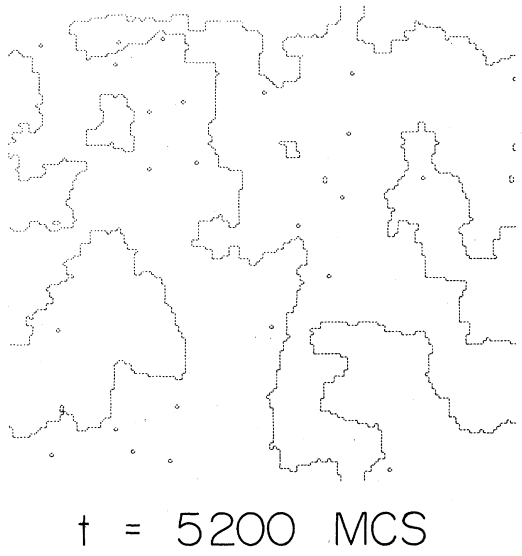

FIG. 3. A late-time spin configuration is shown for $h=0.829$. System size is 150^2.

growth, which is partly attributed to surface heterogeneities. We emphasize though that the experimental slow down can also involve the degeneracy of the ground state. For example, Sahni *et al.*[30] have found that vertices in the Q-state Potts model slow domain growth

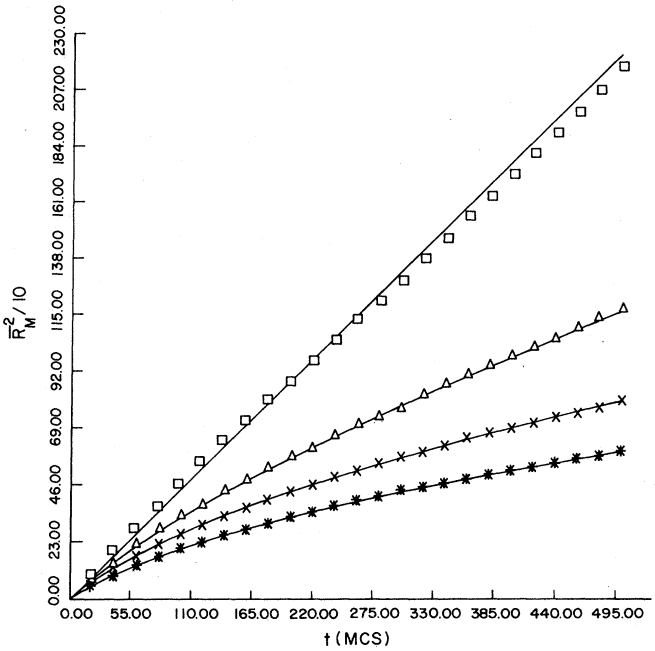

FIG. 4. $\overline{R}_M^2$ vs t. □, △, ×, and * correspond to $h=0$, 0.311, 0.415, 0.518. Every twentieth data point is plotted from $t=5$ Monte Carlo steps. Error bars are 7%. 352 quenches for $h=0$. 450 quenches for each nonzero field. $N=75^2$. Note the time-dependent deviations from the zero-field result ($\overline{R}_M^2 \propto t$) for nonzero random-field strength. Fits to Eq. (11), as given in Table I, are shown.

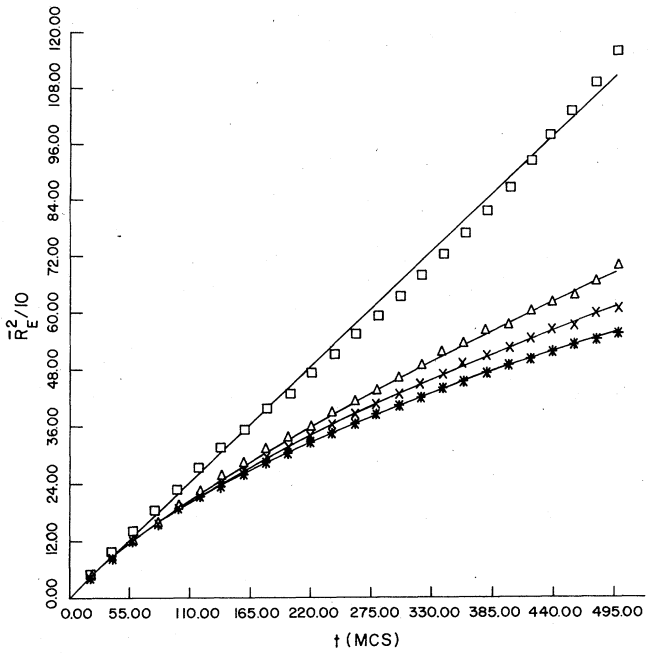

FIG. 5. $\bar{R}_E^2$ vs t. Data as in Fig. 4. Fits to Eq. (11), as given in Table II, are shown.

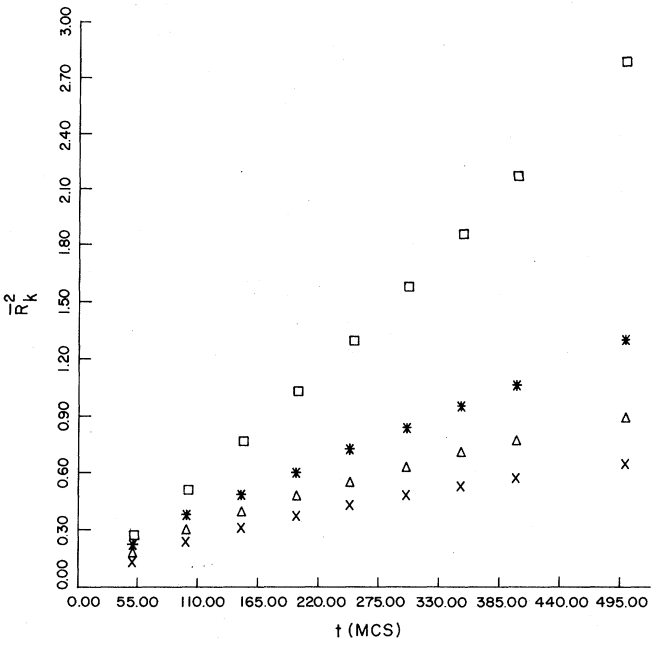

FIG. 7. $\bar{R}_k^2$ vs t. Data only for times as plotted. Remainder of information is the same as in Fig. 4.

through Q-dependent exponents in an effective power law. It may be that a combination of both impurity and degeneracy effects is responsible for the experimental observations.

We now turn to the analysis of our data. Only $\bar{R}_M$ and $\bar{R}_E$ will be considered, since there is much less data for the other length measures. It is worth noting, though, that the data from all length measures, $\bar{R}_M$, $\bar{R}_E$, $\bar{R}_p$, and $\bar{R}_k$, are consistent with one another. We cannot determine the precise functional form of $\bar{R}(h,t)$ at present. Nevertheless, good characterizations of the data by fitting functions are possible. For example, it is possible to find an effective power-law fit $\bar{R}^2 \propto t^n$ to our data, where, however, the *effective* exponent n decreases as the random field increases. That is, for the $\bar{R}_M$ data, we find $n = 1.0$, 0.79, 0.70, and 0.67, while for the $\bar{R}_E$ data we find $n = 1.0$, 0.80, 0.74, and 0.69, for $h = 0$, 0.311, 0.415, and 0.518, respectively.

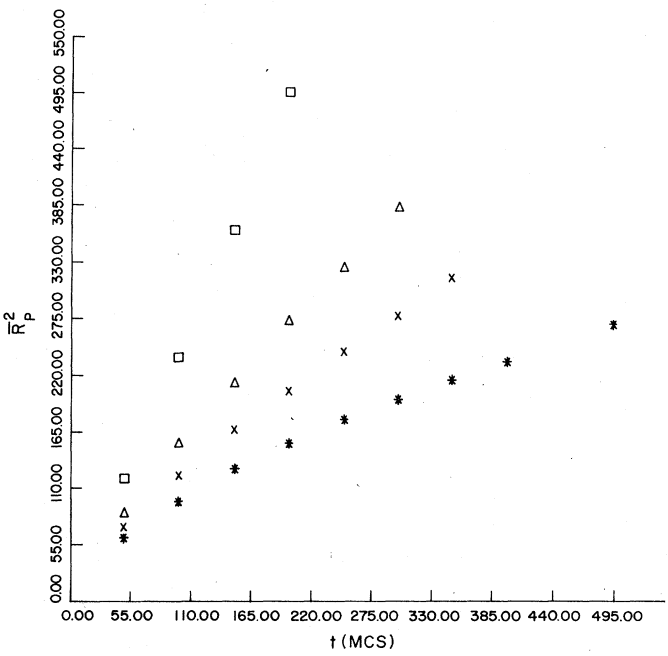

FIG. 6. $\bar{R}_p^2$ vs t. Data only for times as plotted. Remainder of information is the same as in Fig. 4.

We have also found good fits to the result from our previous theory, Eq. (11), where the parameters a and b are fitted to each nonzero field strength. (By the χ^2 criterion, these fits are approximately one order of magnitude better than the power-law fits.) In Tables I and II we present these values for the $\bar{R}_M$ and $\bar{R}_E$ data. As can be seen in Figs. 4 and 5, the log-correlation form gives a very reasonable fit to the time dependence of the data (fits to the $\bar{R}_E$ data are equally good). From Tables I and II, however, the field dependence does not seem to be in agreement with theory in that the coefficients a and b appear to have some h dependence. It is not clear to us whether the origin of this is as noted in Ref. 29, or if it may also involve noise in the data. In any case, there is a clear need for a better theoretical understanding of this problem.

Finally, it may be worth noting the following about the $\bar{R}$ data, and the relationship of $\bar{R}_E$ to $\bar{R}_p$. Note that the energy per spin of a completely ordered domain is $E/J = -2$. Interfaces raise the energy of the system by an amount $2/\bar{R}_p$, while the random field raises it by an

TABLE I. Fitted parameters a and b in Eq. (11), to $\overline{R}_M$ data over the first 500 Monte Carlo steps, for the three small field strengths, and over the first 5000 Monte Carlo steps for $h=0.829$. χ^2 is the normalized sum of squares of the residuals for the fits ($\chi^2<1$ is a good fit). Data fitted using $\partial \overline{R}_M^{\,2}/\partial t \simeq 4.347$, for $h=0$. Fits are shown in Fig. 4.

h	a	b	χ^2
0.311	1.346	13.938	0.30
0.415	0.824	5.757	0.10
0.518	0.432	0.939	0.23
0.829	0.148	0.091	2.78

amount $\sim h/\overline{R}_p$. (This latter result follows straightforwardly from Imry and Ma's[1] original argument.)

Thus we obtain $\overline{R}_E = C(h)\overline{R}_p$, which we have confirmed directly from the simulation data. Although $\overline{R}_E$ involves an unknown field dependence, its time dependence is equivalent to that of $\overline{R}$. Hence we have studied it in some detail. We should also note that it is a somewhat less noisy measure of length than $\overline{R}_M$.

B. Large field strength

To conclude this section, we discuss our results for $h=0.829$. As we mentioned above, the data here are noisier than that for the smaller field strengths, thus our conclusions are more qualitative. In Fig. 8 the data for $\overline{R}_M$ and $\overline{R}_E$ at this field strength are presented.[31] Clearly, over the 5000 Monte Carlo steps studied, the two quantities have different time dependences. This is, we believe, a consequence of the expected breakdown of long-range order in the two-dimensional random-field Ising model.[16] We have attempted to fit the data to the form of Eq. (11), as well as the logarithmic forms proposed in Refs. 17 and 18. Because there is much less data, we cannot make any strong statements concerning the validity of either theory from the fits. We note though that the log-correction form is reasonably good for early times, while the straight log fits are reasonably good for late times. This suggests to us that it would be useful to develop a theory which incorporates the features of both the early-time and late-time theoretical approaches.

The structure factor is presented in Fig. 9 in the form

TABLE II. Fitted parameters a and b in Eq. (11), to $\overline{R}_E$ data over the first 500 Monte Carlo steps, for the three small field strengths, and over the first 5000 Monte Carlo steps for $h=0.829$. χ^2 is the normalized sum of squares of the residuals for the fits ($\chi^2<1$ is a good fit). Data fitted using $\partial \overline{R}_E^{\,2}/\partial t \simeq 2.195$, for $h=0$. Fits are shown in Fig. 5.

h	a	b	χ^2
0.311	1.527	39.45	0.69
0.415	1.048	43.07	0.87
0.518	0.768	45.88	0.87
0.829	0.541	257.9	3.55

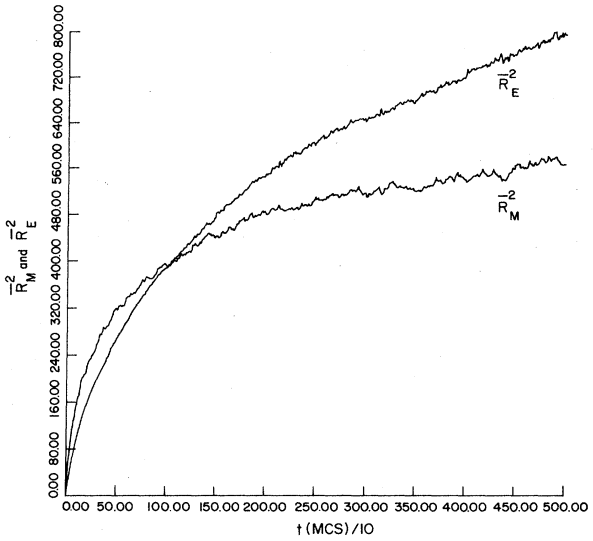

FIG. 8. $\overline{R}_M^{\,2}$ and $\overline{R}_E^{\,2}$ vs t over 5000 Monte Carlo steps for $h=0.829$. $\overline{R}_M^{\,2}$ curve is the lower of the two for later times. Error bars are 10%. 44 quenches on a $N=240^2$ lattice.

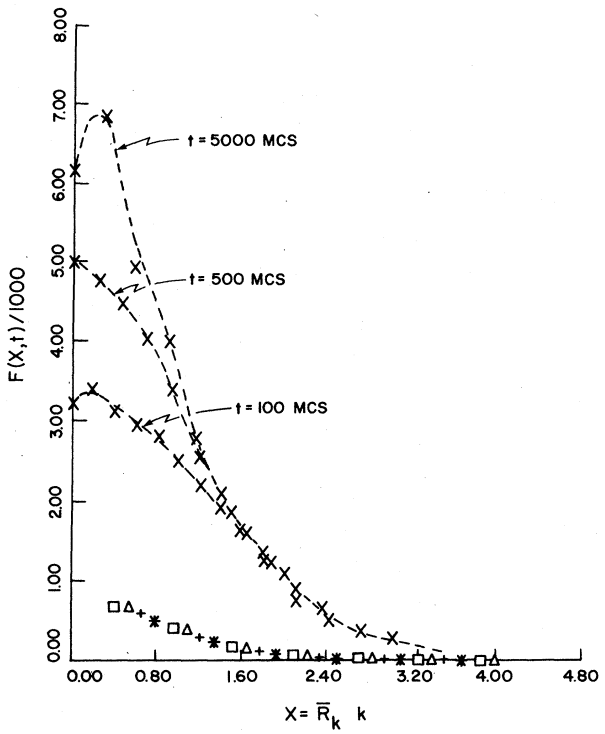

FIG. 9. "Scaling function" $F(x,t)$ vs x, as defined in Eq. (12), where $x \equiv k\overline{R}_k(t)$. Indistinguishable data for bottom curves, with symbols □, △, +, and *, correspond to $h=0$, 0.311, 0.415, and 0.518, respectively, for $t=5$ to 500 Monte Carlo steps. The symbol × corresponds to $h=0.829$ at the times $t=100$, 500, and 5000 Monte Carlo steps, from bottom to top (dashed lines are only a guide for the eye). Error bars for $h=0$—0.518 are 7%, error bars for $h=0.829$ are 10%. Note that all the small-field results scale to the zero-field scaling function, while the large-field result does not. In particular, note the time dependence of the scaling function for $h=0.829$.

$$F(x,t)=\frac{S(k,t)}{\overline{R}_k^2(t)}\ , \tag{12}$$

where $x\equiv k\overline{R}_k(t)$. The scaling function F is time independent in the self-similar scaling regime, if that regime exists. In the absence of a random field, the time-independent form of $F(x)$ is well known from both simulation and theory.[8,11] This function characterizes, basically, the geometric form factor for the morphology of the evolving system. Figure 9 shows that the smaller field strengths, $h=0.311$, 0.415, and 0.518, all scale to the zero-field result over the first 500 Monte Carlo steps. (We note again that finite-size effects preclude an examination of longer times for these field strengths.) The result for the "scaling function" for $h=0.829$, for the times $t=100$, 500, and 5000 Monte Carlo steps, is also shown in Fig. 9 (the later the time, the larger the peak near $x=0$). Clearly the morphology for the late-time, large-field system is considerably different from that of the smaller-field results. We believe this is because the domains are approaching their maximum size, as determined by the random-field strength. In other words, the domains are far from the regime where the dynamics is dominated by a curvature-driven mechanism.

The time dependence of the scaling function for $h=0.829$ is also evident in Fig. 9. This shows the late-time breakdown of scaling. As we have mentioned above, this is to be expected since there is no long-range order in two dimensions: The system becomes ordered over a length scale proportional to the lattice constant (although that proportionality constant can be large) rather than over a length scale given by the size of the system.[16] To our knowledge there is no detailed theory available for this interesting time regime. In contrast to the case for early times, where the random field and thermal fluctuations work together to roughen interfaces, here we would expect these two effects to be competitive. Perhaps this leads to another relevant length scale, as has been discussed by Binder.[3] In any case, we note that the small-x time dependence of F implies that the ratio $\overline{R}_M/\overline{R}_k$ is increasing with time, which may be a useful clue for further theoretical work. We do not know if any significance should be given to the fact that F is peaked at nonzero x in Fig. 9 (the peak is at $k=2\pi/\sqrt{N}$). This may only be an indication of the noise in the data. While the error for nonzero x is relatively small,[32] the error for the $x=0$ point is larger.[11]

To conclude this section, we mention one way to quantitatively characterize $S(k,t)$ [or equivalently $F(x,t)$]. The tail of the structure factor can be fitted to the form

$$\lim_{k\to\infty} S(k,t)\sim 1/k^{d+1-\theta}\ , \tag{13}$$

in d dimensions. This equation, with $\theta=0$, is called Porod's law,[33] and gives the scattering intensity from a sharp (infinitely thin) interface. The phenomenological quantity θ, which we have included in Eq. (13), characterizes the diffuseness of the interface. Other ways to use Porod's law in data analysis are also possible, as are discussed in Ref. 34. We have attempted to fit our data to Eq. (13), but have been unable to draw any definitive conclusions.

IV. CONCLUSIONS

In this paper we have studied the dramatic effect that a random field has on domain growth in the spin-flip kinetic Ising model in two dimensions. Time-dependent deviations from the zero-field Allen-Cahn growth law were observed, during an early-to-intermediate time regime. A qualitative explanation of this slow down appears to be given by the previous theory of two of us. Further theoretical work, however, is necessary for a complete understanding of the problem. We found that self-similar scaling of the structure factor breaks down for long times. This is related to the breakdown of long-range order in two dimensions.

These results should be related to growth in chemisorbed systems in the presence of heterogeneous impurities. An experimental study of such a system in the presence of controlled random impurities would, we believe, be of considerable interest.

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[28]It may be worth noting that, according to theory, if the lower critical dimension were $d_1=3$ one would expect different dynamical behavior. One dimension below d_1 one would naively expect $\overline{R}^2 \propto t(1-B\sqrt{t})$. This is not consistent with our data.

[29]There is some evidence (Refs. 3 and 20) that the transition from continuum to lattice would make $h^2 \rightarrow h^\alpha$ in Eq. (11), where we expect $\alpha < 2$. We have attempted to fit our data to this form, but have been unable to draw any definitive conclusions.

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