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Electromagnetic properties of proximity systems

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Magnetic screening in the proximity system S_α-M_β, where M_β is a normal metal N, semiconductor (semimetal), or a superconductor, is studied. Main attention is paid to the low-temperature region where nonlocality plays an important role. The thermodynamic Green's-function method is employed in order to describe the behavior of the proximity system in an external field. The temperature and thickness dependences of the penetration depth λ are obtained. The dependence $\lambda(T)$ differs in a striking way from the dependence in usual superconductors. The strong-coupling effect is taken into account. A special case of screening in a superconducting film backed by a size-quantizing semimetal film is considered. The results obtained are in good agreement with experimental data.

I. INTRODUCTION

This paper is concerned with the electromagnetic properties of a proximity system. Consider the system S_α-N_β containing a superconducting S_α and a normal N_β film. The proximity effect induces a superconducting state in the β film. However, the behavior of the contact in an external field differs in a striking way from the behavior of usual superconductors. For example, the penetration depth, contrary to the usual case, depends noticeably upon the temperature in the low-temperature region.

The problem of magnetic screening is directly related to the problem of the behavior of a Josephson junction in a magnetic field. As is known, Josephson current oscillates with a change of the external field, and the period of oscillations is related to the penetration depth. Lately, there have appeared many papers (see, e.g., the reviews in Refs. 1 and 2) dealing with tunneling into a proximity system, so investigation of the electromagnetic properties of proximity systems is of considerable interest.

Screening in proximity systems has been studied by several groups[3–8] (see the review in Ref. 6). Theoretical analysis has been based on the Ginzburg-Landau theory, and is therefore applicable in the region near T_c. In the low-temperature region it is necessary to develop a different approach, and not only because of the inapplicability of the Ginzburg-Landau theory. The corresponding London type of electrodynamics can be used if $\lambda \gg \xi$ (λ is the penetration depth, ξ is the coherence length). A decrease in temperature results in an increase of ξ_N,[9–11] where ξ_N is the coherence length in the normal β film. Therefore, in the low-temperature region we are dealing with a situation when the coherence length ξ exceeds the value of λ. This means that it is necessary to use the nonlocal Pippard-type approach in order to describe the screening.

A detailed experimental investigation of magnetic screening in proximity systems has been carried out by Simon *et al.*[4–6] for the systems Pb-Ag, Pb-Al, etc.; A. Mota *et al.*[7] for NbTi-Cu; and Oda *et al.*[8] for Nb-Au.

In this paper, the electrodynamics of the proximity system is studied. The temperature and thickness dependences of screening are evaluated. We focus on the low-temperature region where nonlocality plays an important role. We are mainly concerned with the effect of nonlocality on the properties of the proximity system.

Our approach is based on the method of thermodynamic Green's functions. This method has been applied by the present author[11,1] to study the behavior of the critical temperature and the Josephson current for a proximity system. Reference 1 (below referred to as I) contains equations describing a proximity system in any temperature region.

We consider various systems of the type S_α-M_β, where M_β is either a normal metal (semiconductor, semimetal), or another superconductor (the S_α-S_β system with $T_c^\alpha \neq T_c^\beta$, where T_c^α and T_c^β are the critical temperatures of isolated films). Note that the electron-phonon interaction is directly included in the theory, which allows incorporation of the effects of strong electron-phonon coupling.

The structure of the paper is as follows. Section II addresses the problem of obtaining the main equations. Temperature and thickness dependences of the screening are considered in Secs. III and IV. Electromagnetic properties of S_α-S_β systems are studied in Sec. V. Screening by a superconducting film backed by a normal size-quantizing film is described in Sec. VI.

II. THEORY

A. Proximity system

Let us consider the proximity system S_α-M_β [Fig. 1(a)], where S_α is a superconductor, and M_β is a normal metal (or semiconductor), e.g., Pb-Ag, or a superconductor, e.g., Pb-Al.

Suppose that the thicknesses of the films, L_α and L_β, satisfy the conditions $L_\beta < L_\alpha$, $L_\beta \ll \xi_\beta$, where ξ_β is the coherence length. Moreover, suppose that the α film is "dirty" in the Anderson sense.[12] Then we can use the well-known McMillan tunneling description[13] of the proximity effect. The electron-phonon interaction can be in-

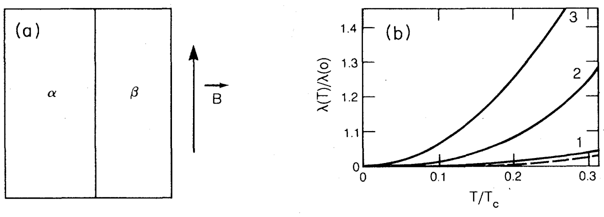

FIG. 1. (a) Proximity system in the external field; (b) temperature dependence of λ for the S_α-N_β system: 1, $t=1$; 2, $t=5$; 3, $t=10$. The dashed line corresponds to BCS theory.

cluded in the McMillan model.[14] Note that results obtained on the basis of the McMillan theory are in good agreement with experimental data (see, e.g., Refs. 15–17 and the review in I).

We consider the general case of an S_α-S_β contact. Equations describing the superconductor—normal-metal (S_α-N_β) contact can be obtained from the theory (see below) by putting $g_\beta=0$ (g_β is the electron-phonon coupling in the β film).

Our approach is based on the method of thermodynamic Green's functions. The corresponding description of the proximity system was given by the author.[1,11] Equations for the order parameters $\Delta_\alpha(\omega_n)$ and $\Delta_\beta(\omega_n)$ can be written in the form [see I, Fig. 1 and Eqs. (4) and (5)]

$$\Delta_\alpha(\omega_n)=Z_\alpha^{-1}\pi T\sum_{\omega_{n'}}\int d\Omega\, g_\alpha(\Omega)D(\Omega,\omega_n-\omega_{n'})\times K_\alpha^{-1}(\omega_{n'})\Delta_\alpha(\omega_{n'})+Z_\alpha^{-1}\Gamma^{\alpha\beta}K_\beta^{-1}(\omega_n)\Delta_\beta(\omega_n)\,, \tag{1}$$

$$\Delta_\beta=Z_\beta^{-1}\pi T\sum_{\omega_{n'}}\int d\Omega\, g_\beta(\Omega)D(\Omega,\omega_n-\omega_{n'})\Delta_\beta(\omega_{n'})+Z_\beta^{-1}\Gamma^{\beta\alpha}K_\alpha^{-1}(\omega_n)\Delta_\alpha(\omega_n)\,. \tag{2}$$

Here $D=\Omega^2/[\Omega^2+(\omega_n-\omega_{n'})^2]$ is the phonon thermodynamic Green's function, $g_{\alpha(\beta)}(\Omega)=a_{\alpha(\beta)}(\Omega)F_{\alpha(\beta)}(\Omega)$ [$a^2_{\alpha(\beta)}(\Omega)$ describes the electron-phonon interaction, $F_{\alpha(\beta)}(\Omega)$ is the phonon density of states], and

$$K_{\alpha(\beta)}(\omega_n)=[\omega_n^2+\Delta^2_{\alpha(\beta)}(\omega_n)]^{1/2}\,. \tag{3}$$

The quantities $\Gamma^{\alpha\beta}$ and $\Gamma^{\beta\alpha}$ were introduced by McMillan[13] and are equal to

$$\Gamma^{\alpha\beta}=\pi\widetilde{T}^2\nu_\beta SL_\beta\,, \tag{4}$$

$$\Gamma^{\beta\alpha}=\pi\widetilde{T}^2\nu_\alpha SL_\alpha \tag{5}$$

(ν_α and ν_β are the densities of states, S is the area of the films, and $\widetilde{T}$ is the tunneling matrix element). Note also that $\Gamma^{\beta\alpha}$ can be written in the form[13]

$$\Gamma^{\beta\alpha}=v_F\sigma/BL_\beta\,. \tag{6}$$

Here v_F is the Fermi velocity, $B\simeq4$,[13] and σ is the barrier-penetration probability. An analogous expression can be written for $\Gamma^{\alpha\beta}$. Z_α and Z_β are the renormalization functions. For example, Z_β is equal to [see I, Eq. (8)]

$$Z_\beta=1+\Gamma^{\beta\alpha}/K_\alpha(\omega_n)-\widetilde{\Sigma}_{1\beta}(\omega_n)\,. \tag{7}$$

Here $\widetilde{\Sigma}_{1\beta}$ is the self-energy part describing electron-phonon scattering. In the weak-coupling approximation, $\widetilde{\Sigma}_{1\beta}/\omega_n=-\lambda_\beta$, where

$$\lambda_\beta=\int d\Omega\, g_\beta(\Omega)\Omega^{-1}\,. \tag{8}$$

Equations (1) and (2) allow us to evaluate the order parameter $\Delta_\beta(\omega_n)$ which can be used to analyze the electromagnetic properties of the proximity system.

B. External field

Consider the proximity system in an external field [see Fig. 1(a)]. Our goal is to study the response of the system to an applied field and to evaluate the screening, that is the penetration depth λ into the β film. We assume that $\lambda<L_\beta$. The current density can be written in the form (see, e.g., Ref. 18):

$$\mathbf{j}=2\left[\frac{ie}{2m}(\nabla_{\mathbf{r}'}-\nabla_{\mathbf{r}})G(x,x')-\frac{e^2\mathbf{A}}{m}G(x,x')\right]\Bigg|_{\substack{\mathbf{r}'\to\mathbf{r}\\ \tau'\to\tau}}\,. \tag{9}$$

Here $G(x,x')$ is the thermodynamic Green's function, $x=\{\mathbf{r},\tau\}$, and τ is the imaginary time. The expression (9) can be written in the form

$$\mathbf{j}(\mathbf{r})=\frac{ie}{m}T\sum_{\omega_n}(\nabla_{\mathbf{r}}-\nabla_{\mathbf{r}'})G^{(1)}(\mathbf{r},\mathbf{r}')\Bigg|_{\mathbf{r}'\to\mathbf{r}}-\frac{e^2\mathbf{A}}{m}\,. \tag{10}$$

Here $\omega_n=(2n+1)\pi T$, $G^{(1)}$ is the correction (in the linear approximation) to the Green's function due to the external field.

The function $G^{(1)}$ should be calculated from the system of equations for the thermodynamic Green's functions. This system, which directly includes the electron-phonon interaction, has been obtained by Litovchenko and the present author,[19] and can be written in the form:

$$\begin{aligned}&[i\omega_n+(\nabla-ie\mathbf{A})^2+\mu]G(\mathbf{r},\mathbf{r}',\omega_n)-\int d\mathbf{r}''G(\mathbf{r}'',\mathbf{r}',\omega_n)\Sigma_1(\mathbf{r},\mathbf{r}'',\omega_n)-\int d\mathbf{r}''F^+(\mathbf{r}'',\mathbf{r}',\omega_n)\Sigma_2(\mathbf{r},\mathbf{r}'',\omega_n)=\delta(\mathbf{r}-\mathbf{r}')\\&[-i\omega_n+(\nabla+ie\mathbf{A})^2+\mu]F^+(\mathbf{r},\mathbf{r}',\omega_n)+\int d\mathbf{r}''F^+(\mathbf{r}'',\mathbf{r}',\omega_n)\Sigma_1(\mathbf{r}'',\mathbf{r}',\omega_n)-\int d\mathbf{r}''G(\mathbf{r}'',\mathbf{r}',\omega_n)\Sigma_2(\mathbf{r}'',\mathbf{r},\omega_n)=0\,,\end{aligned} \tag{11}$$

where

$$\Sigma_1(\mathbf{r},\mathbf{r}',\omega_n)=T\sum_{\omega_{n'}} g^2 G(\mathbf{r},\mathbf{r}',\omega_{n'})D(\mathbf{r},\mathbf{r}',\omega_n-\omega_{n'}) ,$$
$$\Sigma_2(\mathbf{r},\mathbf{r}',\omega_n)=T\sum_{\omega_{n'}} g^2 F(\mathbf{r},\mathbf{r}',\omega_{n'})D(\mathbf{r},\mathbf{r}',\omega_n-\omega_{n'}) , \tag{12}$$

where F is the abnormal Green's function.

Based on Eqs. (10)—(12) one can evaluate the current density, and we obtain for the β film

$$\mathbf{j}_\mathbf{q}=-\frac{c}{4\pi}K(\mathbf{q},T)\mathbf{A}_\mathbf{q} , \tag{13}$$

where

$$K(\mathbf{q},T)=\frac{3\pi^2Ne^2}{c^2m_\beta^*}T\sum_n\int_{-1}^{1}\frac{(1-\mu^2)d\mu}{Z_\beta[\omega_n^2+\Delta_\beta^2(\omega_n,T)]^{1/2}}\frac{Z_\beta^2\Delta_\beta^2(\omega_nT)}{Z_\beta^2[\omega_n^2+\Delta_\beta^2(\omega_{n'})]+(v_{\beta q}^2q^2\mu^2/4)} . \tag{14}$$

Here Δ_β is the order parameter in the β film, m_β^* is the effective mass, and Z_β is the renormalization function [see Eq. (7)]. Equation (12) contains the self-energy part describing the electron-phonon scattering. In a similar way one can include the terms describing the contribution of the tunneling Hamiltonian to the self-energy part. Note that the penetration depth in the nonlocal case [see below, Eq. (16)] does not contain the renormalization function. We consider the effect of the tunneling upon the order parameter Δ_β. One can see from Eq. (2) that its value depends strongly (particularly for S-N systems) upon the value of the parameter Γ.

C. Screening

Equations (13) and (14) can be used in order to evaluate the penetration depth λ. λ is defined in the usual way (see, e.g., Ref. 20):

$$\lambda=\int Bdz/B_0=\left[\frac{2}{\pi}\right]\int_0^\infty dq[q^2+K(q)]^{-1} . \tag{15}$$

This expression is valid for specular reflection, but, as is known (see. e.g., Ref. 20), the dependence on the nature of reflection is weak.

Consider how the β film screens an external field [see Fig. 1(a)]. If $\lambda<L_\beta<\xi_\beta$ [this condition $\xi_\beta>\lambda,L_\beta$ is satisfied in the low-temperature region (see above)], we should use the expression corresponding to nonlocal Pippard-type electrodynamics. In this case (see Refs. 18 and 19) one can neglect the term μ^2 in the numerator of the expression (14). Integrating over μ with the use of the residue theorem, we obtain

$$K(q)=\frac{3\pi^2}{v_\beta q}T\sum_{\omega_n(>0)}\frac{\Delta_\beta^2(\omega_n)}{\omega_n^2+\Delta_\beta^2(\omega_n)} . \tag{16}$$

Based on Eqs. (15) and (16), we arrive, after some manipulations, at the following expression:

$$\lambda=a_\beta\phi^{-1/3} , \tag{17}$$

where

$$a_\beta=\frac{4}{3\sqrt{3}}\left[\frac{m_\beta^*c^2v_F^\beta}{12\pi^2N_\beta e^2}\right]^{1/3} , \tag{18}$$

$$\phi=\pi T\sum_{\omega_n(>0)}\frac{\Delta_\beta^2(\omega_n)}{\omega_n^2+\Delta_\beta^2(\omega_n)} . \tag{19}$$

Here N_β and m_β^* are the electron concentration and the effective mass in the β film, respectively. Hence, the penetration depth depends strongly upon the parameters of the β film. Moreover, the value of the order parameter Δ_β is related to the properties of the proximity system [see Eqs. (2) and (5)].

Equations (1), (2), and (17)—(19) form the basis of the theory. The temperature and thickness dependences of the penetration depth can be obtained from Eq. (17). This equation contains the thermodynamic order parameter $\Delta_\beta(\omega_n)$ which can be evaluated from Eqs. (1) and (2) describing the proximity system. Consider the general case of screening by the S_α-S_β proximity system. The order parameter $\Delta_\beta(\omega_n)$ can be written in the form [see I, Eqs. (20) and (23)]

$$\Delta_\beta(\omega_n)=g(\omega_n)\epsilon_\alpha\delta+S(\omega_n) , \tag{20}$$

where

$$g(\omega_n)=K_\alpha(\omega_n)[\Gamma+K_\alpha(\omega_n)]^{-1} ;$$
$$S(\omega_n)=[1-g(\omega_n)]\Delta_\alpha(\omega_n) ; \tag{21}$$
$$\Gamma=\Gamma^{\beta\alpha}(1+\lambda_\beta) ;$$

the quantities $K_\alpha(\omega_n)$, $\Gamma^{\beta\alpha}$, and λ_β are defined by Eqs. (3) and (4)—(8), ϵ_α is the energy gap in the α film, and δ is the solution of the nonlinear equation

$$\delta=\rho_\beta\frac{\pi T}{\epsilon_\alpha}\sum_n\frac{f_\alpha+\delta t\widetilde{K}}{[x_n^2(1+t\widetilde{K})^2+(f_\alpha+\delta t\widetilde{K})^2]^{1/2}}\chi(x_n\epsilon_\alpha) , \tag{22}$$

where

$$x_n=\omega_n/\epsilon_\alpha,\ \ \rho_\beta=\lambda_\beta/(1+\lambda_\beta),\ \ f_\alpha=\Delta_\alpha(x_n\epsilon_\alpha)/\epsilon_\alpha\ , \tag{23}$$

$$\widetilde{K}=(x_n^2+f_\alpha^2)^{1/2}\alpha,\ \ \alpha=\epsilon_\alpha/\pi T_c^\alpha,\ \ \chi(z)=\Omega_\beta^2/(\Omega_\rho^2+z^2)\ . \tag{24}$$

It is assumed (see I) that the β film is a superconductor with weak electron-phonon coupling; we do not limit the intensity of the electron-phonon interaction in the α film. If we are interested in screening in the S_α-N_β system, then

$$\Delta_\beta=S(\omega_n)\ . \tag{25}$$

The parameter t introduced in I is defined by the relation

$$t=l/S_0\ , \tag{26}$$

where $l=L_\beta/L_0$ is a dimensionless quantity (L_0 is some fixed thickness; we have chosen $L_0=10^2$ Å), and

$$S_0=\Gamma_0/\pi T_c^\alpha\ , \tag{27}$$

where $\Gamma_0=v_F^r\sigma/BL_0$ is the quantity introduced by McMillan[13] ($L=L_0$); $v_F^r=v_F/(1+\lambda)$ is the renormalized Fermi velocity.

Substituting Eqs. (20) and (21) into (19), we obtain after simple manipulations:

$$\phi=\pi T\sum_{x_n(>0)}\frac{[f_\alpha+\alpha t\delta(x_n^2+f_\alpha^2)^{1/2}]^2}{x_n^2[1+\alpha t(x_n^2+f_\alpha^2)^{1/2}]^2+[f_\alpha+\alpha t\delta(x_n^2+f_\alpha^2)^{1/2}]^2}\ . \tag{28}$$

We have introduced the parameters t and x_n (see also I) which allows us to carry out the calculations with dimensionless quantities. If $T\to 0$, one can pass from summation to integration, $(2\pi T/\epsilon_\alpha)\sum_n\to\int dx$ and we obtain

$$\phi(0)=\frac{1}{2}\epsilon_\alpha(0)\int_0^\infty dx\frac{\{f_\alpha(x)+\alpha t\delta[x^2+f_\alpha^2(x)]^{1/2}\}}{x^2\{1+\alpha t[x^2+f_\alpha^2(x)]^{1/2}\}^2+\{f_\alpha+\alpha t\delta[x^2+f_\alpha^2(x)]^{1/2}\}^2}\ . \tag{29}$$

Equations (17), (18), and (28), (29) allow us to investigate magnetic screening by the proximity system. As was mentioned above, for the S_α-N_β system, one should put $\delta=0$ in Eqs. (28) and (29). Now we turn to the analysis of the temperature and thickness dependences of the penetration depth for various systems.

III. TEMPERATURE DEPENDENCE OF THE PENETRATION DEPTH

As is known, the penetration depth of usual superconductors depends very weakly upon the temperature ($\sim[1-(T/T_c)^4]$, see, e.g., Ref. 21), in the low-temperature region. For a proximity system, the picture appears to be entirely different.

The dependence $\lambda(T)$ can be determined from Eqs. (17), (19), and (28) and is described by the relation

$$\lambda(T)/\lambda(0)=[\phi(0)/\phi(T)]^{1/3}\ , \tag{30}$$

where $\phi(T)$ and $\phi(0)$ are defined by Eqs. (28) and (29).

A. S_α-N_β system

Consider magnetic screening by the following proximity system: superconductor-normal metal (semiconductor or semimetal), that is by S_α-N_β (e.g., Sn-Ag). Then $\Delta_\beta(\omega_n)=S(\omega_n)$ [see Eq. (25)] and

$$\phi=\pi T\sum_{x_n(>0)}\{x_n^2[1+\alpha t(x_n^2+1)^{1/2}]^2+1\}^{-1} \tag{31}$$

[see Eq. (28)]. Equation (31) is written in the weak electron-phonon coupling approximation (the effect of strong coupling will be considered below), so that $f_\alpha=1$, $\alpha=\alpha_{\rm BCS}=0.56$. Note also that the thickness of the α film is large enough [$\sim 5\times 10^3$ Å (Ref. 6)], so the quantity $\Gamma^{\alpha\beta}\sim L_\alpha^{-1}$ is small [$\Gamma^{\alpha\beta}\ll\epsilon_\alpha(0)$]; the superconducting state of the α film is caused by the electron-phonon coupling g_α [the function $\Delta_\alpha(\omega_n)$ is described by the usual Eliashberg equation; see the first term in Eq. (1)]. As for the N_β film, the superconducting state in it is due entirely to the proximity effect.

Based on Eqs. (17) and (31), we obtain the following expression describing the temperature dependence of the penetration depth:

$$\frac{\lambda(T)}{\lambda(0)}=\left\{\int_0^\infty dx\,G(x,t)\left[\frac{2}{\alpha_0}\frac{T}{T_c}\sum_{x_n(>0)}G(x_n,t)\right]^{-1}\right\}^{1/3}, \tag{32}$$

where

$$G(x_n,t)=[x_n^2(1+t\widetilde{K})^2+1]^{-1}\ . \tag{33}$$

The penetration depth increases with temperature, and the sharpness of this increase depends parametrically [see Fig. 1(b)] on the value of the parameter t [see Eq. (26)]. This parameter can be increased by an increase of the thickness L_β, or by a decrease of S_0, e.g., by a decrease of the transmission coefficient. Note that the presence of the term $t\widetilde{K}$ [see Eq. (33)] results in a deviation from the BCS dependence of $\lambda(T)$, and this deviation is noticeable for large t. This feature of the proximity system is connected with the temperature dependence of the order parameter Δ_β [see Eq. (2)]. Another interesting property of the proximity system is the dependence of the penetration depth upon the thickness of the β film. This feature will be studied below (see Sec. IV).

It is interesting to note that the dependence $\lambda(T)$ described by Eq. (32) (see Fig. 1) differs noticeably from the dependence obtained in Ref. 3 [see the review (Ref.

22)]. As has been mentioned above, this theoretical approach is based on the expansion in a series in powers of the small quantity Δ, and provides an excellent description in the region near T_c. However, this method is not applicable in the low-temperature region. According to Ref. 3 $\lambda(T) \sim T^{1/2}$ and $\lambda \to 0$ if $T \to 0$ K. This unphysical result (see the discussion in Ref. 22, p. 1023) is connected with the aforementioned expansion which is not valid if $T \to 0$ K. Moreover, the approach in Ref. 3 is based on local electrodynamics, whereas nonlocality plays an important role in the low-temperature region. The approach developed in this paper allows us to consider both this region and the effect of nonlocality.

Note that the value of the parameter S_0 can be determined from independent measurements (e.g., from a measurement of the Josephson current, see I, Sec. III A). On the other hand, one can determine the value of S_0 from a measurement of $\lambda(T)/\lambda(0)$ at some fixed temperature. Then this value can be used in order to describe the dependence $\lambda(T)$ everywhere in this temperature region.

Hence, an increase of the thickness L_B results in a noticeable increase of the slope of the dependence $\lambda(T)$.

B. Strong coupling

If one is interested in the behavior of a proximity system like Pb-Ag, one should take into account the effect of strong coupling in the α film, because lead is a strongly-coupled superconductor. Then the temperature dependence of the penetration depth is described by the expression:

$$\frac{\lambda(T)}{\lambda(0)} = \left\{ \int_0^\infty dx\, G^{\text{str}}(x,t) \times \left[\frac{2}{\alpha_0} \frac{T}{T_c} \sum_{x_n(>0)} G^{\text{str}}(x_n,t) \right]^{-1} \right\}^{1/3}, \tag{34}$$

where

$$G^{\text{str}}(x_n,t) = f_\alpha^2 [x_n^2 (1+t\widetilde{K})^2 + f_\alpha^2]^{-1}, \tag{35}$$

where f_α, $\widetilde{K}$, and α are defined by Eqs. (23) and (24); $\alpha_0 = \alpha(T=0\text{ K})$.

Our method allowing for the effect of strong coupling is described in I (Sec. I, Sec. III C), where it was used to evaluate the strong-coupling corrections to the Josephson current into a proximity system. This method is based on the theory of strong coupling which has been developed by Geilikman, Masharov, and the present author.[23] Note that this theory was used by Litovchenko and the present author[19] in order to describe the electromagnetic properties of superconductors with strong coupling.

The method[23] is based on the theory of the thermodynamic Green's function and allows one directly to evaluate the order parameter $\Delta_\alpha(\omega_n)$. According to Ref. 23, the energy gap $\epsilon(0)$ is equal to

$$\epsilon(0) = 1.76 T_c [1 + 5.3 (T_c/\widetilde{\Omega})^2 \ln(\widetilde{\Omega}/T_c)], \tag{36}$$

where $\widetilde{\Omega}$ is the characteristic frequency, e.g., for Pb, $\widetilde{\Omega} = 4.5$ meV, and corresponds to the frequency of the

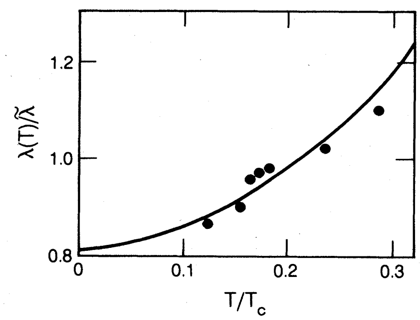

FIG. 2. The dependence $\lambda(T)$ for Pb-Ag. Solid line is the theoretical curve; ●, experimental data (Refs. 4 and 6) [$l=25$, $S_0=2.5$; $\tilde{\lambda} = \lambda(T/T_c = 0.2)$].

lowest peak of the function $g(\Omega)$ [the presence of the highest peak near Ω_h results in a small correction on the order of $(T_c/\Omega_h)^2$]. The function f_α can be written in the form[23] (see also I):

$$f_\alpha = A^2/(x_n^2 + A^2), \tag{37}$$

where $A = \Omega/\epsilon_\alpha$. For example, for Pb, $\epsilon(0) = 2.1 T_c$, $\alpha = 0.65$ [see Eq. (24)], and $A = 3.45$. Substituting this expression for f_α and the values of $\epsilon_\alpha(0)$ and α into Eq. (34), one can evaluate the temperature dependence of screening for a proximity system containing Pb as the α film. We have calculated this dependence for the Pb-Ag proximity system (see Fig. 2). One can see that the dependence described by Eq. (34) is in good agreement with the experimental data obtained by Simon.[6]

IV. THICKNESS DEPENDENCE OF THE PENETRATION DEPTH

Consider the S_α-N_β proximity system. Based on Eqs. (17), (18), and (29), one can get the following expression describing the screening at $T=0$ K in the weak-coupling approximation:

$$\lambda(t) = L_0^\beta \left[\int_0^\infty G(x,t) dx \right]^{-1/3}, \tag{38}$$

where [see Eq. (18)]

$$L_0^\beta = \frac{4}{3\sqrt{3}} [m_\beta^* c^2 v_F^\beta / 6\pi^2 N_\beta e^2 \epsilon_\alpha(0)]^{1/3} \tag{39}$$

and [see Eq. (33)]

$$G(x,t) = \{ x^2 [1 + \alpha_0 t (x^2+1)^{1/2}]^2 + 1 \}^{-1}. \tag{40}$$

The parameter t is defined by Eq. (26). One can see that an increase of the thickness L_β results in an increase of the penetration depth (Fig. 3). Qualitatively, this effect is caused by the thickness dependence of the order parameter Δ_β [see Eqs. (2), (5), and (6)]. The pairing in the β film is induced by the proximity effect and an increase in the thickness L_β results in a decrease of Δ_β. This decrease indicates a weakening of the superconducting properties of the β film, and is accompanied by a decrease of the screening, that is, by an increase of the penetration depth. The parameter S_0 can be determined, for instance, from measurements of the temperature dependence of λ (see

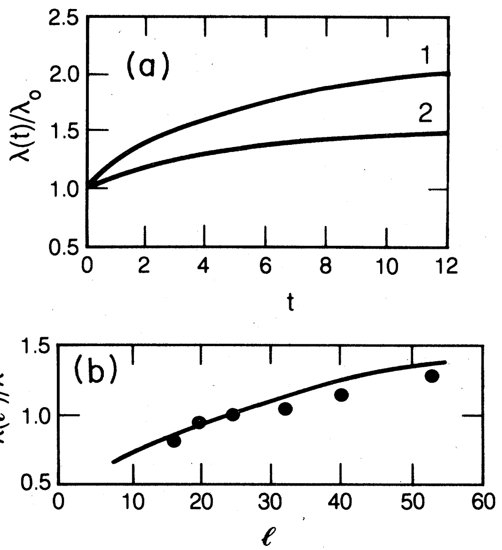

FIG. 3. (a) Thickness dependence of λ. (1) S_α-N_β system; (2) S_α-S_β system. The difference between (1) and (2) is due to inequality $\rho_\beta \neq 0$; (b) thickness dependence for Pb-Ag. Solid line is the theoretical curve; ●, experimental data (Refs. 4 and 6) ($S_0 = 2.5$).

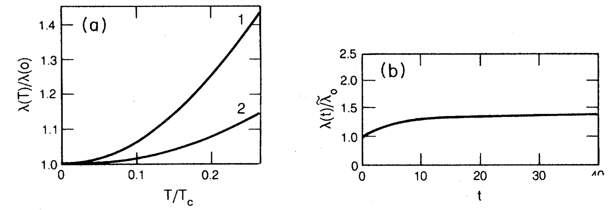

FIG. 4. Screening in S_α-S_β proximity system; (a) temperature dependence of $\lambda(T)$: (1) S_α-N_β system ($t=10$); (2) S_α-S_β system ($S_\alpha \equiv$ Pb, $t=10$). The difference between (1) and (2) is due to the inequality $\rho_\beta \neq 0$; (b) thickness dependence for the system Pb-Al.

above). The quantity L_0^β [see Eq. (39)] depends on parameters describing the properties of a normal metal. Note that the values of these parameters in the thin-film state may differ from their bulk values, and measurements of λ can be used in order to determine them.

If $T \neq 0$ K, one should use the expression

$$\lambda(t) = L_0^\beta \left[\frac{2\pi T}{\epsilon_\alpha} \sum_{x_n(>0)} G(x_n, t) \right]^{-1/3} ; \tag{41}$$

$G(x_n, t)$ is defined by Eq. (33).

In the low-temperature region one can put $\epsilon_\alpha(T) \simeq \epsilon_\alpha(0)$. Note that a decrease of temperature results in a less slanting thickness dependence of λ [this can be seen from Fig. 1(b)].

The effect of strong coupling can be considered by analogy with the analysis of the dependence $\lambda(T)$ (see above, Sec. II B; see also I). We carried out a calculation of the thickness dependence of screening for the Pb-Ag system at $T = 1.2$ K [see Fig. 3(b)]. We use the value $S_0 = 2.5$, obtained during the analysis of the dependence $\lambda(T)$ for this system (see Fig. 2). One can see that our calculations are in good agreement with the experimental data obtained in Refs. 4 and 6.

V. SCREENING BY THE S_α-S_β PROXIMITY SYSTEM

Consider the electromagnetic properties of the proximity system S_α-S_β containing two superconducting films (e.g., Pb-Al). We assume that $T_c^\alpha > T_c^\beta$, where T_c^α and T_c^β are the critical temperatures of isolated films. Moreover, we assume that the β film is a superconductor with weak electron-phonon coupling.

According to Eqs. (17), (28), and (29), the temperature dependence of λ is described by the equation

$$\frac{\lambda(T)}{\lambda(0)} = \left\{ \int_0^\infty dx\, \widetilde{G}(x,t) \left[\frac{2}{\alpha_0} \frac{T}{T_c} \sum_{x_n(>0)} \widetilde{G}(x_n, t) \right]^{-1} \right\}^{1/3} , \tag{42}$$

where

$$\widetilde{G}(x_n, t) = (f_\alpha + \delta t \widetilde{K})^2 \times [x_n^2 (1 + t\widetilde{K})^2 + (f_\alpha + \delta t \widetilde{K})^2]^{-1} .$$

The parameter δ must be determined from the nonlinear equation (22). We carried out a calculation of the dependence $\lambda(T)$ for $t = 10$ and $\rho_\beta = \rho_{\text{Zn}} = 0.145$ [see Fig. 4(a)]. One can see from a comparison of the curves 1 and 2 in Fig. 3(a) and the curves 1 and 2 in Fig. 4(a) that the fact that $\rho_\beta \neq 0$ results in a noticeable decrease of the penetration depth. We would like to emphasize that this decrease takes place even if $T > T_c^\beta$. This result is analogous to the behavior of the Josephson current for the S_α-S_β system (see I, Sec. V) and is due to the induced nature of the superconducting state of the proximity system. If $T > T_c^\beta$, the superconducting state of an isolated β film is destroyed by thermal motion. In our case, the superconducting order parameter is not equal to zero because of the proximity effect, and the fact that $\rho_\beta \neq 0$ also makes a contribution.

Based on Eqs. (17), (28), and (29) we have calculated the thickness dependence of λ [see Fig. 4(b)]. One can see that in the region $10 \leq t \leq 40$ the penetration depth depends very weakly on the thickness L_β. This conclusion about the weak thickness dependence of the screening for the S_α-S_β proximity system is in qualitative agreement with the experimental data obtained in Refs. 4 and 6 for the system Pb-Al. A more detailed analysis of the dependence $\lambda(L_{\text{Al}})$ for this system requires the knowledge of the value of the parameter S_0 for this system. Evaluation of this parameter can be based, for example, on measurements of the dependence $\lambda(T)$. In connection with this, it would be interesting to investigate experimentally the temperature dependence of λ for the system Pb-Al.

VI. SCREENING BY A SUPERCONDUCTING FILM BACKED BY A NORMAL SIZE-QUANTIZING FILM

In this section we study the special case of magnetic screening by the proximity system S_α-N_β. Namely, we consider the penetration of the electromagnetic field into the superconducting film S_α backed by the normal film N_β. The superconducting state in the α film is caused mainly by the electron-phonon interaction in the film. Nevertheless, it is affected also by the β film (proximity effect) which results in some changes of λ. The most interesting case corresponds to N_β being a semimetal size-quantizing film. Then λ becomes an oscillating function of L_β (see below).

Note that an analogous situation may appear if we study the Josephson current into a proximity system. It would be interesting to study the thickness and temperature dependences of the Josephson current for the N_β-S_α-I-S_α system. This problem will be considered elsewhere.

The penetration depth λ for the case of interest can be evaluated by analogy with (17)–(19) and is described by the expression

$$\lambda = a_\alpha \phi_\alpha^{-1/3} , \tag{43}$$

where

$$\phi_\alpha = \pi T \sum_{\omega_n} \frac{\Delta_\alpha^2(\omega_n)}{\omega_n^2 + \Delta_\alpha^2(\omega_n)}, \quad a_\alpha = \left[\frac{m_\alpha c^2 v_F^\alpha}{12\pi^2 N_\alpha e^2} \right]^{1/3} . \tag{44}$$

The thermodynamic order parameter $\Delta_\alpha(\omega_n)$ can be evaluated from Eqs. (1) and (2). We assume that $L_\beta \ll L_\alpha$. Moreover, we consider the case of $\Gamma^{\alpha\beta}, \Gamma^{\beta\alpha} \ll \epsilon_\alpha(0)$. The smallness of these parameters can be due to a small value of the penetration coefficient, that is, it can be connected with the contact quality. In addition, the smallness of $\Gamma^{\alpha\beta} \sim L_\alpha^{-1}$ [cf. Eq. (6)] is due to the large value of the thickness L_α. Under these conditions, the correction to Δ_α due to the presence of the β film is connected with the second term in the renormalization factor Z_α [see Eqs. (1) and (7)]. One can seek the solution of Eq. (1) in the form: $\Delta_\alpha(\omega_n) = \Delta_\alpha^0(\omega_n) + \Delta_\alpha'(\omega_n)$, where $\Delta_\alpha^0(\omega_n)$ is the order parameter in the absence of the β film, and $\Delta_\alpha' \sim \Gamma^{\alpha\beta}$. In the weak electron-phonon coupling approximation, we obtain:

$$\Delta_\alpha'(\omega_n) \simeq -(\Gamma^{\alpha\beta}/K_\beta)\Delta_\alpha^0 . \tag{45}$$

With the use of this expression and Eqs. (43) and (44), one can evaluate the correction to the penetration depth λ' due to the proximity contact with the normal film N_β. We obtain

$$\lambda = \lambda_0 f(L_\beta) , \tag{46}$$

where λ_0 is the usual penetration depth in a Pippard-type superconductor. If $T=0$, the quantity $\lambda_0 = (4/3\sqrt{3}) \times (m_\alpha c^2 v_F^\alpha / 3\pi^3 N_\alpha e^2 \epsilon_\alpha)^{1/3}$ [see, e.g., Ref. 20] and

$$f(L_\beta) = 1 - \phi_\alpha (S_1 - S_2) , \tag{47}$$

where ϕ_α is defined by Eq. (44),

$$S_1 = \pi T \sum_{\omega_n} \frac{\Delta_\alpha^1}{\omega_n^2 + \Delta_{\alpha 0}^2}; \quad S_2 = \pi T \sum_{\omega_n} \frac{\Delta_\alpha^1}{(\omega_n^2 + \Delta_{\alpha 0})^2} \Delta_{\alpha 0}^2 . \tag{48}$$

If $T=0$, one should make the transformation $(2\pi T/\epsilon_\alpha)\sum_n \to \int dx$; $x = \omega/\epsilon_\alpha(0)$. Finally, we arrive at the following expression:

$$\lambda = \lambda_0 [1 - \Gamma^{\alpha\beta} g(L_\beta)] , \tag{49}$$

where $\Gamma^{\alpha\beta}$ is defined by Eq. (4), and

$$g(L_\beta) = [G_1(t) + (\partial G_r/\partial r)|_{r=1}] , \tag{50}$$

where

$$G_r(t) = \int_0^\infty dx \{x^2[1 + \alpha t (x^2+1)^{1/2}]^2 + 1\}^{-1/2} \times (x^2 + r^2)^{-1/2} \tag{51}$$

is the function introduced in I (Sec. I, Sec. III A). This expression is valid if $\Gamma^{\beta\alpha} \ll \Gamma^{\alpha\beta} \ll \epsilon_\alpha(0)$ (see above). It is important to note that $g(L_\beta)$ is a smooth function of the thickness L_β, whereas $\Gamma^{\alpha\beta}$ is a sharply oscillating function for a size-quantizing film (see below).

Consider the case when N_β is a semimetal size-quantizing film. Size quantization (SQ) is observed experimentally in films of Bi,[24,25] Sb,[26] and InSb.[27] A very interesting investigation of SQ in Bi films by tunneling was carried out in Ref. 25. SQ results in a situation when the energy $\epsilon(\boldsymbol{\kappa}, n)$ is determined by the longitudinal two-dimensional quasimomentum $\boldsymbol{\kappa}$ and by the transverse quantum number n. As is known (see, e.g., Refs. 28 and 11), the best conditions for SQ are realized in a semimetal film. Instead of a Fermi surface, we have a set of two-dimensional subbands. From the point of view of the present analysis the most important factor is the behavior of the density of states in the presence of SQ. This quantity is an oscillating function of the film thickness.[28,11] An increase of the thickness results in a decrease of the distance between the transverse levels, and in a possibility to fill higher levels (for a more detailed analysis, see a previous paper by the present author, Ref. 11). For example, for Bi films filling of the next-higher level corresponds to a change in thickness $\Delta L \simeq 2 \times 10^2$ Å. Note that Eq. (4) contains the density of states $\nu_\beta = \sum_i \nu_\beta^i$, where ν_β^i are two-dimensional densities of states (see Ref. 11); summation is taken over the filled subbands. Contrary to the usual situation (see, e.g., Ref. 29), the quantity $p_\perp$ for a SQ film is not a continuous variable, and hence the usual cancellation of $\partial\epsilon/\partial p_\perp$ does not take place. An increase of the thickness L_β leads to oscillations of the quantity $\Gamma^{\alpha\beta}$. According to Eqs. (4) and (49), the penetration depth depends on the density of states in the β film. Oscillations of the density of states with an increase of L_β result in oscillations of the dependence $\lambda(L_\beta)$. If β is a Bi film, the period of oscillations is $\Delta L_\beta \simeq 2 \times 10^2$ Å. It would be interesting to carry out an experimental investigation of screening by such a N_β-S_α system.

VII. SUMMARY

Based on the thermodynamic Green's-function method, we have studied the electromagnetic properties of the proximity system S_α-M_β, where the film M_β can represent a normal metal (semiconductor, semimetal), or a superconductor S_β. We focus on screening in the low-temperature region, where nonlocality plays an important role.

The main results can be summarized as follows:

(1) A general expression describing screening is obtained [see Eqs. (17)–(19)]. This expression contains the thermodynamic order parameter which can be evaluated

from Eqs. (1) and (2) (see also I) describing the proximity effect.

(2) The penetration depth increases with an increase of temperature [see Eq. (32) and Fig. 1(b)]; the sharpness of this increase depends parametrically on the thickness L_β.

(3) The effect of strong electron-phonon coupling on screening has been investigated (see Sec. II B). An increase of coupling results in a decrease of λ.

(4) The dependence $\lambda(T)$ for a proximity system differs noticeably from the temperature dependence of λ for usual superconductors [see Fig. 1(b)]. The obtained dependence is in good agreement with experimental data [see Fig. 2(b)].

(5) The dependence of λ upon the thickness L_β (at fixed temperature) is studied. This dependence is described by Eqs. (38) and (41) (see Fig. 3) and becomes less slanting with decreasing temperature. The calculations of the thickness dependence of λ for the Pb-Ag system are in good agreement with experimental data [see Fig. 3(b)].

(6) Electromagnetic properties of the proximity system S_α-S_β containing two superconductors ($T_c^\alpha > T_c^\beta$) have been studied (Sec. IV). The fact that $\rho_\beta \neq 0$ results in a noticeable decrease of λ and affects the behavior of the system even if $T > T_c^\beta$. It turns out that for the S_α-S_β system the penetration depth depends weakly on L_β (see Fig. 4).

(7) Screening by a superconducting film backed by a normal film is studied (S_α-N_β). A particularly interesting case corresponds to the situation when N_β is a size-quantizing semimetal film (Sec. V). Then λ becomes an oscillating function of the thickness L_β.

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