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Green's-function approach to nonlinear electronic transport for an electron-impurity-phonon system in a strong electric field

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A Green's-function approach to nonlinear electronic transport in a static electric field is developed microscopically for the system composed of interacting electrons with impurities and phonons. The essential idea is to separate the center-of-mass motion from the relative motion of electrons. An electron temperature is introduced as a measurement of the internal energy of the relative electrons without reference to any distribution function. By allowing different temperatures for decoupled electrons and phonons in the initial state, we obtain the density matrix for the electron-lattice system to the first order of interaction but under arbitrarily strong electric field. The frictional force experienced by the center of mass of electrons and the energy transfer rate from electron system to phonon system are derived by means of the Green's-function technique, and the force- and energy-balance equations for steady state are obtained. These equations are applied to the calculations of the ratio of electron temperature to the lattice temperature and the electron resistivity as functions of drift velocity for impurity, acoustic-phonon, and optical-phonon scatterings. The dynamic nature of Coulomb screening by charge carriers is studied numerically. One of the interesting predictions is the possible cooling of electrons at low temperatures in samples with low impurity concentration.

I. INTRODUCTION

Nonlinear electronic transport deals with the problems which arise when the electric field applied to a material is no longer regarded as small and the current density deviates from the results of linear-response theory. Although many classical and quantum methods on this subject have been proposed since the earliest studies in the 1930's,[1-9] so far the semiclassical Boltzmann-equation method still seems to be the only one amenable to practical calculations.[10,11] In principle, within Boltzmann theory, if we know the scattering probabilities, we can obtain the distribution function by solving the transport equation. In practice, however, when the system goes beyond the linear regime, solution of the Boltzmann equation also becomes a laborious task. One has to rely either on numerical technique or on simplified models.[12] One of the most commonly used models in nonlinear transport in semiconductors is the "carrier-temperature model," especially the displaced Maxwellian distribution-function model.[11-15] In it, the concept of the "carrier temperature" plays an important role.

Physically, when an electric field E is applied, energy is supplied to the electron system at a rate $E^2\rho_T$, where ρ_T is the total resistivity of the system. This energy, which increases rapidly with increasing field, must be dissipated to the heat bath. The dissipation is via the interaction between electrons and the heat bath. The energy dissipation process has generally been regarded as conventional heat transfer—energy always transmits from the "hot" portion to the "cold" portion of the system. Thus, in the presence of an electric field a stationary state can be reached only when the electrons are hotter than the heat bath, or the average electron energy is higher than at zero field. That is the origin of so-called "hot-electron" or "heating of electrons." The heating effect of the electrons is generally considered to be one of the major reasons for the deviation from Ohm's law in high-field transport.[5,14,16]

Fröhlich and Paranjape[12] first utilized a displaced Maxwellian distribution function to describe the electrons in insulators and semiconductors, which contains a drift velocity v_d and an electron temperature T_e as parameters. This displaced Maxwellian has since been widely used to discuss the high-field transport in semiconductors. Recently, Arai[15] applied a similar model to describe metals at low temperatures but by means of a Fermi distribution function and showed that at zero-lattice-temperature limit, the electron temperature T_e approaches a finite value.

Strictly speaking, one cannot thermodynamically define a temperature for transport electrons because they are not in an equilibrium state. It is well known that the temperature of a system is a representation of the mean energy of the irregular thermal motion of its constituents. Therefore, it is necessary to distinguish the energy of relative motion of electrons from that associated with drift motion. The latter may be sufficiently large that it should not be neglected in comparison with the former in high-field transport.

In this paper we shall describe a Green's-function approach to steady-state electronic transport for interacting electrons with impurities and phonons in a strong electric field. The method lies in the separation of the center-of-mass motion from the relative motion of the electrons in the Hamiltonian and in the density matrix. After the separation we find that the external electric field acts only on the center of mass, and the electrons which move relative to the center of mass do not directly experience the electric field. They couple with the center of mass and with the lattice only through the electron-impurity and electron-phonon interactions. Thus, if we turn off these interactions, the relative electrons will decouple themselves from the center of mass and from phonons, and as an isolated system, will approach an equilibrium state. This fact enables us to introduce an electron temperature T_e for the nonequilibrium electrons which are in a transport state in the presence of an electric field as a measurement of the mean energy of their relative motion. The existence of electron-impurity and electron-phonon interactions induces frictional forces against the movement of the center of mass. The current density J is related to the center-of-mass velocity, or drift velocity v_d. In steady state v_d is a constant, and the applied electric force must be balanced by the frictional forces due to impurities and phonons. Therefore, a force-balance equation is obtained. On the other hand, to reach a steady state, the power supplied by the electric field to the electron system has to be taken away and transferred to the lattice which is in contact with a heat bath with temperature T. This energy dissipation is due to electron-phonon interaction, and the energy transfer rate can be calculated. An energy-balance equation is then obtained. From the force- and the energy-balance equations the two unknown quantities v_d and T_e as functions of electric field E can be solved. The most important aspect of the present method is the simplicity in mathematical structure. The numerical labor involved is minimal as compared with other existing calculations. Moreover, many electron interactions can be microscopically included into our formalism by the use of the diagramatic method for Green's function. One of our findings is that the drift velocity v_d influences directly the energy transfer rate between relative electrons and phonons. This results in the occurrence of electron cooling at low lattice temperatures.

This paper is organized as follows. In Sec. II we shall give a model Hamiltonian for the system and separate the center-of-mass degree of freedom from those of electrons in relative coordinates. In Sec. III we shall introduce an electron temperature T_e and set the initial condition for the density matrix of the relative electron-phonon system. By starting from the Liouville equation the density matrix is obtained to the first order of electron-impurity and electron-phonon interactions, and the statistical average of a dynamic variable is expressed in terms of retarded Green's function. Section IV is devoted to the derivation of the friction force experienced by the center-of-mass and energy transfer rate from the electron system to the phonon system by means of the Green's-function method. The main results are given there as the force- and energy-balance equations in steady state. These equations are applied to the calculation of the ratio of electron temperature to lattice temperature and the electric resistivity as functions of drift velocity for acoustic phonon plus impurity scattering at low temperatures (Sec. V), for acoustic-phonon scattering (Sec. VI A), for optical-phonon scattering (Sec. VI B), for combined acoustic-phonon and optical-phonon scattering (Sec. VI C) in the general case. Section VII focuses on the discussion of dynamic screening due to carrier-carrier interaction. Finally, in Sec. VIII, we summarize the main points of the present approach and give some discussions.

II. HAMILTONIAN

We consider an electron-lattice system, which consists of N electrons interacting with one another via Coulomb potential. The electrons are coupled with phonons and scattered by n_i randomly distributed impurities. Such a model should be applicable to metals and semiconductors in most cases. We use a single-band effective-mass Hamiltonian for this system, which, under the influence of an applied uniform electric field $\mathbf{E}$, can be written as

$$H=\sum_i \frac{\mathbf{p}_i^2}{2m}+\sum_{\substack{i,j\\ i<j}} \frac{e^2}{4\pi\epsilon_0 |\mathbf{r}_i-\mathbf{r}_j|}+\sum_{i,a} u(\mathbf{r}_i-\mathbf{R}_a) -\sum_{i,l}\mathbf{u}_l\cdot\nabla v_l(\mathbf{r}_i-\mathbf{R}_l)-e\sum_i \mathbf{r}_i\cdot\mathbf{E} \ . \tag{1}$$

Here, $\mathbf{r}_i$ and $\mathbf{P}_i=-i\nabla_i$ are the coordinate and momentum of the ith electron with effective mass m and charge e. $v_l(\mathbf{r}-\mathbf{R}_l)$ denotes the potential by the lth ion at regular lattice position $\mathbf{R}_l$, and $\mathbf{u}_l$ is its displacement from the equilibrium position; $u(\mathbf{r}-\mathbf{R}_a)$ denotes the potential by the impurity at $\mathbf{R}_a$, which is randomly located; and the second term on the right-hand side of Eq. (1) is the Coulomb interaction between electrons.

We shall separate the motion of the center of mass from the relative motion of the electrons by defining the center-of-mass momentum and coordinate variables $\mathbf{P},\mathbf{R}$ and the relative electron momentum and coordinate variables $\mathbf{p}_i',\mathbf{r}_i'$:[17–19]

$$\mathbf{P}=\sum_i \mathbf{p}_i, \quad \mathbf{R}=\frac{1}{N}\sum_i \mathbf{r}_i \ , \tag{2}$$

$$\mathbf{p}_i'=\mathbf{p}_i-\frac{1}{N}\mathbf{P}, \quad \mathbf{r}_i'=\mathbf{r}_i-\mathbf{R} \ , \tag{3}$$

which satisfy the following commutations:

$$[R_\alpha,P_\beta]=i\delta_{\alpha\beta} \ , \tag{4}$$

$$[r_{i\alpha}',p_{j\beta}']=i\delta_{\alpha\beta}(\delta_{ij}-1/N)\simeq i\delta_{\alpha\beta}\delta_{ij} \ , \tag{5}$$

and

$$[\mathbf{r}_i',\mathbf{P}]=[\mathbf{R},\mathbf{p}_i']=0 \ . \tag{6}$$

In Eq. (5) we have neglected $1/N$ for large N system. In terms of the center-of-mass variables and the relative variables we can express the Hamiltonian (1) as

$$H=H_c+H_e+H_{\mathrm{ph}}+H_{\mathrm{ei}}+H_{e\text{-ph}} \ , \tag{7}$$

$$H_c=\frac{\mathbf{P}^2}{2M}-Ne\mathbf{E}\cdot\mathbf{R} \ , \tag{8}$$

$$H_e=\sum_i \frac{(\mathbf{p}_i')^2}{2m}+\sum_{\substack{i,j\\ i<j}} \frac{e^2}{4\pi\epsilon_0|\mathbf{r}_i'-\mathbf{r}_j'|}$$
$$=\sum_{\mathbf{k},\sigma}\varepsilon_{\mathbf{k}}c^\dagger_{\mathbf{k}\sigma}c_{\mathbf{k}\sigma}+\frac{1}{2}\sum_{\substack{\mathbf{k},\mathbf{k}',\mathbf{q},\\ \sigma,\sigma'}} v_C(q)c^\dagger_{\mathbf{k}+\mathbf{q}\sigma}c^\dagger_{\mathbf{k}'-\mathbf{q}\sigma'}c_{\mathbf{k}'\sigma'}c_{\mathbf{k}\sigma}\ , \tag{9}$$

$$H_{\mathrm{ph}}=\sum_{\mathbf{q},\lambda}\Omega_{\mathbf{q}\lambda}b^\dagger_{\mathbf{q}\lambda}b_{\mathbf{q}\lambda}\ , \tag{10}$$

$$H_{ei}=\sum_{\mathbf{q},a}u(q)e^{i\mathbf{q}\cdot(\mathbf{R}-\mathbf{R}_a)}\rho_{\mathbf{q}}\ , \tag{11}$$

$$H_{e\text{-ph}}=\sum_{\mathbf{q},\lambda}M(\mathbf{q},\lambda)(b_{\mathbf{q}\lambda}+b^\dagger_{-\mathbf{q}\lambda})e^{i\mathbf{q}\cdot\mathbf{R}}\rho_{\mathbf{q}}\ , \tag{12}$$

where $M=Nm$ is the total mass of electrons, $c^\dagger_{\mathbf{k}\sigma}$ and $c_{\mathbf{k}\sigma}$ are creation and annihilation operators for relative electrons with wave vector $\mathbf{k}$, spin σ, and energy $\varepsilon_{\mathbf{k}}$, $\rho_{\mathbf{q}}=\sum_{\mathbf{k},\sigma}c^\dagger_{\mathbf{k}+\mathbf{q}\sigma}c_{\mathbf{k}\sigma}$ is electron density operator, $b^\dagger_{\mathbf{q}\lambda}$ and $b_{\mathbf{q}\lambda}$ are creation and annihilation operators for phonons with wave vector $\mathbf{q}$ in branch λ, $v_C(q)$ and $u(q)$ denote the Coulomb potential and impurity potential in momentum space, $v_C(q)=e/\epsilon_0^2q^2$, and $M(\mathbf{q},\lambda)$ is the electron-phonon matrix element, which satisfies $M(\mathbf{q},\lambda)=M^*(-\mathbf{q},\lambda)$.

According to Hamiltonian (7), the time derivatives of the center-of-mass momentum $\mathbf{P}$, of the relative electron Hamiltonian H_e, and of the phonon Hamiltonian H_{ph} are as follows:

$$\dot{\mathbf{P}}=-i[\mathbf{P},H]=Ne\mathbf{E}-i\sum_{\mathbf{q},\lambda}M(\mathbf{q},\lambda)\mathbf{q}e^{i\mathbf{q}\cdot\mathbf{R}}(b_{\mathbf{q}\lambda}+b^\dagger_{-\mathbf{q}\lambda})\rho_{\mathbf{q}}-i\sum_{\mathbf{q},a}u(q)\mathbf{q}e^{i\mathbf{q}\cdot(\mathbf{R}-\mathbf{R}_a)}\rho_{\mathbf{q}}\ , \tag{13}$$

$$\dot{H}_{\mathrm{ph}}=-i[H_{\mathrm{ph}},H]=i\sum_{\mathbf{q},\lambda}M(\mathbf{q},\lambda)\Omega_{\mathbf{q}\lambda}e^{i\mathbf{q}\cdot\mathbf{R}}(b_{\mathbf{q}\lambda}-b^\dagger_{-\mathbf{q}\lambda})\rho_{\mathbf{q}}\ , \tag{14}$$

$$\dot{H}_e=-i[H_e,H]=-i\sum_{\mathbf{q},\lambda}M(\mathbf{q},\lambda)e^{i\mathbf{q}\cdot\mathbf{R}}(b_{\mathbf{q}\lambda}+b^\dagger_{-\mathbf{q}\lambda})\sum_{\mathbf{k},\sigma}(\varepsilon_{\mathbf{k}+\mathbf{q}}-\varepsilon_{\mathbf{k}})c^\dagger_{\mathbf{k}+\mathbf{q}\sigma}c_{\mathbf{k}\sigma}$$
$$-i\sum_{\mathbf{q},a}u(q)e^{i\mathbf{q}\cdot(\mathbf{R}-\mathbf{R}_a)}\sum_{\mathbf{k},\sigma}(\varepsilon_{\mathbf{k}+\mathbf{q}}-\varepsilon_{\mathbf{k}})c^\dagger_{\mathbf{k}+\mathbf{q}\sigma}c_{\mathbf{k}\sigma}\ . \tag{15}$$

In obtaining Eq. (15) we have utilized the property that the second part of Eq. (9) (electron-electron interaction) commutes with the electron density operator $\rho_{\mathbf{q}}$. Thus,

$$[H_e,\rho_{\mathbf{q}}]=\sum_{\mathbf{k},\sigma}[H_e,c^\dagger_{\mathbf{k}+\mathbf{q}\sigma}c_{\mathbf{k}\sigma}]$$
$$=\sum_{\mathbf{k},\sigma}(\varepsilon_{\mathbf{k}+\mathbf{q}}-\varepsilon_{\mathbf{k}})c^\dagger_{\mathbf{k}+\mathbf{q}\sigma}c_{\mathbf{k}\sigma}\ . \tag{16}$$

III. DENSITY MATRIX AND STATISTICAL AVERAGE OF DYNAMIC VARIABLES

The Hamiltonian (7) features the separation of the center-of-mass variables from the relative electron variables. In the absence of H_{ei} and $H_{e\text{-ph}}$, the relative electrons are independent of the center of mass. The motion of the center of mass and that of the relative electrons are different in nature. The former is a pure mechanic movement of a single body with enormous mass. The motion of the relative electron system, which consists of a large number of interacting particles, is essentially a statistical movement and, together with the phonon system, should be described by a statistical density matrix ρ. The center-of-mass variable $\mathbf{R}$ enters the density matrix and the statistical average of a dynamical variable of the relative electron-phonon system via H_{ei} and $H_{e\text{-ph}}$. Because of its enormous mass, the center of mass almost obeys the classical equation of motion, and we shall regard $\mathbf{R}$ as the time-dependent coordinate of the center of mass, i.e., its expectation value $\mathbf{R}(t)$.

Therefore, H_{ei}, $H_{e\text{-ph}}$, $\dot{\mathbf{P}}$, $\dot{H}_e$, and $\dot{H}_{\mathrm{ph}}$ are time-dependent operators in the relative electron-phonon system. In the following, H_{It} will denote electron-impurity and electron-phonon interaction:

$$H_{It}=H_{ei}+H_{e\text{-ph}}\ . \tag{17}$$

The density matrix ρ for the relative electron-phonon system satisfies the Liouville equation[20]

$$i\frac{\partial\rho}{\partial t}=[H_e+H_{\mathrm{ph}}+H_{It},\rho]\ . \tag{18}$$

The solutions of the Liouville equation, which describe the time evolution of a statistical ensemble at any time after applying a perturbation, generally depend on the initial condition. However, we are interested only in steady state, i.e., the evolution of the system after a time interval which is larger than the characteristic time of the microrelaxation in the system, in which the memory on initial condition disappears. This enables us to choose a convenient initial condition for the steady-state solution of the Liouville equation. We will choose an equilibrium state, which seems as close to the steady state as possible, as the initial condition. To reach this end we imagine that the electron-phonon and electron-impurity interactions are turned off together with the electric field after the system has already reached the steady state. Then the center of mass, the relative electrons, and the phonons are decoupled from each other. The center of mass continues to move with a constant speed as in the steady state. The relative electrons, as an isolated system, will approach a thermal equilibrium state with temperature T_e. At the same time the phonon system will also approach its thermal equilibrium state with temperature T (for simplicity, T is considered to be the same as the heat-bath temperature). T_e obviously depends on the steady state before turning off the interaction and is not necessarily identical to T. Thus, the density matrix for the decoupled relative electron-phonon system takes the form

$$\rho_0=\frac{1}{Z_e}e^{-H_e/T_e}\frac{1}{Z_{\rm ph}}e^{-H_{\rm ph}/T}=\frac{1}{Z}e^{-H_0/T_e} . \tag{19}$$

Here $Z=Z_eZ_{\rm ph}$ and Z_e and $Z_{\rm ph}$ are the partition functions for equilibrium relative electrons and phonons, respectively. In Eq. (19) we have defined

$$H_0\equiv H_e+\alpha H_{\rm ph} \tag{20}$$

with

$$\alpha\equiv T_e/T . \tag{21}$$

Now we turn on the electron-phonon and electron-impurity interactions adiabatically and at the same time turn on the electric field. The occurrence of $H_{e\text{-ph}}$ and H_{ei} couples the relative electrons both with the center of mass and with phonons. The statistical density matrix will change from ρ_0. Therefore we need to solve the Liouville equation (18) with the initial condition

$$\rho(t=-\infty)=\rho_0 . \tag{22}$$

It is worth noting that although the steady-state solution ρ of the Liouville equation is independent of the choice of ρ_0 and the way to turn on the interactions and the electric field, the time required for the system to reach such a steady state does depend on them. Then it seems best to turn on the interactions and electric field in such a way that the center-of-mass velocity keeps almost unchanged during the process. Equation (18) together with the initial condition (22) can be written in the form of an integration equation:

$$\rho(t)=\rho_0-i\int_{-\infty}^{t}dt'\,e^{i(H_e+H_{\rm ph})(t'-t)}[H_{It'},\rho(t')]e^{-i(H_e+H_{\rm ph})(t'-t)} . \tag{23}$$

To the first order of the impurity scattering potential and the electron-phonon interaction the solution is

$$\rho(t)=\rho_0-i\int_{-\infty}^{t}e^{i(H_e+H_{\rm ph})(t'-t)}[H_{It'},\rho_0]e^{-(H_e+H_{\rm ph})(t'-t)}dt' . \tag{24}$$

Defining

$$H_{It'}(t)\equiv e^{iH_0t}H_{It'}e^{-iH_0t} . \tag{25}$$

We can rewrite Eq. (24) as

$$\rho(t)=\rho_0-i\int_{-\infty}^{t}e^{i(1-\alpha)H_{\rm ph}(t'-t)}[H_{It'}(t'-t),\rho_0]e^{-i(1-\alpha)H_{\rm ph}(t'-t)}dt' . \tag{26}$$

The statistical average of a dynamic variable A_t at time t is defined as

$$\begin{aligned}\langle A_t\rangle&=\mathrm{Tr}\{\rho(t)A_t\}=\mathrm{Tr}\{\rho_0A_t\}-i\int_{-\infty}^{t}\mathrm{Tr}\{e^{i(1-\alpha)H_{\rm ph}(t'-t)}[H_{It'}(t'-t),\rho_0]e^{-i(1-\alpha)H_{\rm ph}(t'-t)}A_t\}dt'\\&=\langle A_t\rangle_0-i\int_{-\infty}^{t}\mathrm{Tr}\{[H_{It'}(t'-t),\rho_0]A_t^{(\alpha-1)(t'-t)}\}dt'\\&=\langle A_t\rangle_0-i\int_{-\infty}^{t}\langle[A_t^{(\alpha-1)(t'-t)},H_{It'}(t'-t)]\rangle_0dt'\\&=\langle A_t\rangle_0+\int_{-\infty}^{\infty}\langle\langle A_t^{(\alpha-1)(t'-t)}(t)\,|\,H_{It'}(t')\rangle\rangle dt' .\end{aligned} \tag{27}$$

Here, we have used the notations

$$\langle(\cdots)\rangle_0\equiv\mathrm{Tr}\{\rho_0(\cdots)\} , \tag{28}$$

$$A_t^{(\alpha-1)\tau}(t')\equiv\exp(iH_0t')A_t^{(\alpha-1)\tau}\exp(-iH_0t') , \tag{29}$$

$$A_t^{(\alpha-1)\tau}\equiv\exp[i(\alpha-1)H_{\rm ph}\tau]A_t\exp[-i(\alpha-1)H_{\rm ph}\tau] , \tag{30}$$

and

$$\langle\langle B(t)\,|\,C(t')\rangle\rangle\equiv-i\Theta(t-t')\langle[B(t),C(t')]\rangle_0 \tag{31}$$

is a retarded Green's function for dynamical variables B and C calculated in the system described by density matrix ρ_0. It is easily seen that the statistical average of a Hermitian operator must be real.

IV. FORCE- AND ENERGY-BALANCE EQUATIONS

If we recognize that the current density J is related to the center-of-mass velocity v_d by $J=Nev_d$, then our essential task is to calculate v_d. In the presence of an electric field E, which is assumed to be along the x direction, the center-of-mass experiences an electric force due to E and frictional forces due to impurities and phonons. According to Eqs. (13) and (27), the statistical expectation value of the total force $\langle\dot{P}_x\rangle=\mathrm{Tr}\{\rho(t)\dot{P}_x\}$ can be written as

$$\begin{aligned}\langle\dot{P}_x\rangle=NeE&-in_i\sum_{\mathbf{q}}|u(q)|^2q_x\int_{-\infty}^{\infty}e^{i\mathbf{q}\cdot[\mathbf{R}(t)-\mathbf{R}(t')]}\langle\langle\rho_{\mathbf{q}}(t)\,|\,\rho_{-\mathbf{q}}(t')\rangle\rangle dt'\\&-i\sum_{\mathbf{q},\lambda}|M(\mathbf{q},\lambda)|^2q_x\int_{-\infty}^{\infty}e^{i\mathbf{q}\cdot[\mathbf{R}(t)-\mathbf{R}(t')]}[e^{-i(1-\alpha)\Omega_{\mathbf{q}\lambda}(t-t')}\langle\langle b_{\mathbf{q}\lambda}(t)\rho_{\mathbf{q}}(t)\,|\,b_{\mathbf{q}\lambda}^{\dagger}(t')\rho_{-\mathbf{q}}(t')\rangle\rangle\\&\qquad+e^{i(1-\alpha)}\Omega_{\mathbf{q}\lambda}(t-t')\langle\langle b_{-\mathbf{q}\lambda}^{\dagger}(t)\rho_{\mathbf{q}}(t)\,|\,b_{-\mathbf{q}\lambda}(t')\rho_{-\mathbf{q}}(t')\rangle\rangle]dt' ,\end{aligned} \tag{32}$$

with

$$b_{\mathbf{q}\lambda}(t)=e^{iH_0t}b_{\mathbf{q}\lambda}e^{-iH_0t}=b_{\mathbf{q}\lambda}e^{-i\alpha\Omega_{\mathbf{q}\lambda}t}\ , \tag{33}$$

$$b^{\dagger}_{\mathbf{q}\lambda}(t)=e^{iH_0t}b^{\dagger}_{\mathbf{q}\lambda}e^{-H_0t}=b^{\dagger}_{\mathbf{q}\lambda}e^{i\alpha\Omega_{\mathbf{q}\lambda}t}\ , \tag{34}$$

and

$$\rho_{\mathbf{q}}(t)=e^{iH_0t}\rho_{\mathbf{q}}e^{-iH_0t}\ . \tag{35}$$

On the right-hand side of Eq. (32), NeE is the electric force, and therefore the rest parts are the frictional forces. In steady state, the center of mass moves with a constant speed v_d along the x direction:

$$\mathbf{R}(t)-\mathbf{R}(t')=v_d(t-t')\mathbf{i}\ . \tag{36}$$

Thus, the frictional forces are functions of the drift velocity v_d, which can be written as

$$\begin{aligned}F(v_d)=&-in_i\sum_{\mathbf{q}}|u(q)|^2\mathbf{q}_x\hat{\Pi}(\mathbf{q},\omega_0)\\&-i\sum_{\mathbf{q},\lambda}|M(\mathbf{q},\lambda)|^2\mathbf{q}_x\\&\quad\times[\Lambda^-(\mathbf{q}\lambda,\omega_0-(1-\alpha)\Omega_{\mathbf{q}\lambda})\\&\qquad+\Lambda^+(\mathbf{q}\lambda,\omega_0+(1-\alpha)\Omega_{\mathbf{q}\lambda})]\end{aligned} \tag{37}$$

with

$$\omega_0\equiv\mathbf{q}_xv_d\ .$$

Here, $\hat{\Pi}(\mathbf{q},\omega)$, $\Lambda^-(\mathbf{q}\lambda,\omega)$, and $\Lambda^+(q\lambda,\omega)$ are, respectively, the Fourier transformations of the retarded Green's functions $\hat{\Pi}(\mathbf{q},t)$, $\Lambda^-(\mathbf{q}\lambda,t)$, and $\Lambda^+(\mathbf{q}\lambda,t)$:

$$\begin{aligned}\hat{\Pi}(\mathbf{q},\omega)&=\int_{-\infty}^{\infty}e^{i\omega t}\hat{\Pi}(\mathbf{q},t)dt\ ,\\\Lambda^-(\mathbf{q}\lambda,\omega)&=\int_{-\infty}^{\infty}e^{i\omega t}\Lambda^-(\mathbf{q}\lambda,t)dt\ ,\\\Lambda^+(\mathbf{q}\lambda,\omega)&=\int_{-\infty}^{\infty}e^{i\omega t}\Lambda^+(\mathbf{q}\lambda,t)dt\ ,\end{aligned} \tag{38}$$

and

$$\hat{\Pi}(\mathbf{q},t)=\langle\langle\rho_{\mathbf{q}}(t)\,|\,\rho_{-\mathbf{q}}(0)\rangle\rangle\ , \tag{39}$$

$$\begin{aligned}\Lambda^-(\mathbf{q}\lambda,t)&=\langle\langle b_{\mathbf{q}\lambda}(t)\rho_{\mathbf{q}}(t)\,|\,b^{\dagger}_{\mathbf{q}\lambda}(0)\rho_{-\mathbf{q}}(0)\rangle\rangle\ ,\\\Lambda^+(\mathbf{q}\lambda,t)&=\langle\langle b^{\dagger}_{-\mathbf{q}\lambda}(t)\rho_{\mathbf{q}}(t)\,|\,b_{-\mathbf{q}\lambda}(0)\rho_{-\mathbf{q}}(0)\rangle\rangle\ .\end{aligned} \tag{40}$$

$\hat{\Pi}(\mathbf{q},\omega)$, $\Lambda^-(\mathbf{q}\lambda,\omega)$, and $\Lambda^+(\mathbf{q}\lambda,\omega)$ can be calculated via the corresponding Matsubara Green's functions:

$$\hat{\pi}(\mathbf{q},\tau)=-\langle T_\tau[\rho_{\mathbf{q}}(\tau)\rho_{-\mathbf{q}}(0)]\rangle_0\ , \tag{41}$$

$$\begin{aligned}\lambda^-(\mathbf{q}\lambda,\tau)&=-\langle T_\tau[b_{\mathbf{q}\lambda}(\tau)\rho_{\mathbf{q}}(\tau)b^{\dagger}_{\mathbf{q}\lambda}(0)\rho_{-\mathbf{q}}(0)]\rangle_0\ ,\\\lambda^+(\mathbf{q}\lambda,\tau)&=-\langle T_\tau[b^{\dagger}_{-\mathbf{q}\lambda}(\tau)\rho_{\mathbf{q}}(\tau)b_{-\mathbf{q}\lambda}(0)\rho_{-\mathbf{q}}(0)]\rangle_0\ ,\end{aligned} \tag{42}$$

and their Fourier transformations:

$$\begin{aligned}\hat{\pi}(\mathbf{q},i\omega_\nu)&=\int_0^{1/T_e}\pi(\mathbf{q},\tau)e^{i\omega_\nu\tau}d\tau\ ,\\\lambda^-(\mathbf{q}\lambda,i\omega_\nu)&=\int_0^{1/T_e}\lambda^-(\mathbf{q}\lambda,\tau)e^{i\omega_\nu\tau}d\tau\ ,\\\lambda^+(\mathbf{q}\lambda,i\omega_\nu)&=\int_0^{1/T_e}\lambda^+(\mathbf{q}\lambda,\tau)e^{i\omega_\nu\tau}d\tau\ .\end{aligned} \tag{43}$$

Here,

$$b_{\mathbf{q}\lambda}(\tau)=e^{H_0\tau}b_{\mathbf{q}\lambda}e^{-H_0\tau}=b_{\mathbf{q}\lambda}e^{-\alpha\Omega_{\mathbf{q}\lambda}\tau}\ , \tag{44}$$

$$b^{\dagger}_{\mathbf{q}\lambda}(\tau)=e^{H_0\tau}b^{\dagger}_{\mathbf{q}\lambda}e^{-H_0\tau}=b^{\dagger}_{\mathbf{q}\lambda}e^{\alpha\Omega_{\mathbf{q}\lambda}\tau}\ , \tag{45}$$

$$\rho_{\mathbf{q}}(\tau)=e^{H_0\tau}\rho_{\mathbf{q}}e^{-H_0\tau}\ , \tag{46}$$

and

$$\omega_\nu=2\nu\pi T_e,\quad \nu=0,\ \pm1,\ldots\ . \tag{47}$$

$\langle\ \rangle_0$ means that the statistical average over the thermodynamic equilibrium system described by the density of matrix ρ_0 [Eq. (28)]. There is a standard graphical technique to calculate Matsubara Green's function,[21] and $\hat{\Pi}(\mathbf{q},\omega)$, $\Lambda^-(\mathbf{q}\lambda,\omega)$, and $\Lambda^+(\mathbf{q}\lambda,\omega)$ can be obtained by making the following analytical continuations:

$$\begin{aligned}\hat{\Pi}(\mathbf{q},\omega)&=\hat{\pi}(\mathbf{q},i\omega_\nu)\,|_{i\omega_\nu\to\omega+i\delta}\ ,\\\Lambda^-(\mathbf{q}\lambda,\omega)&=\lambda^-(\mathbf{q}\lambda,i\omega_\nu)\,|_{i\omega_\nu\to\omega+i\delta}\ ,\\\Lambda^+(\mathbf{q}\lambda,\omega)&=\lambda^+(\mathbf{q}\lambda,i\omega_\nu)\,|_{i\omega_\nu\to\omega+i\delta}\ .\end{aligned} \tag{48}$$

It is easy to verify

$$\lambda^-(\mathbf{q}\lambda,i\omega_\nu)=-T_e\sum_{\omega_\mu}\hat{\pi}(\mathbf{q},\ i\omega_\nu-i\omega_\mu)D(\mathbf{q}\lambda,i\omega_\mu)\ , \tag{49}$$

$$\lambda^+(\mathbf{q}\lambda,i\omega_\nu)=-T_e\sum_{\omega_\mu}\hat{\pi}(\mathbf{q},\ i\omega_\nu-i\omega_\mu)D(-\mathbf{q}\lambda,-i\omega_\mu)\ , \tag{50}$$

where $D(\mathbf{q}\lambda,i\omega_\mu)$ is the Fourier transformation of the phonon Green's function $D(\mathbf{q}\lambda,\tau)$, defined as

$$D(\mathbf{q}\lambda,\tau)=-\langle T_\tau[b_{\mathbf{q}\lambda}(\tau)b^{\dagger}_{\mathbf{q}\lambda}(0)]\rangle_0\ , \tag{51}$$

and

$$D(\mathbf{q}\lambda,i\omega_\mu)=\frac{1}{i\omega_\mu-a\Omega_{\mathbf{q}\lambda}}\ , \tag{52}$$

with $\omega_\mu=2\mu\pi T_e$ and $\alpha=T_e/T$. The expression (37) for frictional forces can be represented graphically by Fig. 1. The analytical continuation should follow the rule:

$$\begin{aligned}&i\omega\to\omega_0+i\delta\ ,\\&i\omega^-\to\omega_0-(1-\alpha)\Omega_{\mathbf{q}\lambda}+i\delta\ ,\\&i\omega^+\to\omega_0+(1-\alpha)\Omega_{\mathbf{q}\lambda}+i\delta\ .\end{aligned} \tag{53}$$

The summation over the imaginary frequencies $i\omega_\mu$ in Eqs. (49) and (50) can be carried out by means of contour integrations. After performing the analytical continuation from the Matsubara Green's functions we find the imaginary part of the function $\Lambda^-(\mathbf{q}\lambda,\omega)$ can be expressed as

$$\begin{aligned}\Lambda_2^-(\mathbf{q}\lambda,\omega)&\equiv\mathrm{Im}\Lambda^-(\mathbf{q}\lambda,\omega)\\&=\hat{\Pi}_2(\mathbf{q},\ \omega-\alpha\Omega_{\mathbf{q}\lambda})\\&\quad\times[n(\Omega_{\mathbf{q}\lambda}/T)-n((\alpha\Omega_{\mathbf{q}\lambda}-\omega)/T_e)]\ .\end{aligned} \tag{54}$$

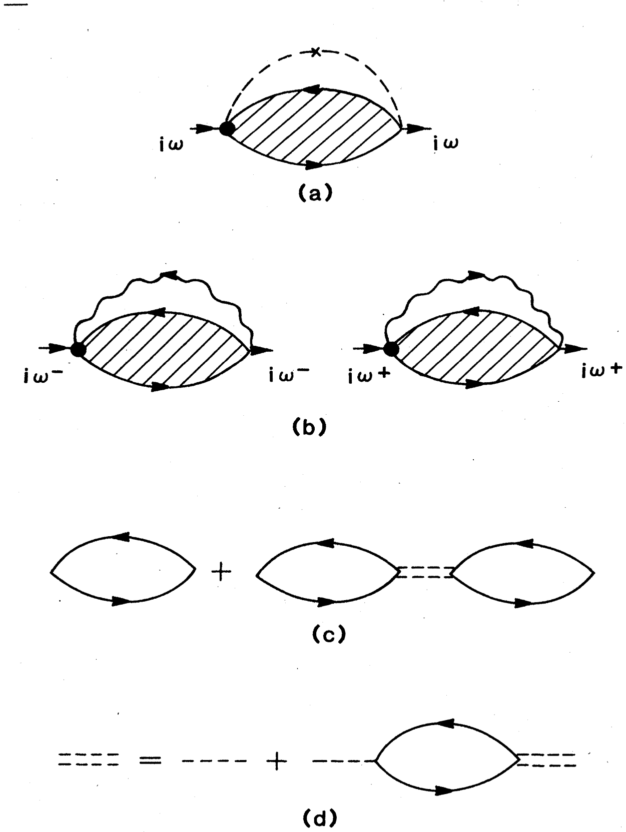

FIG. 1. (a) and (b) represent the frictional forces due to impurities and due to phonons to the first order of interactions. The dotted vertices denote momenta along the field direction. The dashed line with a cross represents impurity and the wavy line represents the phonon Green's function defined by Eq. (52). The shaded bubble is the electron density-density correlation function, which can be represented by (c) under the random-phase approximation (RPA). The double-dashed line in (c) is the dynamically screened Coulomb interaction, which satisfies the Dyson equation (d).

Similarly,

$$\Lambda_2^+(\mathbf{q}\,\lambda,\omega)\equiv \mathrm{Im}\Lambda^+(\mathbf{q}\,\lambda,\omega) = \hat{\Pi}_2(\mathbf{q},\,\omega+\alpha\Omega_{\mathbf{q}\lambda}) \times[n(\Omega_{\mathbf{q}\lambda}/T)-n((\alpha\Omega_{\mathbf{q}\lambda}+\omega)/T_e)]\ . \tag{55}$$

In these equations

$$n(x/T)\equiv\frac{1}{e^{x/T}-1} \tag{56}$$

is the Bose function and

$$\hat{\Pi}_2(\mathbf{q},\omega)\equiv \mathrm{Im}\hat{\Pi}(\mathbf{q},\omega) \tag{57}$$

is the imaginary part of the electron density-density correlation function calculated at electron temperature $T_e=\alpha T$. In the absence of Coulomb interaction between electrons the real part and the imaginary part of the density-density correlation function can be expressed as

$$\Pi_1(\mathbf{q},\omega)\equiv \mathrm{Re}\Pi(\mathbf{q},\omega) =2\sum_{\mathbf{k}} f(\varepsilon_{\mathbf{k}},T_e)\left[\frac{1}{\omega-\varepsilon_{\mathbf{k}+\mathbf{q}}+\varepsilon_{\mathbf{k}}}-\frac{1}{\omega+\varepsilon_{\mathbf{k}+\mathbf{q}}-\varepsilon_{\mathbf{k}}}\right], \tag{58}$$

$$\Pi_2(\mathbf{q},\omega)\equiv \mathrm{Im}\Pi(\mathbf{q},\omega) =2\pi\sum_{\mathbf{k}}[f(\varepsilon_{\mathbf{k}},T_e)-f(\varepsilon_{\mathbf{k}+\mathbf{q}},T_e)] \times\delta(\omega+\varepsilon_{\mathbf{k}+\mathbf{q}}-\varepsilon_{\mathbf{k}})\ , \tag{59}$$

with $f(\varepsilon_{\mathbf{k}},T_e)$ being the Fermi function at temperature T_e:

$$f(\varepsilon_{\mathbf{k}},T_e)=\frac{1}{\exp[(\varepsilon_{\mathbf{k}}-\varepsilon_f)/T_e]+1} \tag{60}$$

and ε_f is the chemical potential. The Coulomb interaction between electrons can be included by the use of the graphical technique for Matsubara Green's function. Under the random-phase approximation we obtain the well-known expression

$$\hat{\Pi}_2(\mathbf{q},\omega)=\frac{\Pi_2(\mathbf{q},\omega)}{[1-v_C(q)\Pi_1(\mathbf{q},\omega)]^2+[v_C(q)\Pi_2(\mathbf{q},\omega)]^2}\ , \tag{61}$$

in which

$$v_C(q)=\frac{e^2}{\epsilon_0 q^2} \tag{62}$$

is the Coulomb potential. It is easily seen that $\Pi_2(\mathbf{q},\omega)$ is an odd function of ω:

$$\hat{\Pi}_2(\mathbf{q},\omega)=-\hat{\Pi}_2(\mathbf{q},-\omega)\ . \tag{63}$$

Since $\dot{P}_x$ is Hermitian and its average value must be real, only the imaginary parts of Π, Λ^-, and Λ^+ are relevant to $F(v_d)$. By substituting the expressions (54) and (55) into the right-hand side of Eq. (37) and using Eq. (63), one can express the frictional forces experienced by the center of mass as

$$F(v_d)=n_i\sum_{\mathbf{q}}|u(q)|^2 q_x\hat{\Pi}_2(\mathbf{q},\omega_0) +2\sum_{\mathbf{q},\lambda}|M(\mathbf{q},\lambda)|^2 q_x\hat{\Pi}_2(\mathbf{q},\,\omega_0+\Omega_{\mathbf{q}\lambda}) \times[n(\Omega_{\mathbf{q}\lambda}/T)-n((\omega_0+\Omega_{\mathbf{q}\lambda})/\alpha T)]\ . \tag{64}$$

The first term on the right-hand side is the frictional force due to impurity scattering, and the second term is the frictional force due to phonons. In steady state, the total force acting on the center of mass should be zero: $\langle\dot{P}_x\rangle=0$, and thus we arrive at the momentum-balance equation

$$NeE+F(v_d)=0\ . \tag{65}$$

To derive the energy-balance equation we should calculate the statistical average of the time derivative of the energy operator for relative electrons $\langle \dot{H}_e \rangle$ (see Appendix A). In steady state, the total energy of the relative electron system must be constant: $\langle \dot{H}_e \rangle = 0$. This gives the energy-balance equation:

$$0 = n_i v_d \sum_{\mathbf{q}} |u(q)|^2 q_x \hat{\Pi}_2(\mathbf{q}, \omega_0 + \Omega_{\mathbf{q}\lambda}) + 2\sum_{\mathbf{q},\lambda} |M(\mathbf{q},\lambda)|^2 (\omega_0 + \Omega_{\mathbf{q}\lambda}) \hat{\Pi}_2(\mathbf{q}, \omega_0 + \Omega_{\mathbf{q}\lambda}) \times [n(\Omega_{\mathbf{q}\lambda}/T) - n((\omega_0 + \Omega_{\mathbf{q}\lambda})/\alpha T)] \ . \tag{66}$$

The energy-balance equation can also be derived if we calculate the increment of the phonon energy $\langle \dot{H}_p \rangle$, which, according to Hamiltonian (7), is just the energy transfer rate from electron system to phonon system. This calculation is described in Appendix B. The result is

$$\langle \dot{H}_{\text{ph}} \rangle \equiv W(v_d) = 2\sum_{\mathbf{q},\lambda} |M(\mathbf{q},\lambda)|^2 \Omega_{\mathbf{q}\lambda} \hat{\Pi}_2(\mathbf{q}, \omega_0 + \Omega_{\mathbf{q}\lambda}) \times [n(\Omega_{\mathbf{q}\lambda}/T) - n((\omega_0 + \Omega_{\mathbf{q}\lambda})/\alpha T)] \ . \tag{67}$$

In steady state, the energy supplied per unit time by the electric field to the electron system must be equal to the energy transfer rate from the electron system to the phonon system: $JE = \langle \dot{H}_p \rangle$. By combining with the momentum-balance equation (65), this relation gives the same equation as Eq. (66), which can be written as

$$v_d F(v_d) + W(v_d) = 0 \ . \tag{68}$$

In expression (67), the factor

$$n(\Omega_{\mathbf{q}\lambda}/T) - n((\omega_0 + \Omega_{\mathbf{q}\lambda})/T_e)$$

changes sign at $T_e = T$ when $v_d = 0$. This means that by neglecting the relative motion of the center of mass with respect to the lattice, the energy transmits from electron system to phonon system if $T_e > T$; from phonon system to electron system if $T_e < T$; no energy exchange if $T_e = T$, in agreement with the conventional idea of heat transfer. In the presence of an electric field, $v_d \neq 0$, however, the situation is different. The direction of energy transfer cannot be determined solely by temperature difference. It is possible for energy to transfer from electrons to lattice even when $T_e < T$. Electron cooling may occur in steady state in the presence of an electric field.

The electron-electron interaction has been included in the electron density-density correlation function $\Pi_2(\mathbf{q},\omega)$. One of the important features of our balance equations is that the drift velocity v_d enters the Π_2 factors dynamically in Eqs. (64) and (67), so that the high electric field may have a significant influence on the screening. For large v_d in most cases the denominator of the right-hand side of the expression (61) would reduce almost to 1, as if the high electric field acts to break up the screening. This is consistent with the high-field descreening effect discussed by Barker.[22]

The force- and energy-balance equations (65) and (68) are the main results of this work. If we define the total electric resistivity ρ_T as the ratio of the electric field to the current density

$$\rho_T \equiv \frac{E}{J} = \frac{E}{Nev_d} \ , \tag{69}$$

the force-balance equation (65) can be written as

$$\rho_T = \rho_i + \rho \ , \tag{70}$$

where

$$\rho_i = -\frac{n_i}{N^2 e^2 v_d} \sum_{\mathbf{q}} |u(q)|^2 q_x \hat{\Pi}_2(\mathbf{q}, \omega_0) \ , \tag{71}$$

is the resistivity due to impurity scattering and

$$\rho = -\frac{2}{N^2 e^2 v_d} \sum_{\mathbf{q},\lambda} |M(\mathbf{q},\lambda)|^2 q_x \hat{\Pi}_2(\mathbf{q}, \omega_0 + \Omega_{\mathbf{q}\lambda}) \times [n(\Omega_{\mathbf{q}\lambda}/T) - n((\omega_0 + \Omega_{\mathbf{q}\lambda})/\alpha T)] \tag{72}$$

is the resistivity due to phonons. We see that the contributions to resistivity by impurities and by phonons are additive. The impurity resistivity has been discussed in detail in Ref. 19. All the results obtained there are valid as long as the temperatures used there are considered to be electron temperature T_e. The energy-balance equation (68) can be rewritten in the form

$$\rho_T - \rho_E = 0 \tag{73}$$

with

$$\rho_E = \frac{2}{N^2 e^2 v_d^2} \sum_{\mathbf{q},\lambda} |M(\mathbf{q},\lambda)|^2 \Omega_{\mathbf{q}\lambda} \hat{\Pi}_2(\mathbf{q}, \omega_0 + \Omega_{\mathbf{q}\lambda}) \times [n(\Omega_{\mathbf{q}\lambda}/T) - n((\omega_0 + \Omega_{\mathbf{q}\lambda})/\alpha T)] \tag{74}$$

being equivalent energy transfer resistivity. The physical meaning of ρ_E is that $J^2 \rho_E$ is the energy transfer rate from electron system to phonon system.

From coupled equations (70) and (73) we can determine α, the ratio of the electron temperature T_e to the lattice temperature T, and the impurity- and phonon-induced resistivities ρ_i and ρ for given lattice temperature T as functions of drift velocity v_d. Then T_e, ρ_i, ρ, and current J as functions of the applied electric field are automatically obtained.

In the following sections we shall apply the balance equations (65) and (68) or (70) and (73) to a system composed of N electrons in a single parabolic band:

$$\varepsilon_k = k^2/2m \ , \tag{75}$$

where m is the effective mass. Such a model system can serve as an approximate description of some metals, semimetals, and semiconductors. Most of the results are also valid for carrier (electron or hole) conduction in conventional intrinsic or doped semiconductors, if the electron density N is considered to be temperature-dependent carrier density. For a single-band system with electron energy described by Eq. (75), the integrations in Eq. (58) can easily be carried out for a bulk crystal, and we have the following expression for $\Pi_2(q,\omega)$ [see Eq. (59)]:

$$\Pi_2(q,\omega)=-\frac{m^2T_e}{2\pi q}P_2(q,\omega)\ , \tag{76}$$

with

$$P_2(q,\omega)=\ln\left[\frac{1+\exp\left[-\frac{1}{2mT_e}\left[\frac{q}{2}-\frac{m\omega}{q}\right]^2+\frac{\epsilon_f}{T_e}\right]}{1+\exp\left[-\frac{1}{2mT_e}\left[\frac{q}{2}+\frac{m\omega}{q}\right]^2+\frac{\epsilon_f}{T_e}\right]}\right]\ . \tag{77}$$

The effects of the carrier-carrier interaction are included dynamically in the electron density-density correlation function $\hat{\Pi}_2(q,\omega)$ [Eq. (61)]. It's role in electron-impurity scattering is important and has been discussed in Ref. 19. However, care must be taken to consider the influence of the Coulomb interaction between carriers on electron-phonon scattering. For metals, the drift velocity v_d is often much less than the Fermi velocity v_F, and the static approximation should be sufficiently good for interelectron screening and is generally included in the electron-phonon matrix elements. For semiconductors, the electron-phonon interaction is usually studied in the absence of doping (or charge carriers). In order to compare with previous theories,[5,13–15] where the screening due to charge carrier has not been considered, the interaction between carriers in semiconductors is neglected until Sec. VII, where its full effects will be taken into account in the calculation.

V. IMPURITY- AND ACOUSTIC-PHONON SCATTERINGS

In this section we shall discuss some consequences of the balance equations (64) and (65) at low electron temperatures, assuming the primary scattering mechanisms are electron-impurity and electron—acoustic-phonon interactions.

When electron temperature $T_e \ll \epsilon_F$ (degenerate case), the expression (76) for $\Pi_2(q,\omega)$ reduces to

$$\Pi_2(q,\omega)=-\frac{m^3v_F^2}{4\pi q}\left\{\left[1-\left[\frac{q}{2k_F}-\frac{\omega}{qv_F}\right]^2\right]\Theta\left[1-\left[\frac{q}{2k_F}-\frac{\omega}{qv_F}\right]^2\right]-\left[1-\left[\frac{q}{2k_F}+\frac{\omega}{qv_F}\right]^2\right]\Theta\left[1-\left[\frac{q}{2k_F}+\frac{\omega}{qv_F}\right]^2\right]\right\}, \tag{78}$$

and the impurity-induced resistivity has been shown to be insensitive to temperature.[19] We shall use deformation potential for electron—acoustic-phonon interaction,[23] by which only the longitudinal phonon contributes for the nondegenerate electron band. The electron-phonon matrix element takes the form

$$|M(q,\lambda)|^2=\frac{E_1^2q^2}{2\Omega_q d}\ , \tag{79}$$

where d is the mass density of the lattice and E_1 is the shift of the band edge per unit dilatation.[23] For the dispersion relation of the longitudinal-acoustic phonon we shall use the Debye spectrum

$$\Omega_q=\Omega_s=v_sq \tag{80}$$

with v_s denoting the sound speed for longitudinal waves, which is connected with the Debye temperature Θ_D by

$$\Theta_D=v_sq_D\ , \tag{81}$$

where q_D is the Debye wave vector. To simplify the expressions we shall use the following notations:

$$\Theta_F\equiv 2k_F\Theta_D/q_D\ , \tag{82}$$

$$t_F\equiv T/\Theta_F\ . \tag{83}$$

A. Ohmic resistivity, Bloch-Grüneisen formula

First, we discuss the solution to Eqs. (53) and (56) in the zero-field limit. The impurity part of the resistivity has been given in Ref. 19:

$$\rho_i(v_d\to 0)\equiv\rho_{i0}=\frac{m^2n_i}{12\pi^3N^2e^2}\int_0^{2k_F}|\bar{u}(q)|^2q^3dq\ . \tag{84}$$

Here

$$\bar{u}(q)=u(q)/[1-v_C(q)\Pi_1(q,0)]$$

is the statically screened electron-impurity potential. For small drift velocity we can expand the factors with the Bose function in these equations in terms of ω_0 and $(\alpha-1)$. Moreover, if $v_s/v_F\ll 1$, we can use the approximate expression for $\Pi_2(q,\,\omega_0+\Omega_q)$:

$$\Pi_2(q,\,\omega_0+\Omega_q)\simeq-\frac{m^2}{2\pi q}\Theta(1-(q/2k_F)^2)(\omega_0+\Omega_q)\ . \tag{85}$$

To the lowest order in v_d, the solution for energy-balance equation is

$$\alpha=1 \tag{86}$$

and the phonon part of the resistivity is

$$\rho_B=\rho^*g(t_F) \tag{87}$$

with

$$\rho^*=\frac{8m^7v_F^4E_1^2}{3\pi^3dN^2e^2} \tag{88}$$

and

$$g(t)=\frac{1}{t}\int_0^1 dy\frac{y^5}{(e^{y/t}-1)(1-e^{-y/t})}\ . \tag{89}$$

Equation (87) is the well-known Bloch-Grüneisen formula for lattice resistivity, which behaves like T^5 and T, respectively, at low and high temperatures. When the condition $v_s/v_F \ll 1$ is not satisfied, we have to be careful in expanding Π_2. In linear limit the result (86), that is $T_e = T$, is always true, but the Ohmic resistivity due to phonon

$$\rho_0 \equiv \rho(v_d \to 0) \tag{90}$$

will be dependent on v_s/v_F, and therefore different from the Bloch-Grüneisen expression.

B. Solution to the order of $(v_d/v_s)^2$

In a similar way we can obtain α and resistivity as functions of lattice temperature to the order of $(v_d/v_s)^2$ for small v_s/v_F. The results can be written as

$$\alpha = 1 + A(v_d/v_s)^2 , \tag{91}$$

$$\rho = \rho_B[1 + B(v_d/v_s)^2] , \tag{92}$$

with

$$A = \tfrac{1}{3}\left\{-1 + \frac{1}{2g(t_F)}\left[-\frac{1}{t_F}n'\left(\frac{1}{t_F}\right)\right] + \frac{\rho_{i0}}{\rho^*}\frac{1}{g(t_F)}\right\} , \tag{93}$$

and

$$\begin{aligned} B = A &\left\{4 - \frac{1}{g(t_F)}\left[-\frac{1}{t_F}n'\left(\frac{1}{t_F}\right)\right]\right\} \\ &+ \tfrac{1}{10}\left\{24 - \frac{4}{g(t_F)}\left[-\frac{1}{t_F}n'\left(\frac{1}{t_F}\right)\right]\right. \\ &\left. - \frac{1}{g(t_F)}\frac{1}{t_F^2}n''\left(\frac{1}{t_F}\right)\right\} . \end{aligned} \tag{94}$$

Here, ρ_{i0} is the Ohmic resistivity due to impurity scattering. The total resistivity $\rho_T = \rho_i + \rho \simeq \rho_{i0} + \rho$ when $v_s/v_F \ll 1$. At high temperature $t_F \gg 1$,

$$\begin{aligned} &A \simeq \tfrac{1}{3} , \\ &B \simeq 0 . \end{aligned} \tag{95}$$

For $t_F \ll 1$,

$$\begin{aligned} &A \simeq \tfrac{1}{3}\left(-1 + \frac{\rho_{i0}}{\rho^*}\frac{1}{g(t_F)}\right) , \\ &B \simeq \tfrac{1}{3}\left(\tfrac{16}{5} + \frac{\rho_{i0}}{\rho^*}\frac{1}{g(t_F)}\right) . \end{aligned} \tag{96}$$

The most interesting feature is that for small impurity resistivity the coefficient A is negative in a certain temperature range, i.e., the electron temperature is lower than lattice temperature. The values of A and B for several different impurity resistivities have been shown in Ref. 24 as functions of T/Θ_F in the case of $v_s/v_F \ll 1$.

C. Zero-lattice-temperature limit

It is also interesting to discuss the zero-lattice-temperature limit. When $T \to 0$, the terms with $n(\Omega_{\mathbf{q}\lambda}/T)$ in force- and energy-balance equations vanish. Only the electron temperature T_e exists in balance equations. We find the electron temperature T_e approaches a finite value $T^* = T_e$ $(T \to 0)$, which depends on electric field and impurity resistivity. For $v_s \ll v_F$, the energy-balance equation and the phonon-induced resistivity at $T \to 0$ can be written, respectively, as

$$\int_0^1 dy \int_{-1}^1 dx\, y^4 \left(1 + \frac{v_d}{v_s}x\right)^2 n\left[\frac{y}{t_F}\left(1 + \frac{v_d}{v_s}x\right)\right] = \frac{2}{3}\frac{\rho_i}{\rho^*}\left(\frac{v_d}{v_s}\right)^2 \tag{97}$$

and

$$\frac{\rho}{\rho^*} = -\frac{3}{2}\frac{v_s}{v_d}\int_0^1 dy \int_{-1}^1 dx\, y^4 x \left(1 + \frac{v_d}{v_s}x\right) \times n\left[\frac{y}{t_F}\left(1 + \frac{v_d}{v_s}x\right)\right] . \tag{98}$$

If $v_d/v_s < 1$, the integrations in Eqs. (97) and (98) are easily carried out and we have the solution

$$\left(\frac{T^*}{\Theta_F}\right)^5 = 0.0134\frac{\rho_{i0}}{\rho^*}\left(\frac{v_d}{v_s}\right)^2\left[1 - \left(\frac{v_d}{v_s}\right)^2\right]^2 \tag{99}$$

and

$$\frac{\rho}{\rho^*} = \frac{4}{3}\frac{\rho_{i0}}{\rho^*}\frac{(v_d/v_s)^2}{1 - (v_d/v_s)^2} . \tag{100}$$

The impurity resistivity ρ_i is almost equal to its weak-field value ρ_{i0}. The electric field $E = (\rho + \rho_i)J$ can be expressed as

$$\frac{E}{E^*} = \frac{\rho_{i0}}{\rho^*}\frac{v_d}{v_s}\left(1 + \frac{4}{3}\frac{(v_d/v_s)^2}{1 - (v_d/v_s)^2}\right) \tag{101}$$

with $E^* = \rho^* Nev_s$. It is easily seen from Eqs. (99) and (101) that for $v_d/v_s \ll 1$, or $E \ll E^*$,

$$\frac{T^*}{\Theta_F} = 0.422\left(\frac{\rho^*}{\rho_{i0}}\right)^{1/5}\left(\frac{E}{E^*}\right)^{2/5} \tag{102}$$

and

$$\frac{\rho}{\rho^*} = \frac{4}{3}\frac{\rho^*}{\rho_{i0}}\left(\frac{E}{E^*}\right)^2 . \tag{103}$$

Equation (102) is in agreement with the result obtained recently by Arai[15] from carrier temperature model. It should be noticed that Eqs. (102) and (103) are valid only in a rather narrow electric field range, depending on the impurity resistivity ρ_{i0}/ρ^*. When $\rho_{i0} = 0$ they are completely invalid. However, Eqs. (99)—(101) can describe a zero-impurity system at $v_d/v_s < 1$, i.e., when $\rho_{i0} = 0$ we have $T^* = 0$, $\rho = 0$, and $E = 0$ for $v_d < v_s$. This means that in the absence of impurity scattering the current can flow without resistance up to the drift velocity $v_d = v_s$ at $T = 0$

K. But once $v_d > v_s$ the resistance shows up in our theory. The calculated results of T^*/Θ_F and the phonon-induced resistivity ρ/ρ^* have been presented in Ref. 25 as functions of an electric field for different values of impurity scatterings. It is worth noticing that in the presence of a finite electric field the phonon contribution to resistivity does not vanish at $T=0$ K.

D. Electron cooling at low temperatures

The lowering of the electron temperature T_e below the lattice temperature occurs also at relatively large drift velocities. In Fig. 2 we show the calculated temperature ratio $\alpha = T_e/T$ and the dimensionless phonon resistivity ρ/ρ_0 as functions of v_d/v_s for several different impurity resistivities at $v_s/v_F=0.02$ and $T/\Theta_F=0.05$. There exist minima in T_e/T curves at $v_d \simeq v_s$ for relatively clean samples. For a given lattice temperature T, the lowest T_e/T occurs when the impurity resistivity is zero. The impurity scattering always increases T_e/T, or makes electrons "hotter," and therefore the electron cooling happens only in samples with low impurity concentrations.

The electron cooling occurs usually at low lattice temperatures. In the presence of a finite-impurity scattering, the lattice temperature range for electron cooling not only has an upper limit, but also has a lower limit $T_m > 0$. In Fig. 3 we plot T_e/T as a function of dimensionless electric field E/E^* at different temperatures for a system with $v_s/v_F=0.02$ and a fixed impurity scattering $\rho_{i0}/\rho^*=0.001$. We see that when $T/\Theta_F < 2.2\times10^{-2}$, the cooling disappears. For zero-impurity samples, the range of the electric field in which the electron cooling exists vanishes when $T\rightarrow0$. Therefore, at finite electric field the electrons always become hot and T_e approaches a finite limit T^* when $T\rightarrow0$.

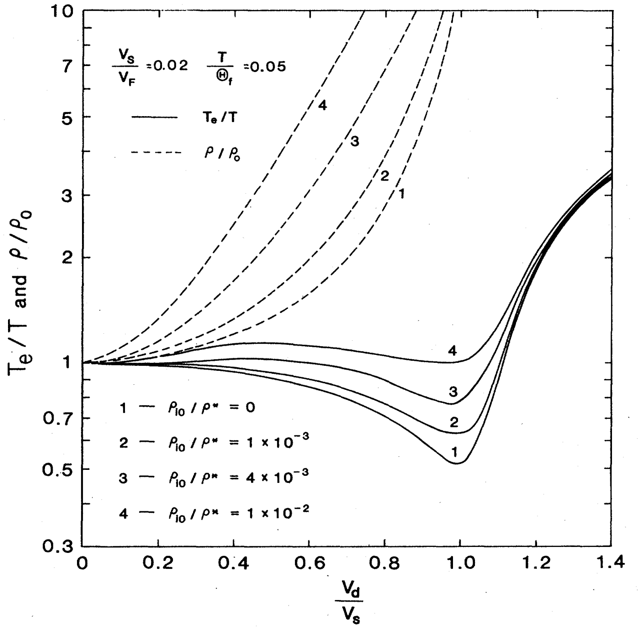

FIG. 2. Temperature ratio T_e/T and the dimensionless phonon-induced resistivity are shown as functions of v_d/v_s at several different values of impurity resistivity for systems with $v_s/v_F=0.02$ at $T/\Theta_F=0.05$. Zero-field phonon resistivity $\rho_0=1.94\times10^{-3}\rho^*$.

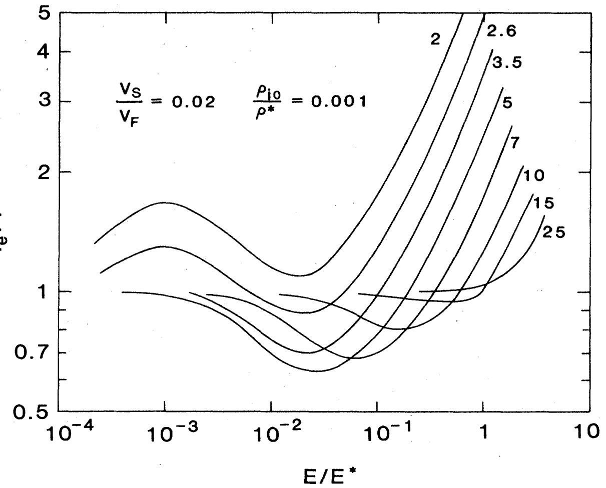

FIG. 3. Temperature ratio T_e/T is shown as a function of dimensionless electric field E/E^* for a system with $v_s/v_F=0.02$ and $\rho_{i0}/\rho^*=0.001$. The number $\times10^{-2}$ is the value of T/Θ_F for a nearby curve.

Another interesting feature is the impurity dependence of the phonon-induced resistivity, as can be seen in Fig. 2. This results from the variation of the electron temperature with impurity.

VI. RESULTS DUE TO ACOUSTIC- AND OPTICAL-PHONON SCATTERINGS

In this section we shall discuss acoustic-phonon scattering, optical-phonon scattering, and combined acoustic- and optical-phonon scattering in a wide range of temperatures. In these cases the variation of chemical potential with electron temperature T_e must be taken into account. For a single-band system with fixed electron density N, the chemical potential ϵ_f can be determined as a function of T_e by the equation

$$N=2\sum_k \frac{1}{\exp[(\varepsilon_k-\epsilon_f)/T_e]+1} . \tag{104}$$

A. Acoustic-phonon scattering

For acoustic-phonon scattering it is convenient to introduce a temperature-dimension parameter:

$$\Theta_s = mv_s^2 , \tag{105}$$

which is about 1 K if $v_s=5\times10^5$ cm/sec and $m=0.6m_e$ (m_e is the free-electron mass). We shall also use the dimensionless temperature

$$t_s \equiv T/\Theta_s \tag{106}$$

in the formula.

The electric resistivity ρ and the equivalent energy transfer resistivity ρ_E due to acoustic phonon can be written in the form

$$\rho=\rho_s^*\alpha t_s\left(\frac{v_s}{v_d}\right)\int d\left(\frac{q}{mv_s}\right)\left(\frac{q}{mv_s}\right)^3\int dx\,xP_2(q,\omega_0+\Omega_s)[n(\Omega_s/T)-n((\omega_0+\Omega_s)/\alpha T)]\ , \tag{107}$$

$$\rho_E=-\rho_s^*\alpha t_s\left(\frac{v_s}{v_d}\right)^2\int d\left(\frac{q}{mv_s}\right)\left(\frac{q}{mv_s}\right)^3\int dx\,P_2(q,\omega_0+\Omega_s)[n(\Omega_s/T)-n((\omega_0+\Omega_s)/\alpha T)]\ , \tag{108}$$

where $\omega_0=qv_dx,x$ is the cosine of the angle between $\mathbf{q}$ and $\mathbf{v}_d$, and

$$\rho_s^*=\frac{m^7v_s^4E_1^2}{8\pi^3dN^2e^2}=\frac{3}{64}\left(\frac{v_s}{v_F}\right)^4\rho^*\ . \tag{109}$$

In writing Eqs. (107) and (108) we have used the deformation potential for the electron-acoustic phonon matrix element and the Debye spectrum for phonon dispersion: $\Omega_s=v_sq$. In this section we assumed the acoustic phonons are the only scatters and therefore the energy-balance equation is

$$\rho-\rho_E=0\ . \tag{110}$$

Most of the experimental investigations on nonlinear transport were carried out for semiconductors, in which the carriers in thermal equilibrium obey the Maxwell-Boltzmann distribution. For a single-band system with fixed electron density N, this happened at high electron temperature $T_e=\alpha T\gg\epsilon_F=\frac{1}{2}mv_F^2$ (ϵ_F is the Fermi energy at $T_e=0$ K) so that the chemical potential ϵ_f becomes negative and $|\epsilon_f|/T_e\gg1$. In this case,

$$e^{-|\epsilon_f|/T_e}=\frac{N}{2}\left(\frac{2\pi}{mT_e}\right)^{3/2} \tag{111}$$

and the function $P_2(q,\omega)$ reduces to

$$P_2(q,\omega)=e^{-|\epsilon_f|/T_e}\left\{\exp\left[-\frac{1}{2mT_e}\left(\frac{q}{2}-\frac{m\omega}{q}\right)^2\right]-\exp\left[-\frac{1}{2mT_e}\left(\frac{q}{2}+\frac{m\omega}{q}\right)^2\right]\right\}. \tag{112}$$

For conventional semiconductors, Eq. (111) is also valid if $|\epsilon_f|$ is considered to be the distance between the bottom (or top) of the conduction band and the chemical potential.

We discuss the solutions of the energy-balance equation (110) under the condition that $\alpha t_s\gg1$ in the Maxwell-Boltzmann case. To the lowest order of the expansion in $(\alpha t_s)^{-1}$, the energy-balance equation reduces to

$$\left(\frac{v_d}{v_s}\right)^2=\frac{6\alpha^3t_s^3}{S(\alpha,t_s)}-3\ , \tag{113}$$

where

$$S(\alpha,t_s)=\int_0^\infty e^{-y^2/(2\alpha t_s)}y^4\coth\left(\frac{y}{t_s}\right)dy\ . \tag{114}$$

If, at the same time $\alpha/t_s\gg1$, we have

$$S(\alpha,t_s)\simeq3\frac{\pi}{\sqrt{2}}(\alpha t_s)^{5/2} \tag{115}$$

and then

$$\alpha=\frac{\pi}{8t_s}\left(\frac{v_d}{v_s}\right)^4. \tag{116}$$

The dimensionless resistivity ρ/ρ_0 for a single-band model at $\alpha t_s\gg1$ can be expressed as

$$\frac{\rho}{\rho_0}=\frac{S(\alpha,t_s)}{2\alpha^{3/2}g_s(t_s)} \tag{117}$$

with

$$g_s(t_s)=\int_0^\infty\left[\exp\left(-\frac{(y-1)^2}{2t_s}\right)\right][-n'(2y/t_s)]dy\ , \tag{118}$$

which, for $t_s\gg1$, can reduce to

$$g_s(t_s)\simeq t_s^3\ . \tag{119}$$

Therefore, when $\alpha/t_s\gg1$ and $t_s\gg1$, we have

$$\frac{\rho}{\rho_0}\simeq1.88\frac{\alpha}{t_s^{1/2}} \tag{120}$$

for a single-band system. By combining Eqs. (116) and (120) we can easily obtain the relation between $\alpha=T_e/T$ and the electric field $E=\rho Nev_d$:

$$\frac{T_e}{T}=0.5\left(\frac{T}{\Theta_s}\right)^{1/5}\left(\frac{E}{E_0}\right)^{4/5}, \tag{121}$$

which is valid for $T/\Theta_s\gg1$ and $E/E_0\gg1$. Here $E_0=\rho_0Nev_s=v_s/\mu_0$, μ_0 is the carrier mobility in the zero-field limit. Equation (121) is different from that deduced by carrier temperature model. The latter predicts a linear dependence of T_e/T on electric field with a temperature-independent coefficient.[13]

In the general case we have, for a single-band system with parameters $v_s/v_F=0.02$ and 0.5, calculated the temperature ratio $\alpha=T_e/T$ and dimensionless phonon resistivity ρ/ρ_0 as functions of v_d/v_s at different temperatures. The results are shown in Figs. 4 and 5. The temperature variation of the chemical potential is included in the calculation. In Fig. 5 the curves for $v_d/v_s\geq2$ are almost identical with those calculated in the Maxwell-Boltzmann limit, while most curves of Fig. 4 are in degenerate and intermediate cases. In both cases the electron temperature can increase to more than 100 times the lattice temperature when $v_d/v_s\sim5$. Yet, at relative low-temperature electron cooling occurs for $v_d\lesssim v_s$.

B. Optical-phonon scattering

The spectra of an optical phonon and an acoustic phonon are quite different and the q dependence of the matrix element of the electron—optical-phonon interaction is also different from that of the electron—acoustic-phonon in-

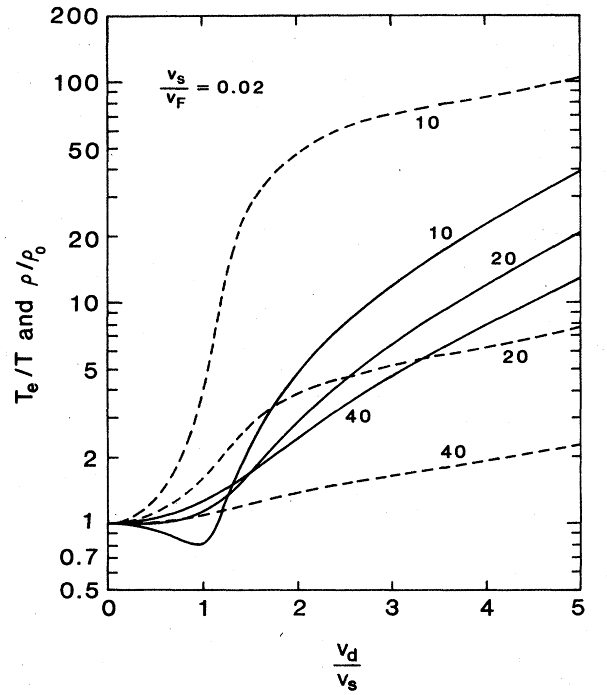

FIG. 4. Temperature ratio T_e/T (solid lines) and the dimensionless phonon-induced resistivity (dashed lines) are shown as functions of v_d/v_s at several different lattice temperatures for a system with $v_s/v_F=0.02$. The scattering is entirely due to acoustic phonons. The numbers near the curves are the values of T/Θ_s.

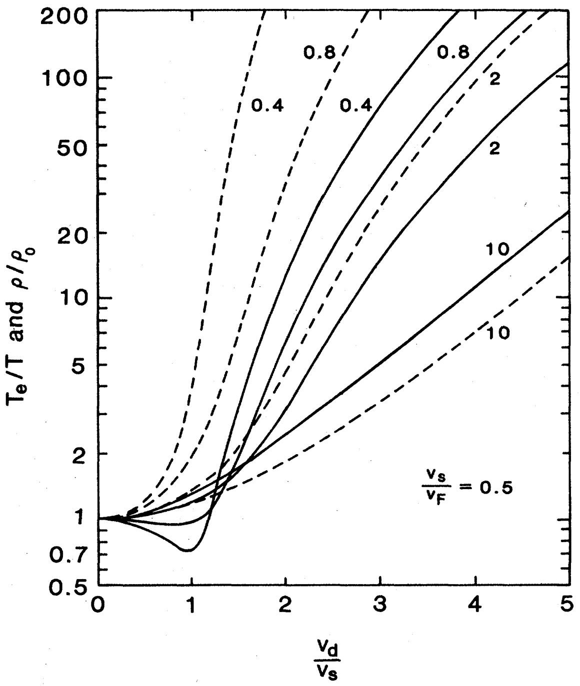

FIG. 5. Temperature ratio T_e/T (solid lines) and the dimensionless phonon-induced resistivity (dashed lines) are shown as functions of v_d/v_s at several different lattice temperatures for a system with $v_s/v_F=0.5$. The scattering is entirely due to acoustic phonons. The numbers near the curves are the values of T/Θ_s.

teraction. These make the behavior of the temperature ratio and resistivity due to optical-phonon scattering as functions of drift velocity significantly distinct from those discussed in the above subsection. As is usually done in the discussion of optical-phonon scattering, we take the optical mode frequency as a constant (the Einstein model):

$$\Omega_{q\lambda}=\Omega_o \,. \tag{122}$$

For the matrix element of electron-nonpolar optical-phonon interaction we use[24]

$$|M(q,\lambda)|^2=\frac{D^2}{2\Omega_o d} \,, \tag{123}$$

where D is the shift of the band edge per unit relative displacement of the two sublattices relative to the optical mode.

In describing optical-phonon scattering, it is convenient to introduce temperature and velocity parameters Θ_o and v_o, defined by

$$\Theta_o=mv_o^2=\Omega_o \,, \tag{124}$$

in which m is the effective mass of the charge carriers. We shall also use a dimensionless temperature

$$t_o\equiv T/\Theta_o \tag{125}$$

in the following expressions.

The electric resistivity ρ and the equivalent energy transfer resistivity ρ_E due to optical-phonon scattering are, respectively,

$$\rho=\rho_o^*\alpha t_o\left[\frac{v_o}{v_d}\right]\int d\left[\frac{q}{mv_o}\right]\left[\frac{q}{mv_o}\right]^2\int dx\, xP_2(q,\omega_o+\Omega_o)[n(\Omega_o/T)-n((\omega_0+\Omega_o)/\alpha T)] \tag{126}$$

and

$$\rho_E=-\rho_o^*\alpha t_o\left[\frac{v_o}{v_d}\right]^2\int d\left[\frac{q}{mv_o}\right]\frac{q}{mv_o}\int dx\, P_2(q,\omega_0+\Omega_o)[n(\Omega_o/T)-n((\omega_0+\Omega_o)/\alpha T)] \,. \tag{127}$$

Here,

$$\rho_o^*=\frac{m^5v_o^2D^2}{8\pi^3dN^2e^2} \,. \tag{128}$$

If we assume optical phonons are the dominant scatterers, the energy-balance equation will be the same form as Eq. (110):

$$\rho-\rho_E=0 \, .$$

The significant feature of the optical-phonon scattering is the saturation of the current density at high electric field. That is, the solution of the energy-balance equation exists only when $v_d < v_m$. In order to determine the saturation value of the drift velocity, we consider the asymptotic behavior of the energy-balance equation at large αt_o. To the leading term of its expansion in $(\alpha t_o)^{-1}$, the energy-balance equation gives

$$\left[\frac{v_d}{v_o}\right]^2=6\left[2-\frac{K_1'(1/2\alpha t_o)}{\alpha t_o K_1(1/2\alpha t_o)}\right]\tanh\left[\frac{1}{2t_o}\right] , \qquad (129)$$

where $K_1(x)$ and $K_1'(x)$ are the modified Bessel function and its derivative. The saturation value v_m can be obtained by letting $\alpha t_o \to \infty$ in Eq. (129):

$$\left[\frac{v_m}{v_o}\right]^2=\tfrac{3}{4}\tanh\left[\frac{1}{2t_o}\right] , \qquad (130)$$

which is temperature dependent, having a maximum $v_m=0.866 v_o$. In the limit $E\to\infty$ the above result is exactly equal to that obtained from the phenomenological Boltzmann equation (see page 197 of Ref. 13). It is also consistent with the result of Thornber and Feynmann[6] as far as the current saturation at high field is concerned.

The calculated temperature ratio $\alpha=T_e/T$ and electric resistivity due to optical phonon as functions of v_d/v_o are shown in Fig. 6 for $v_o/v_F=1$ at several different values of T/Θ_o by assuming optical phonons are the only scatterers. The change of the chemical potential ϵ_f with varying electron temperature T_e is included in the calculation. The saturation of the current density is clearly shown. Moreover, electron cooling also appears in the lowest temperature curve.

C. Combined acoustic- and optical-phonon scatterings

Usually, both acoustic-phonon scattering and optical-phonon scattering are existent in a system. At low lattice and electron temperatures optical phonons can hardly be excited, and acoustic-phonon scattering (together with impurity scattering) dominates the process. When lattice temperature becomes high or electrons become hot, both acoustic phonon and optical phonon should be taken into account. Here, we shall give a brief discussion and some numerical results on the nonlinear transport due to combined acoustic-phonon and nonpolar optical-phonon scatterings.

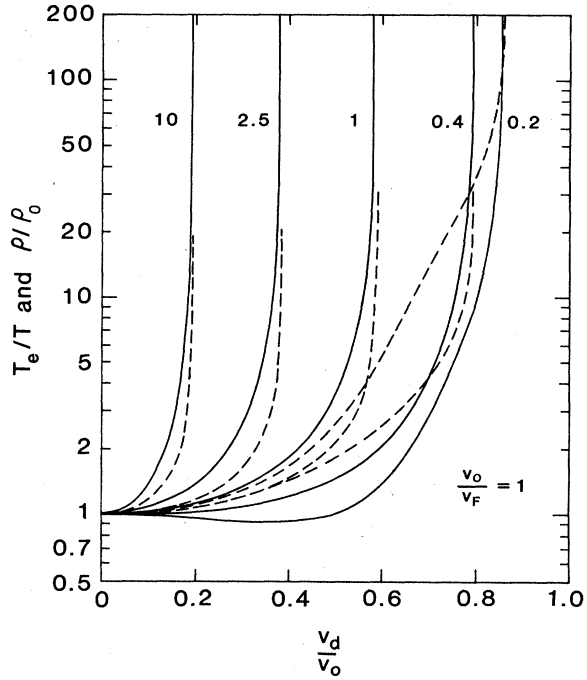

FIG. 6. Temperature ratio T_e/T (solid lines) and the dimensionless phonon-induced resistivity (dashed lines) are shown as functions of v_d/v_s at several different lattice temperatures for a system with $v_o/v_F=1$. The scattering is entirely due to nonpolar-optical phonons. The numbers near the curves are the values of T/Θ_o.

The total electric resistivity is the sum of those contributed by acoustic phonons and by optical phonons:

$$\rho=\rho_a+\rho_o \, , \qquad (131)$$

and the equivalent energy transfer resistivity should be

$$\rho_E=\rho_{Ea}+\rho_{Eo} \, . \qquad (132)$$

The expressions for ρ_a, ρ_{Ea}, ρ_o, and ρ_{Eo}, are (107), (108), (126), and (127), respectively. The energy-balance equation now becomes

$$\rho_a-\rho_{Ea}+\rho_o-\rho_{Eo}=0 \, . \qquad (133)$$

The saturation of the current density in high field, which happens in the case of optical-phonon scattering, now disappears. This is easily seen from the balance equation at $\alpha t_o \gg 1$:

$$\left[\frac{v_d}{v_o}\right]^2=\frac{\frac{3}{4}\left[\frac{v_s}{v_F}\right]^4[(\alpha t_s)^3-\frac{1}{2}S(\alpha,t_s)]+4K_1(1/2\alpha t_o)\rho_o^*/\rho^*}{\frac{3}{8}\frac{v_s^2 v_o^2}{v_F^4}s(\alpha,t_s)+\frac{8}{3}\left[K_1(1/2\alpha t_o)-\frac{K_1'(1/2\alpha t_o)}{2\alpha t_o}\right]\coth\left[\frac{1}{2t_o}\right]\frac{\rho_o^*}{\rho^*}} \, . \qquad (134)$$

For very large α, the right-hand side of Eq. (134) behaves like $(\alpha t_s)^{1/2}$, as Eq. (113) in the pure acoustic-phonon case. Nevertheless, within the region we are usually interested in, the behavior of the temperature ratio $\alpha=T_e/T$ and the resistivity as functions of drift velocity depend on the parameters involved. If the electron—optical-phonon interaction is relatively strong, there will exist a rather wide field range in which the current density is almost unchanged. In Fig. 7 we plot the temperature ratio $\alpha=T_e/T$, the acoustic-phonon-induced resistivity ρ_a and the optical-phonon-induced resistivity ρ_o as functions of v_d/v_o at $T/\Theta_o=0.2$ for a single-band system with $v_o/v_F=1$, $v_s/v_o=0.04$, and $\rho_o^*/\rho^*=80$. At small current density ($v_d/v_o<0.3$), $\rho_o<\rho_a$ and the electron-acoustic-phonon interaction is the major scattering mechanism, at relatively strong current density ($0.4<v_d/v_o<0.9$) optical phonon dominates the contribution to resistivity, in agreement with conventional understanding. However, if v_d/v_o gets higher, the role of acoustic phonon will once more become important.

VII. DYNAMIC COULOMB SCREENING FOR SEMICONDUCTORS

One of the major advantages of the present approach to nonlinear transport is the easy inclusion of carrier-carrier interaction. In this section we shall present some results obtained from the force- and energy-balance equations (64) and (67) for semiconductors by including the electron-electron interaction dynamically under the random-phase approximation. In semiconductors the carrier-carrier interaction is weakened due to the large dielectric constant ϵ of the lattice. For these systems the Coulomb potential $v_C(q)$ in Eqs. (9) and (62) should be

$$v_C(q)=\frac{e^2}{\epsilon\epsilon_0 q^2}\ . \qquad (135)$$

In the following we shall first discuss the case for acoustic-phonon scattering and then the case for combined charged impurity and acoustic-phonon scattering.

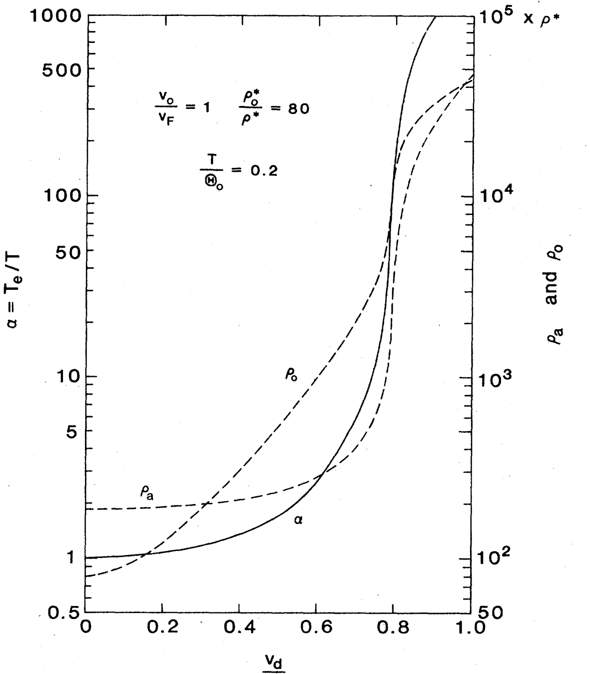

FIG. 7. Temperature ratio T_e/T, acoustic-phonon-induced resistivity ρ_a and optical-phonon-induced resistivity ρ_o are shown as functions of v_d/v_o for a system with $v_o/v_F=1$ and $\rho_o^*/\rho^*=80$ at lattice temperature $T=0.2\Theta_o$. The scatterings are due to combined acoustic and optical phonons.

A. Pure acoustic-phonon scattering

For acoustic-phonon scattering the temperature ratio $\alpha=T_e/T$ and dimensionless resistivity ρ/ρ_0 [$\rho_0=\rho(v_d\rightarrow 0)$] as functions of v_d/v_s for systems with $v_s/v_F=0.02$ and 0.5 are calculated at several different temperatures. The results are shown in Figs. 8 and 9. The bare electron-phonon matrix element (it includes the screening by all the electrons except the current carriers) is taken to be the deformation potential, and the variation of the chemical potential ϵ_f with electron temperature T_e is included according to Eq. (104). The magnitude of the parameter $\lambda_s=e^2/(4\pi^2\epsilon\epsilon_0 v_s)$ is selected to be 15.0, which is about the values for Ge and Si. By comparing Figs. 8 and 9 with Figs. 4 and 5 obtained by neglecting carrier-carrier interaction, we see that the appreciable change happens mainly at relatively low drift velocities. For example, the weak current resistivities ρ_0 in the case without Coulomb screening are equal to 3.36, 2.06, and 1.84 times those with Coulomb screening for $T/\Theta_s=10$, 20, and 40, respectively, at $v_s/v_F=0.02$, and equal to 17.8, 3.44, and 1.16 times for $T/\Theta_s=0.8$, 2.0, and 10.0, respectively, at

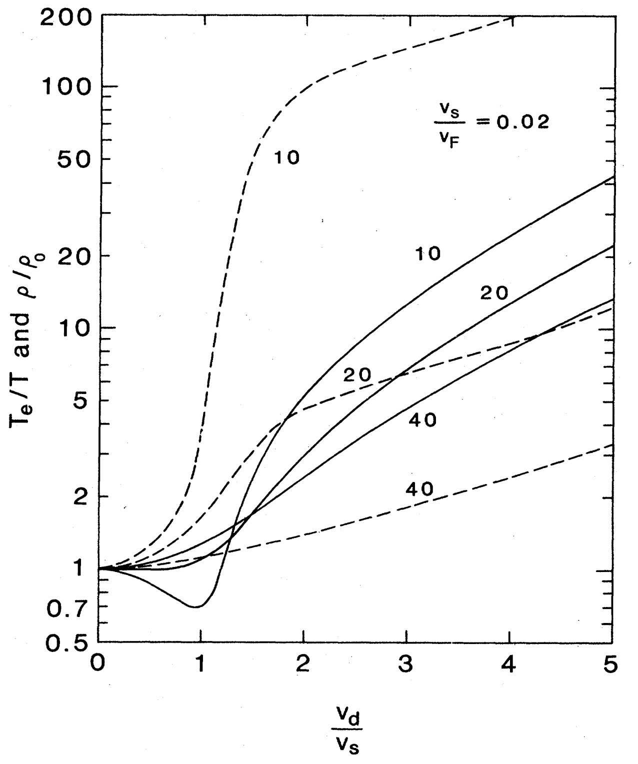

FIG. 8. Temperature ratio T_e/T and the dimensionless phonon-induced resistivity are shown as functions of v_d/v_s for a system with $v_s/v_F=0.02$ at several different lattice temperatures. The scattering is entirely due to acoustic phonons. The dynamic effects of carrier-carrier interaction are included with the parameter $\lambda_s=e^2(4\pi^2\epsilon\epsilon_0 v_s)=15$. The solid lines are for T_e/T and the dashed lines for ρ/ρ_0. The numbers near the curves are the values of T/Θ_s.

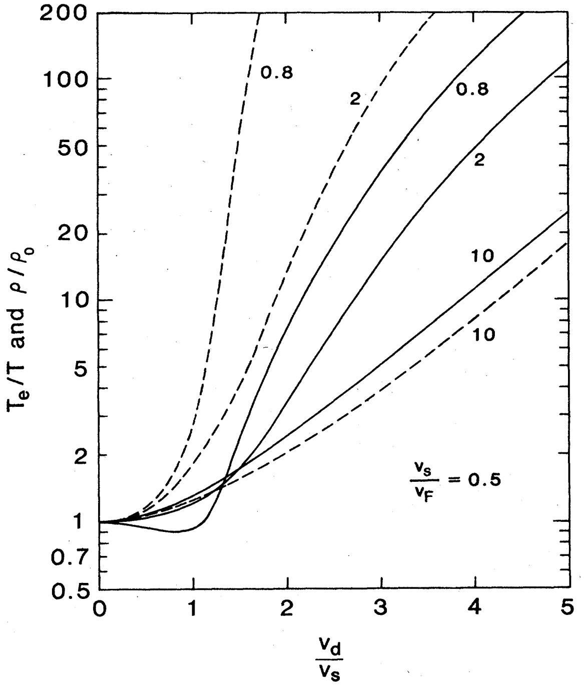

FIG. 9. Temperature ratio T_e/T and the dimensionless phonon-induced resistivity are shown as functions of v_d/v_s for a system with $v_s/v_F=0.5$ at several different lattice temperatures. The scattering is entirely due to acoustic phonons. The dynamic effects of carrier-carrier interaction are included with the parameter $\lambda_s=e^2/(4\pi^2\epsilon\epsilon_0 v_s)=15$. The solid lines are for T_e/T and the dashed lines for ρ/ρ_0. The numbers near the curves are the values of T/Θ_s.

$v_s/v_F=0.5$. The electron cooling in the case with Coulomb interaction is more pronounced than in the case without Coulomb screening. On the other hand, the high-field descreening effect clearly exhibits when $v_d/v_F>1.5$: both the temperature ratio T_e/T and the electric resistivity ρ are almost identical in Fig. 5 and Fig. 9 for $3<v_d/v_s<5$.

B. Combined charged-impurity and acoustic-phonon scattering

It is also interesting to investigate the combined charge-impurity and acoustic-phonon scattering in a wide range of electron temperature with dynamic Coulomb screening. The bare-electron—impurity potential is assumed to be $u(q)=Ze^2/(\epsilon\epsilon_0 q^2)$. In Fig. 10 the temperature ratio α, impurity-induced resistivity ρ_i, phonon-induced resistivity ρ, and total resistivity $\rho_T=\rho_i+\rho$ are plotted as functions of v_d/v_s at two different temperatures $T/\Theta_s=30$ and 88 for a system with $v_s/v_F=1.3$, $\lambda_s=15$, and $\rho_i^*/\rho_s^*=10^5$. Here,

$$\rho_i^*=\frac{n_i m^2 Z^2 e^2}{16\pi^3\epsilon^2\epsilon_0^2 N^2} \tag{136}$$

and ρ_s^* has been given in Eq. (109). The variation of the chemical potential ϵ_f with electron temperature is included. The impurity-induced resistivity decreases with increasing v_d/v_s and phonon-induced resistivity increases. For weak current density and at low temperatures the impurity scattering may dominate the contribution to the total resistivity, but for high current density the phonon always plays a major role.

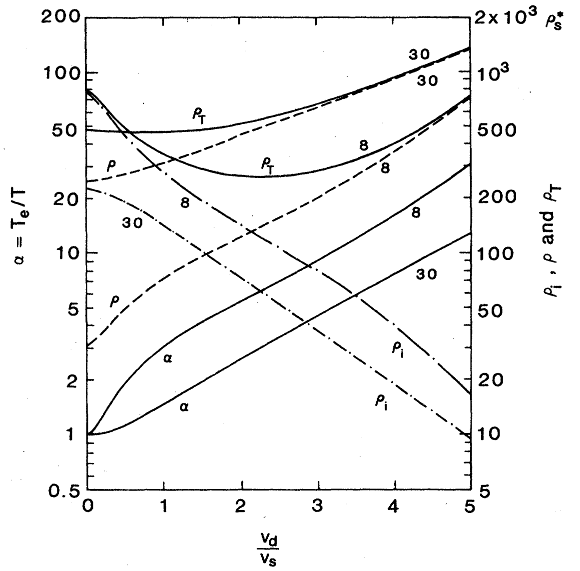

FIG. 10. Temperature ratio $\alpha=T_e/T$, impurity-induced resistivity ρ_i, acoustic-phonon-induced resistivity ρ, and total resistivity $\rho_T=\rho_i+\rho$ are shown as functions of v_d/v_s for a system with $v_s/v_F=1.3$, $\lambda_s=15$ and $\rho_i^*/\rho_s^*=10^5$ at two different lattice temperatures. The numbers near the curves are the values of T/Θ_s.

As was discussed in Ref. 19 this kind of behavior for impurity resistivity occurs only in the case of $T_e \gtrsim \varepsilon_F$ (Maxwell-Boltzmann case). For degenerate case $T_e \ll \varepsilon_F$, impurity resistivity increases with increasing v_d.[19] In Fig. 11 we plot α, ρ_i, ρ, and ρ_T as functions of v_d/v_s for a system with $v_s/v_F=0.5$ and $\rho_i^*/\rho_s^*=10^4$ at $T/\Theta_s=2$ and 4. Impurity resistivity shows an increase at beginning, then reaches a maximum before it decreases with increasing v_d/v_s. This behavior indicates a transition from degenerate case to Maxwell-Boltzmann case as v_d/v_s increases.

VIII. DISCUSSIONS

We have developed a new approach to nonlinear electronic transport in the presence of a static electric field for systems in which the electron-electron interaction, electron-phonon interaction, and electron-impurity scattering are assumed to exist. The starting point of our method is to separate the center-of-mass variables from the relative variables of the electrons in the Hamiltonian. This separation enables us to differentiate the thermal motion of the electron system from its drift motion. The fact that the electric field acts only on the center of mass and the relative electrons are coupled with the center of mass and with phonons only via the electron-impurity and electron-phonon interaction make it possible to introduce an electron temperature T_e for the nonequilibrium electron system, which is in a steady transport state in the presence of an electric field, as a measurement of its inter-

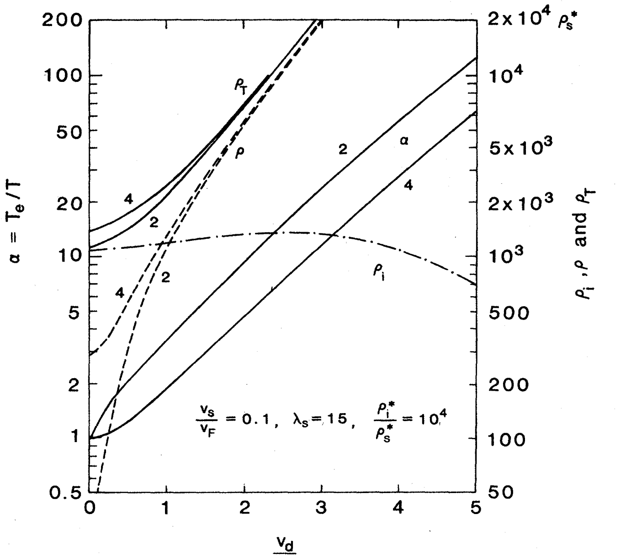

FIG. 11. Temperature ratio $\alpha = T_e/T$, impurity-induced resistivity ρ_i, acoustic-phonon-induced resistivity ρ, and total resistivity $\rho_T = \rho_i + \rho$ are shown as functions of v_d/v_s for s systems with $v_s/v_F = 0.1$, $\lambda_s = 15$, and $\rho_i^*/\rho_s^* = 10^4$ at two different lattice temperatures. The numbers near the curves are the values of T/Θ_s.

nal energy. The electron temperature T_e is defined as the thermodynamical temperature of an isolated system, which can be obtained from the electrons in relative coordinates by turning off the electron-phonon and electron-impurity interaction after the system has reached the steady state. For these decoupled electron and phonon systems it is natural to have two different temperatures (T_e and T), and this is used as the initial condition of the density matrix describing relative electrons and phonons. To the first order of electron-impurity and electron-phonon interactions the solution of the Liouville equation for density matrix is obtained and the momentum- and energy-balance equations are derived.

Unlike the case of the carrier-temperature model, in which the "electron temperature" is introduced as a parameter in an *ad hoc* distribution function (usually an equilibrium-form distribution function) for nonequilibrium transport electrons, we have not assumed any distribution function for them. The functions $f(x)$ and $n(x)$ in the expressions (64) and (67) for frictional force and energy transfer rate come from the summation of Matsubara frequencies over the electron and phonon Green's functions. It is worth noticing that the Hamiltonian we used for the decoupled electrons and phonons is $H_e + H_{\text{ph}}$ [see Eqs. (7) and (18)], not $H_0 \equiv H_e + \alpha H_{\text{ph}}$. The use of H_0 is only a mathematical trick which permits us to apply the standard Green's function technique in deriving the relevant equations. The resultant equations are not dependent on how to choose H_0. In fact, the initial density matrix ρ_0 can also be written as

$$\rho_0 = \frac{1}{Z} e^{-\overline{H}_0/T} \tag{137}$$

with $\overline{H}_0 = \gamma H_e + H_{\text{ph}}$ and $\gamma = T/T_e$. In Appendix C we show that exactly the same equations can be derived when starting from Eq. (137) and by the use of $\overline{H}_0$.

In the conventional Boltzmann theory of phonon-induced resistivity, the phonon system is generally assumed to be in the equilibrium state. The present approach is in fact free of this assumption. The phonon temperature T, which can be defined as the thermodynamical equilibrium temperature of the phonon system when the electron-phonon interaction and lattice—heat-bath interaction are turned off, is a measurement of its internal energy. This temperature can, in principle, be determined in the same way as we did for relative electrons as long as we take the phonon—heat-bath interaction into account and deduce the energy-balance equation for the phonon system. If the phonon system is near enough to equilibrium that we can approximately define a thermodynamical temperature for it, our T will be equal to that temperature. Otherwise, the thermodynamical temperature loses its meaning. Our definition for T, however, is still valid, which acts as one of the important features of a steady-state-transport electron-phonon system.

One of the interesting predictions from the present approach is the possible cooling of electrons for low impurity samples in a certain range at low temperatures when the drift velocity is near the sound speed. This arises from the direct influence of drift velocity (therefore, electric field) on the frictional force and the energy transfer rate from electrons to phonons. Previously, it was generally believed that the energy can transmit from electrons to lattice only when the electron temperature is higher than the lattice temperature and the energy transfer rate increases with increasing electron temperature so that T_e is always higher than T in the presence of an electric field.[14,15] This is a traditional heat-transfer idea. In the present theory, this is true only when $v_d = 0$. As we pointed out in Sec. IV, when $v_d = 0$, the energy transfers from electrons to phonons if $T_e > T$ and from phonons to electrons if $T_e < T$, in agreement with the concept of heat transfer. When $v_d \neq 0$, however, both the frictional force and the energy transfer from electrons to phonons depend on the temperatures and drift velocity in such a way that under certain conditions the energy transfer from electrons to phonons is still possible even when the electron temperature T_e is lower than the lattice temperature T and the energy balance is achievable only when $T_e < T$.

The essential feature of the present approach is its simplicity in mathematical structure, so that the numerical labor involved is kept minimal. Moreover, since many-body (electron-electron) interactions only affect H_e and thus are entirely included in the electron density-density correlation function $\Pi(\mathbf{q},\omega)$, its effect can easily be taken into account by the diagrammatic method. The calculation presented in Sec. VII under the random-phase approximation has clearly shown the simplicity and effectiveness of the present approach.

In this paper we performed numerical calculations only for an isotropic electron system with parabolic band, assuming the simplest phonon dispersion and electron-phonon matrix element. For convenience and definiteness we presented the results only for the systems with fixed carrier density N. They are easily modified to be applicable to, for example, semiconductors where the carrier density is temperature dependent, if the temperature-dependence of the carrier density $N(T_e)$ is included in the calculation. Nor are there principle difficulties in applying the present method to more complicated band-structure, phonon-dispersion, and electron-phonon interactions.

Finally, the present approach can be extended to investigate the nonlinear electronic transport in the presence of both an electric field and a magnetic field. This project is in progress and the results will be presented in a separate publication.

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APPENDIX A: CALCULATION OF $\langle \dot{H}_e \rangle$

According to Eqs. (15), (27), and (29), we can express $\langle \dot{H}_e \rangle$ as

$$\begin{aligned}\langle \dot{H}_e \rangle = & -in_i \sum_{\mathbf{q},\mathbf{k}} |u(q)|^2(\varepsilon_{\mathbf{k}+\mathbf{q}}-\varepsilon_{\mathbf{k}})\hat{\Pi}(\mathbf{k},\mathbf{q},\omega_0) \\ & -i \sum_{\mathbf{q},\lambda,\mathbf{k}} |M(\mathbf{q},\lambda)|^2(\varepsilon_{\mathbf{k}+\mathbf{q}}-\varepsilon_{\mathbf{k}}) \\ & \times[\Lambda^-(\mathbf{k},\mathbf{q}\lambda,\omega_0-(1-\alpha)\Omega_{\mathbf{q}\lambda}) \\ & +\Lambda^+(\mathbf{k},\mathbf{q}\lambda,\omega_0+(1-\alpha)\Omega_{\mathbf{q}\lambda})] \,. \end{aligned} \tag{A1}$$

Here $\omega_0 \equiv q_x v_d$ and

$$\hat{\Pi}(\mathbf{k},\mathbf{q},\omega)=\int_{-\infty}^{\infty} e^{i\omega t}\hat{\Pi}(\mathbf{k},\mathbf{q},t)dt \,, \tag{A2}$$

$$\begin{aligned}\Lambda^-(\mathbf{k},\mathbf{q}\lambda,\omega)&=\int_{-\infty}^{\infty} e^{i\omega t}\Lambda^-(\mathbf{k},\mathbf{q}\lambda,t)dt \,, \\ \Lambda^+(\mathbf{k},\mathbf{q}\lambda,\omega)&=\int_{-\infty}^{\infty} e^{i\omega t}\Lambda^+(\mathbf{k},\mathbf{q}\lambda,t)dt \,, \end{aligned} \tag{A3}$$

with

$$\hat{\Pi}(\mathbf{k},\mathbf{q},t)=\sum_{\sigma}\langle\langle c^{\dagger}_{\mathbf{k}+\mathbf{q}\sigma}(t)c_{\mathbf{k}\sigma}(t)\,|\,c^{\dagger}_{\mathbf{k}}(0)c_{\mathbf{k}+\mathbf{q}}(0)\rangle\rangle \,, \tag{A4}$$

$$\begin{aligned}\Lambda^-(\mathbf{k},\mathbf{q}\lambda,t)&=\sum_{\sigma}\langle\langle b_{\mathbf{q}\lambda}(t)c^{\dagger}_{\mathbf{k}+\mathbf{q}\sigma}(t)c_{\mathbf{k}\sigma}(t)\,|\,b^{\dagger}_{\mathbf{q}\lambda}(0)c^{\dagger}_{\mathbf{k}\sigma}(0)c_{\mathbf{k}+\mathbf{q}\sigma}(0)\rangle\rangle \,, \\ \Lambda^+(\mathbf{k},\mathbf{q}\lambda,t)&=\sum_{\sigma}\langle\langle b^{\dagger}_{-\mathbf{q}\lambda}(t)c^{\dagger}_{\mathbf{k}+\mathbf{q}\sigma}(t)c_{\mathbf{k}\sigma}(t)\,|\,b_{-\mathbf{q}\lambda}(0)c^{\dagger}_{\mathbf{k}\sigma}(0)c_{\mathbf{k}+\mathbf{q}\sigma}(0)\rangle\rangle \,. \end{aligned} \tag{A5}$$

By introducing the corresponding Matsubara Green's functions, performing the summation over the imaginary frequency, and doing the analytical continuation, we find, for the imaginary part of Λ^- and Λ^+,

$$\begin{aligned}\Lambda_2^-(\mathbf{k},\mathbf{q}\lambda,\omega_0-(1-\alpha)\Omega_{\mathbf{q}\lambda}) &\equiv \mathrm{Im}\Lambda^-(\mathbf{k},\mathbf{q}\lambda,\omega_0-(1-\alpha)\Omega_{\mathbf{q}\lambda}) \\ &=\hat{\Pi}_2(\mathbf{k},\mathbf{q},\omega_0-\Omega_{\mathbf{q}\lambda})[n(\Omega_{\mathbf{q}\lambda}/T)-n((\Omega_{\mathbf{q}\lambda}-\omega_0)/\alpha T)] \,, \end{aligned} \tag{A6}$$

$$\begin{aligned}\Lambda_2^+(\mathbf{k},\mathbf{q}\lambda,\omega_0+(1-\alpha)\Omega_{\mathbf{q}\lambda}) &\equiv \mathrm{Im}\Lambda^+(\mathbf{k},\mathbf{q}\lambda,\omega_0+(1-\alpha)\Omega_{\mathbf{q}\lambda}) \\ &=\hat{\Pi}_2(\mathbf{k},\mathbf{q},\omega_0+\Omega_{\mathbf{q}\lambda})[n(\Omega_{\mathbf{q}\lambda}/T)-n((\Omega_{\mathbf{q}\lambda}+\omega_0)/\alpha T)] \,. \end{aligned} \tag{A7}$$

Here, $\hat{\Pi}_2(\mathbf{k},\mathbf{q},\omega)$ is the imaginary part of $\hat{\Pi}(\mathbf{k},\mathbf{q},\omega)$. Under the random-phase approximation it can be expressed as

$$\hat{\Pi}_2(\mathbf{k},\mathbf{q},\omega)=\frac{\Pi_2(\mathbf{k},\mathbf{q},\omega)}{[1-v_C(q)\Pi_1(\mathbf{q},\omega)]^2+[v_C\Pi_2(\mathbf{q},\omega)]^2} \,, \tag{A8}$$

where

$$\Pi_1(\mathbf{q},\omega)=\sum_{\mathbf{k}}\Pi_1(\mathbf{k},\mathbf{q},\omega)$$

and

$$\Pi_2(\mathbf{q},\omega)=\sum_{\mathbf{k}}\Pi_2(\mathbf{k},\mathbf{q},\omega) \tag{A9}$$

are just the real part and the imaginary part of the electron density-density correlation function without Coulomb interaction. The expression for $\Pi_2(\mathbf{k},\mathbf{q},\omega)$ is

$$\begin{aligned}\Pi_2(\mathbf{k},\mathbf{q},\omega)=&2\pi[f(\varepsilon_{\mathbf{k}},T_e)-f(\varepsilon_{\mathbf{k}+\mathbf{q}},T_e)] \\ &\times\delta(\omega-\varepsilon_{\mathbf{k}}+\varepsilon_{\mathbf{k}+\mathbf{q}}) \,. \end{aligned} \tag{A10}$$

Only the imaginary parts of $\hat{\Pi}$, Λ^-, and Λ^+, are relevant to Eq. (A1), and the δ function in Eq. (A10) enables us to write Eq. (A1) as

$$\begin{aligned}\langle \dot{H}_e \rangle = & -n_i \sum_{\mathbf{q},\mathbf{k}} |u(q)|^2 \omega_0 \hat{\Pi}_2(\mathbf{k},\mathbf{q},\omega_0) - \sum_{\mathbf{q},\lambda,\mathbf{k}} |M(\mathbf{q},\lambda)|^2 [(\omega_0 - \Omega_{\mathbf{q}\lambda}) \Lambda_2^-(\mathbf{k},\mathbf{q}\lambda, \omega_0 - (1-\alpha)\Omega_{\mathbf{q}\lambda}) \\ & + (\omega_0 + \Omega_{\mathbf{q}\lambda}) \Lambda_2^+(\mathbf{k},\mathbf{q}\lambda, \omega_0 + (1-\alpha)\Omega_{\mathbf{q}\lambda})] \\ = & -n_i \sum_{\mathbf{q}} |u(q)|^2 \omega_0 \hat{\Pi}_2(\mathbf{q},\omega_0) + \sum_{\mathbf{q},\lambda} [(\Omega_{\mathbf{q}\lambda} - \omega_0) \Lambda_2^-(\mathbf{q}\lambda, \omega_0 - (1-\alpha)\Omega_{\mathbf{q}\lambda}) \\ & - (\Omega_{\mathbf{q}\lambda} + \omega_0) \Lambda_2^+(\mathbf{q}\lambda, \omega_0 + (1-\alpha)\Omega_{\mathbf{q}\lambda})] \ . \end{aligned} \tag{A11}$$

Here,

$$\begin{aligned} \Lambda_2^-(\mathbf{q}\lambda,\omega) &= \sum_{\mathbf{k}} \Lambda_2^-(\mathbf{k},\mathbf{q}\lambda,\omega) \ , \\ \Lambda_2^+(\mathbf{q}\lambda,\omega) &= \sum_{\mathbf{k}} \Lambda_2^+(\mathbf{k},\mathbf{q}\lambda,\omega) \ . \end{aligned} \tag{A12}$$

We finally obtain

$$\langle \dot{H}_e \rangle = -n_i \sum_{\mathbf{q}} |u(q)|^2 \omega_0 \hat{\Pi}_2(\mathbf{q},\omega_0) - 2\sum_{\mathbf{q},\lambda} |M(\mathbf{q},\lambda)|^2 (\omega_0 + \Omega_{\mathbf{q}\lambda}) \hat{\Pi}_2(\mathbf{q}, \omega_0 + \Omega_{\mathbf{q}\lambda}) [n(\Omega_{\mathbf{q}\lambda}/T) - n((\omega_0 + \Omega_{\mathbf{q}\lambda})/\alpha T)] \ . \tag{A13}$$

APPENDIX B: CALCULATION OF $\langle \dot{H}_{\text{ph}} \rangle$

From Eqs. (14), (27), and (29)

$$\begin{aligned}\langle \dot{H}_{\text{ph}} \rangle = & \, i \sum_{\mathbf{q},\lambda} |M(\mathbf{q},\lambda)|^2 \Omega_{\mathbf{q}\lambda} \int_{-\infty}^{\infty} e^{i\mathbf{q}\cdot[\mathbf{R}(t)-\mathbf{R}(t')]} \\ & \times [e^{-i(1-\alpha)\Omega_{\mathbf{q}\lambda}(t-t')} \langle\langle b_{\mathbf{q}\lambda}(t)\rho_{\mathbf{q}}(t) \,|\, b_{\mathbf{q}\lambda}^{\dagger}(t')\rho_{-\mathbf{q}}(t') \rangle\rangle \\ & - e^{i(1-\alpha)\Omega_{\mathbf{q}\lambda}(t-t')} \langle\langle b_{-\mathbf{q}\lambda}^{\dagger}(t)\rho_{\mathbf{q}}(t) \,|\, b_{-\mathbf{q}\lambda}(t')\rho_{-\mathbf{q}}(t') \rangle\rangle] dt' \\ = & \, i \sum_{\mathbf{q},\lambda} |M(\mathbf{q},\lambda)|^2 \Omega_{\mathbf{q}\lambda} [\Lambda^-(\mathbf{q}\lambda, \omega_0 - (1-\alpha)\Omega_{\mathbf{q}\lambda}) - \Lambda^+(\mathbf{q}\lambda, \omega_0 + (1-\alpha)\Omega_{\mathbf{q}\lambda})] \\ = & -\sum_{\mathbf{q},\lambda} |M(\mathbf{q},\lambda)|^2 \Omega_{\mathbf{q}\lambda} [\Lambda_2^-(\mathbf{q}\lambda, \omega_0 - (1-\alpha)\Omega_{\mathbf{q}\lambda}) - \Lambda_2^+(\mathbf{q}\lambda, \omega_0 + (1-\alpha)\Omega_{\mathbf{q}\lambda})] \ , \end{aligned} \tag{B1}$$

with $\omega_0 \equiv \mathbf{q}_x v_d$. By the use of Eqs. (54) and (55) and the symmetrical property of $\Pi_2(\mathbf{q},\omega)$, we have

$$\begin{aligned}\langle \dot{H}_{\text{ph}} \rangle = & \, 2\sum_{\mathbf{q},\lambda} |M(\mathbf{q},\lambda)|^2 \Omega_{\mathbf{q}\lambda} \hat{\Pi}_2(\mathbf{q}, \omega_0 + \Omega_{\mathbf{q}\lambda}) \\ & \times [n(\Omega_{\mathbf{q}\lambda}/T) - n((\omega_0 + \Omega_{\mathbf{q}\lambda})/\alpha T)] \ . \end{aligned} \tag{B2}$$

APPENDIX C: DERIVATION OF BALANCE EQUATIONS BY A DIFFERENT SELECTION OF H_0

The initial density matrix ρ_0 can also be written in the form

$$\rho_0 = \frac{1}{Z_e} e^{-H_e/T_e} \frac{1}{Z_{\text{ph}}} e^{-H_{\text{ph}}/T} = \frac{1}{Z} e^{-\bar{H}_0/T} \ , \tag{C1}$$

with

$$\bar{H}_0 \equiv \gamma H_e + H_{\text{ph}} \tag{C2}$$

and

$$\gamma \equiv \frac{1}{\alpha} = \frac{T}{T_e} \ . \tag{C3}$$

By defining

$$\bar{H}_{It'}(t) \equiv e^{i\bar{H}_0 t} H_{It'} e^{-i\bar{H}_0 t} \tag{C4}$$

Eq. (24) can be written as

$$\begin{aligned}\rho(t) = \rho_0 - i \int_{-\infty}^{t} & e^{i(1-\gamma)H_e(t'-t)} [\bar{H}_{It'}(t'-t), \rho_0] \\ & \times e^{-i(1-\gamma)H_e(t'-t)} dt' \end{aligned} \tag{C5}$$

and the statistical average of a dynamic variable A_t at time t can be expressed as

$$\langle A_t \rangle = \langle A_t \rangle_0 + \int_{-\infty}^{\infty} \langle\langle \bar{A}_t^{(\gamma-1)(t'-t)}(t) \,|\, \bar{H}_{It'}(t') \rangle\rangle dt' \ . \tag{C6}$$

Here,

$$\bar{A}_t^{(\gamma-1)\tau}(t') \equiv \exp(i\bar{H}_0 t') \bar{A}_t^{(\gamma-1)\tau} \exp(-i\bar{H}_0 t') \tag{C7}$$

and

$$\bar{A}_t^{(\gamma-1)\tau} = \exp[i(\gamma-1)H_e\tau] A_t \exp[-i(\gamma-1)H_e\tau] \ . \tag{C8}$$

The expectation value of the total force acting on the center of mass now can be written as

$$\begin{aligned}\langle \dot{P}_x \rangle = NeE - in_i \sum_{\mathbf{q},\mathbf{k},\sigma} |u(q)|^2 q_x \int_{-\infty}^{\infty} e^{i\mathbf{q}\cdot[\mathbf{R}(t)-\mathbf{R}(t')]} e^{i(1-\gamma)(\epsilon_{\mathbf{k}+\mathbf{q}}-\epsilon_{\mathbf{k}})(t-t')} \langle\langle \bar{c}^{\dagger}_{\mathbf{k}+\mathbf{q}\sigma}(t)\bar{c}_{\mathbf{k}\sigma}(t) \,|\, \bar{c}^{\dagger}_{\mathbf{k}\sigma}(0)\bar{c}_{\mathbf{k}+\mathbf{q}\sigma}(0)\rangle\rangle \\ - i \sum_{\substack{\mathbf{q},\lambda,\\ \mathbf{k},\sigma}} |M(\mathbf{q},\lambda)|^2 q_x \int_{-\infty}^{\infty} e^{i\mathbf{q}\cdot[\mathbf{R}(t)-\mathbf{R}(t')]} e^{i(1-\gamma)(\varepsilon_{\mathbf{k}+\mathbf{q}}-\varepsilon_{\mathbf{k}})(t-t')} \\ \times [\, \langle\langle \bar{b}_{q\lambda}(t)\bar{c}^{\dagger}_{\mathbf{k}+\mathbf{q}\sigma}(t)\bar{c}_{k\sigma}(t) \,|\, \bar{b}^{\dagger}_{q\lambda}(t')\bar{c}_{k\sigma}(t')\bar{c}_{\mathbf{k}+\mathbf{q}\sigma}(t')\rangle\rangle \\ + \langle\langle \bar{b}^{\dagger}_{-\mathbf{q}\lambda}(t)\bar{c}^{\dagger}_{\mathbf{k}+\mathbf{q}\sigma}(t)\bar{c}_{\mathbf{k}\sigma}(t) \,|\, \bar{b}_{-\mathbf{q}\lambda}(t')\bar{c}^{\dagger}_{\mathbf{k}\sigma}(t')\bar{c}_{\mathbf{k}+\mathbf{q}\sigma}(t')\rangle\rangle] \,, \end{aligned} \tag{C9}$$

with

$$\bar{b}_{\mathbf{q}\lambda}(t) = e^{i\bar{H}_0 t} b_{\mathbf{q}\lambda} e^{-i\bar{H}_0 t} \,, \qquad \bar{b}^{\dagger}_{\mathbf{q}\lambda}(t) = e^{i\bar{H}_0 t} b^{\dagger}_{\mathbf{q}\lambda} e^{-i\bar{H}_0 t} \,, \tag{C10}$$

$$\bar{c}_{\mathbf{k}\sigma}(t) = e^{iH_0 t} c_{\mathbf{k}\sigma} e^{-i\bar{H}_0 t} \,, \qquad \bar{c}^{\dagger}_{\mathbf{k}\sigma}(t) = e^{iH_0 t} c^{\dagger}_{\mathbf{k}\sigma} e^{-i\bar{H}_0 t} \,. \tag{C11}$$

In writing Eq. (C9) we have utilized the property that

$$\sum_{\mathbf{k},\sigma} e^{iH_e t} c^{\dagger}_{\mathbf{k}+\mathbf{q}\sigma} c_{\mathbf{k}\sigma} e^{-iH_e t} = \sum_{\mathbf{k},\sigma} e^{i(\varepsilon_{\mathbf{k}+\mathbf{q}}-\varepsilon_{\mathbf{k}})t} c^{\dagger}_{\mathbf{k}+\mathbf{q}\sigma} c_{\mathbf{k}\sigma} \,, \tag{C12}$$

which can be verified from the commutation relation (16). In steady state, Eq. (C9) can be expressed as

$$\langle \dot{P}_x \rangle = NeE + F(v_d)$$

with

$$\begin{aligned} F(v_d) = -in_i \sum_{\mathbf{q},\mathbf{k}} |u(q)|^2 q_x \widetilde{\Pi}(\mathbf{k},\mathbf{q},\, \omega_0 + (1-\gamma)(\varepsilon_{\mathbf{k}+\mathbf{q}}-\varepsilon_{\mathbf{k}})) \\ - i \sum_{\mathbf{q},\lambda,\mathbf{k}} |M(\mathbf{q},\lambda)|^2 q_x [\overline{\Lambda}^{-}(\mathbf{k},\mathbf{q}\,\lambda,\, \omega_0 + (1-\gamma)(\varepsilon_{\mathbf{k}+\mathbf{q}}-\varepsilon_{\mathbf{k}})) + \overline{\Lambda}^{+}(\mathbf{k},\mathbf{q}\,\lambda,\, \omega_0 + (1-\gamma)(\varepsilon_{\mathbf{k}+\mathbf{q}}-\varepsilon_{\mathbf{k}}))] \,. \end{aligned} \tag{C13}$$

Here,

$$\widetilde{\Pi}(\mathbf{k},\mathbf{q},\omega) = \int_{-\infty}^{\infty} e^{i\omega t} \widetilde{\Pi}(\mathbf{k},\mathbf{q},t)\, dt \,, \tag{C14}$$

$$\overline{\Lambda}^{-}(\mathbf{k},\mathbf{q}\,\lambda,\omega) = \int_{-\infty}^{\infty} e^{i\omega t} \overline{\Lambda}^{-}(\mathbf{k},\mathbf{q}\,\lambda,t)\, dt \,, \qquad \overline{\Lambda}^{+}(\mathbf{k},\mathbf{q}\,\lambda,\omega) = \int_{-\infty}^{\infty} e^{i\omega t} \overline{\Lambda}^{+}(\mathbf{k},\mathbf{q}\,\lambda,t)\, dt \,, \tag{C15}$$

with

$$\widetilde{\Pi}(\mathbf{k},\mathbf{q},t) = \sum_{\sigma} \langle\langle \bar{c}^{\dagger}_{\mathbf{k}+\mathbf{q}\sigma}(t)\bar{c}_{\mathbf{k}\sigma}(t) \,|\, \bar{c}^{\dagger}_{\mathbf{k}\sigma}(0)\bar{c}_{\mathbf{k}+\mathbf{q}\sigma}(0)\rangle\rangle \,, \tag{C16}$$

$$\begin{aligned} \overline{\Lambda}^{-}(\mathbf{k},\mathbf{q},t) = \sum_{\sigma} \langle\langle \bar{b}^{\dagger}_{\mathbf{q}\lambda}(t)\bar{c}^{\dagger}_{\mathbf{k}+\mathbf{q}\sigma}(t)\bar{c}_{\mathbf{k}\sigma}(t) \,|\, \bar{b}_{\mathbf{q}\lambda}(0)\bar{c}^{\dagger}_{\mathbf{k}\sigma}(0)\bar{c}_{\mathbf{k}+\mathbf{q}\sigma}(0)\rangle\rangle \,, \\ \overline{\Lambda}^{+}(\mathbf{k},\mathbf{q},t) = \sum_{\sigma} \langle\langle \bar{b}_{-\mathbf{q}\lambda}(t)\bar{c}^{\dagger}_{\mathbf{k}+\mathbf{q}\sigma}(t)\bar{c}_{\mathbf{k}\sigma}(t) \,|\, \bar{b}^{\dagger}_{-\mathbf{q}\lambda}(0)\bar{c}^{\dagger}_{\mathbf{k}\sigma}(0)\bar{c}_{\mathbf{k}+\mathbf{q}\sigma}(0)\rangle\rangle \,. \end{aligned} \tag{C17}$$

Similarly, $\widetilde{\Pi}(\mathbf{k},\mathbf{q},\omega)$, $\overline{\Lambda}^{-}(\mathbf{k},\mathbf{q}\,\lambda,\omega)$, and $\overline{\Lambda}^{+}(\mathbf{k},\mathbf{q}\,\lambda,\omega)$ can be calculated by means of corresponding Matsubara Green's functions: $\tilde{\pi}(\mathbf{k},\mathbf{q},i\omega_\nu)$, $\bar{\lambda}^{-}(\mathbf{k},\mathbf{q}\,\lambda,i\omega_\nu)$, and $\bar{\lambda}^{+}(\mathbf{k},\mathbf{q}\,\lambda,i\omega_\nu)$, whose definitions are almost the same as those in Sec. IV, except that now $\omega_\nu = 2\pi\nu T$ ($\nu = 0, \pm 1, \ldots$) and H_0 there should be replaced by $\bar{H}_0$. Now

$$\begin{aligned} \lambda^{-}(\mathbf{k},\mathbf{q}\,\lambda,i\omega_\nu) = -T \sum_{\omega_\mu} \tilde{\pi}(\mathbf{k},\mathbf{q}\,\lambda,\, i\omega_\nu - i\omega_\mu)\bar{D}(\mathbf{q}\,\lambda,i\omega_\mu) \,, \\ \lambda^{+}(\mathbf{k},\mathbf{q}\,\lambda,i\omega_\nu) = -T \sum_{\omega_\mu} \tilde{\pi}(\mathbf{k},\mathbf{q}\,\lambda,\, i\omega_\nu - i\omega_\mu)\bar{D}(-\mathbf{q}\,\lambda,-i\omega_\mu) \,, \end{aligned} \tag{C18}$$

with

$$\bar{D}(\mathbf{q},i\omega_\mu)=\frac{1}{i\omega_\mu-\Omega_{\mathbf{q}\lambda}}\,. \tag{C19}$$

After performing the summation over the imaginary frequencies and performing the analytical continuation

$$\begin{aligned}
&\tilde{\Pi}(\mathbf{k},\mathbf{q},\omega)=\tilde{\pi}(\mathbf{k},\mathbf{q},i\omega_\nu)\,|_{i\omega_\nu\to\omega+i\delta}\,,\\
&\bar{\Lambda}^-(\mathbf{k},\mathbf{q}\lambda,\omega)=\bar{\lambda}^-(\mathbf{k},\mathbf{q}\lambda,i\omega_\nu)\,|_{i\omega_\nu\to\omega+i\delta}\,,\\
&\bar{\Lambda}^+(\mathbf{k},\mathbf{q}\lambda,\omega)=\bar{\lambda}^+(\mathbf{k},\mathbf{q}\lambda,i\omega_\nu)\,|_{i\omega_\nu\to\omega+i\delta}\,,
\end{aligned} \tag{C20}$$

we obtain the expression for the imaginary parts of $\bar{\Lambda}^-$ and $\bar{\Lambda}^+$:

$$\begin{aligned}
&\bar{\Lambda}_2^-(\mathbf{k},\mathbf{q}\lambda,\omega)\equiv\mathrm{Im}\bar{\Lambda}^-(\mathbf{k},\mathbf{q}\lambda,\omega)=\tilde{\pi}_2(\mathbf{k},\mathbf{q},\,\omega-\Omega_{\mathbf{q}\lambda})[n(\Omega_{\mathbf{q}\lambda}/T)-n((\Omega_{\mathbf{q}\lambda}-\omega)/T)]\,,\\
&\bar{\Lambda}_2^+(\mathbf{k},\mathbf{q}\lambda,\omega)\equiv\mathrm{Im}\bar{\Lambda}^+(\mathbf{k},\mathbf{q}\lambda,\omega)=\tilde{\pi}_2(\mathbf{k},\mathbf{q},\,\omega+\Omega\,\mathbf{q}\,\lambda)[n(\Omega_{\mathbf{q}\lambda}/T)-n((\Omega_{\mathbf{q}\lambda}+\omega)/T)]\,.
\end{aligned} \tag{C21}$$

Now, $\tilde{\pi}_2(\mathbf{k},\mathbf{q}\lambda,\omega)$ is the imaginary part of the electron density-density correlation function $\tilde{\pi}(\mathbf{k},\mathbf{q}\lambda,\omega)$ in the equilibrium system described by Hamiltonian γH_e and at temperature T (lattice temperature). In the absence of electron-electron Coulomb interaction the real part and the imaginary part of this function are, respectively,

$$\bar{\Pi}_1(\mathbf{k},\mathbf{q},\omega)=2f(\gamma\varepsilon_{\mathbf{k}},T)\left[\frac{1}{\omega-\gamma\varepsilon_{\mathbf{k}+\mathbf{q}}+\gamma\varepsilon_{\mathbf{k}}}-\frac{1}{\omega+\gamma\varepsilon_{\mathbf{k}+\mathbf{q}}-\gamma\varepsilon_{\mathbf{k}}}\right], \tag{C22}$$

$$\bar{\Pi}_2(\mathbf{k},\mathbf{q},\omega)=2\pi[f(\gamma\varepsilon_{\mathbf{k}},T)-f(\gamma\varepsilon_{\mathbf{k}+\mathbf{q}},T)]\delta(\omega-\gamma\varepsilon_{\mathbf{k}}+\gamma\varepsilon_{\mathbf{k}+\mathbf{q}})\,. \tag{C23}$$

When including the Coulomb interaction between carriers under random-phase approximation, we have

$$\tilde{\Pi}_2(\mathbf{k},\mathbf{q},\omega)=\frac{\bar{\Pi}_2(\mathbf{k},\mathbf{q},\omega)}{[1-v_C(q)\bar{\Pi}_1(\mathbf{q},\omega)]^2+[v_C(q)\bar{\Pi}_2(\mathbf{q},\omega)]^2}\,, \tag{C24}$$

in which

$$\begin{aligned}
&\bar{\Pi}_1(\mathbf{q},\omega)=\sum_{\mathbf{k}}\bar{\Pi}_1(\mathbf{k},\mathbf{q},\omega)\,,\\
&\bar{\Pi}_2(\mathbf{q},\omega)=\sum_{\mathbf{k}}\bar{\Pi}_2(\mathbf{k},\mathbf{q},\omega)\,.
\end{aligned} \tag{C25}$$

From Eqs. (C22)—(C24) and $f(\gamma\varepsilon_{\mathbf{k}},T)=f(\varepsilon_{\mathbf{k}},T_e)$, it is easily seen that

$$\tilde{\Pi}_2(\mathbf{k},\mathbf{q},\omega+(1-\gamma)(\varepsilon_{\mathbf{k}+\mathbf{q}}-\varepsilon_{\mathbf{k}}))=\hat{\Pi}_2(\mathbf{k},\mathbf{q},\omega)\,, \tag{C26}$$

the electron density-density correlation function defined in Sec. IV. On the other hand, according to Eq. (C21) we have

$$\begin{aligned}
&\sum_{\mathbf{k}}\bar{\Lambda}_2^-(\mathbf{k},\mathbf{q}\lambda,\,\omega_0+(1-\gamma)(\varepsilon_{\mathbf{k}+\mathbf{q}}-\varepsilon_{\mathbf{k}}))\\
&\quad=\sum_{\mathbf{k}}\tilde{\Pi}_2(\mathbf{k},\mathbf{q},\,\omega_0-\Omega_{\mathbf{q}\lambda}+(1-\gamma)(\varepsilon_{\mathbf{k}+\mathbf{q}}-\varepsilon_{\mathbf{k}}))[n(\Omega_{\mathbf{q}\lambda}/T)-n([\Omega_{\mathbf{q}\lambda}-\omega_0-(1-\gamma)(\varepsilon_{\mathbf{k}+\mathbf{q}}-\varepsilon_{\mathbf{k}})]/T)]\\
&\quad=\sum_{\mathbf{k}}\hat{\Pi}_2(\mathbf{k},\mathbf{q},\,\omega_0-\Omega_{\mathbf{q}\lambda})[n(\Omega_{\mathbf{q}\lambda}/T)-n(\gamma(\Omega_{\mathbf{q}\lambda}-\omega_0)/T)]=\hat{\Pi}_2(\mathbf{q},\,\omega_0-\Omega_{\mathbf{q}\lambda})[n(\Omega_{\mathbf{q}\lambda}/T)-n((\Omega_{\mathbf{q}\lambda}-\omega_0)/\alpha T)]
\end{aligned} \tag{C27}$$

and

$$\begin{aligned}
&\sum_{\mathbf{k}}\bar{\Lambda}_2^+(\mathbf{k},\mathbf{q}\lambda,\omega_0+(1-\gamma)(\varepsilon_{\mathbf{k}+\mathbf{q}}-\varepsilon_{\mathbf{k}}))\\
&\quad=\sum_{\mathbf{k}}\tilde{\Pi}_2(\mathbf{k},\mathbf{q},\,\omega_0+\Omega_{\mathbf{q}\lambda}+(1-\gamma)(\varepsilon_{\mathbf{k}+\mathbf{q}}-\varepsilon_k))[n(\Omega_{\mathbf{q}\lambda}/T)-n([\Omega_{\mathbf{q}\lambda}+\omega_0+(1-\gamma)(\varepsilon_{\mathbf{k}+\mathbf{q}}-\varepsilon_{\mathbf{k}})]/T)]\\
&\quad=\sum_{\mathbf{k}}\hat{\Pi}_2(\mathbf{k},\mathbf{q},\,\omega_0+\Omega_{\mathbf{q}\lambda})[n(\Omega_{\mathbf{q}\lambda}/T)-n(\gamma(\Omega_{\mathbf{q}\lambda}+\omega_0)/T)]=\hat{\Pi}_2(\mathbf{q},\,\omega_0+\Omega_{\mathbf{q}\lambda})[n(\Omega_{\mathbf{q}\lambda}/T)-n((\Omega_{\mathbf{q}\lambda}+\omega_0)/\alpha T)]\,.
\end{aligned} \tag{C28}$$

Therefore, Eq. (C13) is exactly the same expression as Eq. (64) in Sec. IV.

Similarly, we can verify that the expressions of $\langle\dot{H}_{\mathrm{ph}}\rangle$ and $\langle\dot{H}_e\rangle$ calculated according to Eqs. (C5) and (C6) are exactly the same as those obtained in Sec. IV. This confirms that the balance equations we obtained there are not dependent on the way to derive them.

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