New kind of noise in photoconductors

N. V. Joshi
Centro de Estudios Avanzados en Optica, Facultad de Ciencias, Universidad de Los Andes, 5101 Mérida, Venezuela
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Transient behavior of the photocurrent near the equilibrium point is examined by considering the stability criteria of Liapunov. It is found that in the presence of recombination centers the system is asymptotically stable. This suggests that as the photocurrent reduces below some critical value (in the decay curve), its trajectory becomes uncertain. Hence, a new type of noise is created, caused by the behavior of the electrons near the critical point of the set of simultaneous nonlinear differential equations.

It is well known[1,2] that the forms of the relaxation curves observed in photoconductivity studies have not been fully explained. The basic mechanism of the time-dependent photocurrent is understood.[3] However, this involves many parameters of the charge carriers and traps, such as density of traps and their distribution in energy, capture cross sections for electrons and for holes with corresponding temperature dependence, probability of emission of electrons from traps to the conduction band, density of the recombination centers with their efficiencies, and so on. Unfortunately, it is not easy to estimate these parameters experimentally and therefore to predict the precise form of the decay curves.

In addition to the above-mentioned aspects, the complexity is enhanced enormously due to the presence of several simultaneous and competitive processes that finally turn into a set of simultaneous nonlinear differential equations.[4,5] To obtain a solution for such a system is rather difficult, and hence alternative computer-aided techniques such as Monte Carlo[6] and numerical integration[7] have been developed. Even though these methods supply adequate information for specific problems, they rarely succeed in providing global and meaningful information for understanding transient behavior in general.

To extract the maximum possible information from the model proposed earlier requires a solution of a set of nonlinear differential equations. Such a solution has been obtained[5] only in a series form for a decay curve. Its success is limited because it does not give any information about behavior near the mathematical equilibrium points. Since the solution of the system of differential equations is not available in a closed form, some partial aspects of the solution can be examined by following Liapunov's direct method[8,9,10] near the mathematical equilibrium points. This will throw light on the stability behavior of the system and the trajectory of the excess of charge carriers (electrons in n-type semiconductors) near the mathematical equilibrium point. No similar study has been reported in this system so far. The purpose of the present investigation is, therefore, to exploit this approach and obtain additional information about the trajectory of the photocurrent in the time domain near the mathematical equilibrium point (near maximum photocurrent in the rise curve and minimum in the decay curve).

BACKGROUND

Fundamental mechanisms of the generation and annihilation of the photoexcited charge carriers are well understood[4] and, hence, the basic theory for time-dependent photoconductivity is known with reasonable accuracy.

In the presence of traps and recombination centers, non-steady-state equations governing the photocurrent for electrons and holes can be written as[4]

$$\frac{dn}{dt}=G-\alpha_1 n(N_t-n_c)+r_1 n_c-c_1 n , \tag{1a}$$

$$\frac{dn_c}{dt}=\alpha_1 n(N_t-n_c)-\delta_0 n_c p-r_1 n_c , \tag{1b}$$

$$\frac{dp}{dt}=G-\delta_0 n_c p-c_2 p , \tag{1c}$$

where c_1 and c_2 are the capture cross sections of recombination centers for electrons and holes, and n and p are the number of excess electrons and holes in the conduction and valence band, respectively. A number of occupied traps is n_c and N_t is the total number of traps. α_1 and δ_0 are the rates at which electrons from the conduction band and holes from the valence band are captured. When G, the generation rate of electrons and holes produced by an incident photon flux is zero, this system of equations represents the dynamical behavior of the electrons in a decay curve.

If one assumes that the major contribution in the decay process comes from Eq. (1a), and the role of traps and recombination centers is not predominant, then the solution is given by

$$n=n_{\max} e^{-t/\tau} , \tag{2}$$

where

$$\tau=[\alpha_1(N_t-n_{c\,\max})]^{-1} .$$

The time-dependent photocurrent is given by the classical decay curve

$$I_{\text{ph}}(t)=n(t)e\mu\tau=\mu n_{\max}e\tau e^{-t/\tau}=I_0 e^{-t/\tau} , \tag{3}$$

where μ is the mobility of the majority charge carriers and I_0 is the steady-state photoconductivity defined as $\mu n_{\max} e\tau$.

It is clear from the above discussion that the classical decay curve (3) is a solution of only a single differential equation of the set of simultaneous equations and, hence, in no case can it represent the trajectory of the photocurrent near the mathematical equilibrium point. Such a study needs to be understood with the help of a stability consideration, along with the phase-plane analysis of the system.

The system of equations (1a) to (1c) is evidently a nonlinear differential equation. Stable points of the system can be obtained by considering dn/dt, dn_c/dt, and dp/dt, each equal to zero. It is clear that for the present system there exists only one critical point, and that is $n=n_c=p=0$. Therefore, the stability should be examined by Liapunov's direct method.[8]

MATHEMATICAL ANALYSIS AND DISCUSSION

Eigenvalues of the matrix corresponding to the linear part of the present system of differential equations can be obtained[8,9] by linearizing about the equilibrium point, which can be obtained by simply neglecting the quadratic and product terms like nn_c, n_cp, etc.,

$$\begin{bmatrix} n' \\ n_c' \\ p \end{bmatrix} = \underline{M} \begin{bmatrix} n \\ n_c \\ p \end{bmatrix} , \tag{4}$$

where

$$\underline{M} = \begin{bmatrix} -\alpha_1 N_t - c_1 & r_1 & 0 \\ \alpha_1 N_t & -r_1 & 0 \\ 0 & 0 & -c_2 \end{bmatrix} .$$

The eigenvalues λ of $\underline{M}$ are given by

$$|\underline{M} - \lambda \underline{I}| = 0, \quad \lambda_1 = -c_2 ,$$

$$\lambda_2 = \tfrac{1}{2}\{ -(\alpha_1 N_t + c_1 + r_1) - [(\alpha_1 N_t + c_1 + r_1)^2 - 4r_1c_1]^{1/2}\} ,$$

$$\lambda_3 = \tfrac{1}{2}\{ -(\alpha_1 N_t + c_1 + r_1) + [(\alpha_1 N_t + c_1 + r_1)^2 - 4r_1c_1]^{1/2}\} .$$

All three roots are real and negative and, therefore, the present system is asymptotically stable.

According to the theorem on stability,[10] an asymptotic nature is valid even after including the nonlinear terms. Physically, this means that when an excess of electrons or holes falls to some critical value near equilibrium, the solution goes to the equilibrium point in an undetermined trajectory. This suggests that the long time limit approach to the equilibrium part of the relaxation curve is not reproducible, even though all the experimental parameters are kept constant.

On the basis of Liapunov's stability consideration, it is not possible to estimate the threshold value of the electrons (or holes) at which such behavior is predominant. However, it is possible to estimate roughly how the trajectory would look near the mathematical equilibrium point in terms of relative values of the parameters of the traps and recombination centers.

Behavior of the excess of electrons near the mathematical equilibrium point can be understood by examining the phase-plane analysis of the system.[9,10] In fact, a real situation demands a phase diagram in three dimensions. However, analyses of these diagrams are not mathematically developed to the extent required for a complicated system such as this. We, therefore, consider a phase diagram for n and n_c. This is a realistic approach since the photocurrent in n-type semiconductors reflects the behavior of excess electrons and traps that are closely related to the conduction band.

Equations (1a) and (1b) for phase-plane analysis can be written as

$$\frac{dn}{dn_c} = \frac{-n[c_1 + \alpha_1(N_t - n_c)] + r_1 n_c}{n[\alpha_1(N_t - n_c)] - r_1 n_c} . \tag{5}$$

The major interest in the present work is the analysis of the independence of n and n_c near the critical point ($n=n_c=0$). First, let us suppose that n_c is sufficiently small so that

$$r_1 n_c \ll n[c_1 + \alpha_1(N_t - n_c)] .$$

In this region, the equation then becomes

$$\frac{dn}{dn_c} = -\frac{1 + A(N_t - n_c)}{A(N_t - n_c)} , \tag{6}$$

where $A = \alpha_1/c_1$. The solution of the equation is

$$n = (N_t - n_c) + A^{-1}\ln(N_t - n_c) . \tag{7}$$

The value of the constant of integration is not evaluated since the value of n corresponding to the value of n_c is not known at any point in the phase diagram. Moreover, the constant of integration just shifts the trajectory in phase space and does not modify any argument in the discussion. Since N_t is a constant, n_c increases as n reduces. This behavior can be understood with the help of Fig. 1. The conclusion obtained is against normal expectation, namely, that the number of electrons in the traps and in the conduction band should both be reduced as time advances. This apparent paradox can be explained easily. As n_c, the number of occupied traps, is reduced, the probability of occupation of electrons from the conduction band increases. As expected, the form of the curve of the phase diagram depends on the ratio of c_1/r_1.

After a certain time, the value of n decreases and n_c increases so that

$$r_1 n_c \gg n\alpha_1(N_t - n_c) .$$

The equation controlling the trajectory in the phase plane then becomes

$$\frac{dn}{dn_c} = B\frac{n}{n_c} - 1 , \tag{8}$$

where B is the numerical quantity c_1/r_1. The solution of this equation is given by

$$n = \frac{n_c^B + n_c}{B-1} + K , \tag{9}$$

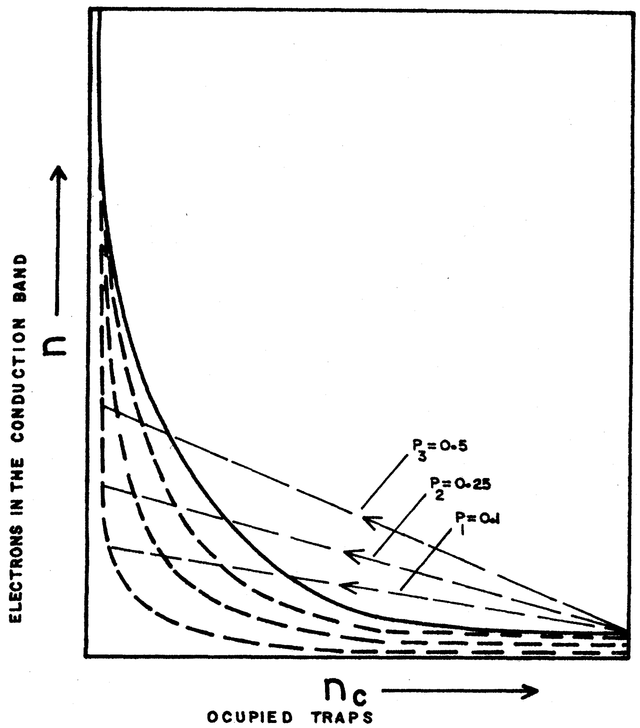

FIG. 1. Phase-plane diagram for free electrons and occupied traps. Parts P_1, P_2, and P_3 are simulated with the help of Eq. (11) for $B=0.1$, 0.25, and 0.5, respectively. The values of Δn and n_{c0} are chosen arbitrarily.

where K is a constant of integration. The value of K is calculated from the initial condition of the process and found to be

$$K=\Delta n-\frac{1}{B-1}(n_{c0}^{B}+n_{c0}) , \tag{10}$$

where n_{c0} is the number of occupied traps for which Eq. (9) is valid and Δn is the number of electrons in the conduction band at that time; we have

$$n=\Delta n+\frac{1}{B-1}[(n_c^B+n_c)-(n_{c0}^B+n_{c0})] . \tag{11}$$

In order to simplify a visualization of the variation of n with respect to n_c, trajectories in phase space are simulated with the help of a computer (TRS model III) for a few representative values of B ($B<1$) and are shown in Fig. 1.

It is clear from this figure that as time advances, the number of electrons in the conduction band again increases and the number of occupied traps decreases. This can be repeated several times, depending upon the relative values of c_1 and r_1 as compared to the value $\alpha_1(N_t-n_c)$. It is worth pointing out that the system approaches the critical value ($n=n_c=0$) since it is an asymptotically stable system.

This process causes a type of noise in photoconductive and photovoltaic cells, whose physical origin is the consequence of the asymptotical stability of the nonlinear set of differential equations involved in the process. It is different from all kinds of noise mentioned earlier in photodetectors, at a very low injection level.[11] Moreover, the transference of electrons from the conduction band to the traps, and vice versa, suggests that long time is needed to reach an equilibrium value.

A slow component observed in the relaxation curves is considered as a photomemory effect or persistent photoconductivity (PPC). Experimental detection of PPC is actually quite old and several aspects of it have already been summarized.[12] Because of its technological importance in radiation detection and its appearance in two-dimensional electron gas,[13] interest has been renewed.[14]

On the basis of complex empirical evidence, three major types of models have been proposed to explain persistent photoconductivity. All of the models essentially consider the existence of a potential barrier which separates electrons from holes either in real space or in momentum space. This ultimately reduces the velocity of the recombination, or in terms of the parameters c_1 and c_2 discussed above, results in both having a very small value.

It is worth pointing out that exactly the same conclusion has been obtained with the help of the present mathematical approach. When $c_1/r_1<1$ the electrons are injected into the conduction band (see Fig. 1) and PPC is expected. Careful observation[15] shows that in most of the cases, PPC drives the system to the saturation value of the photocurrent and always possesses fluctuations, which are difficult to avoid. All these aspects can be summarized merely by stating that the system is asymptotically stable.

It is true that in Volterra's nonlinear equations, some fluctuations or noise also appear at the stable value, originating from the simultaneous nature of the nonlinear

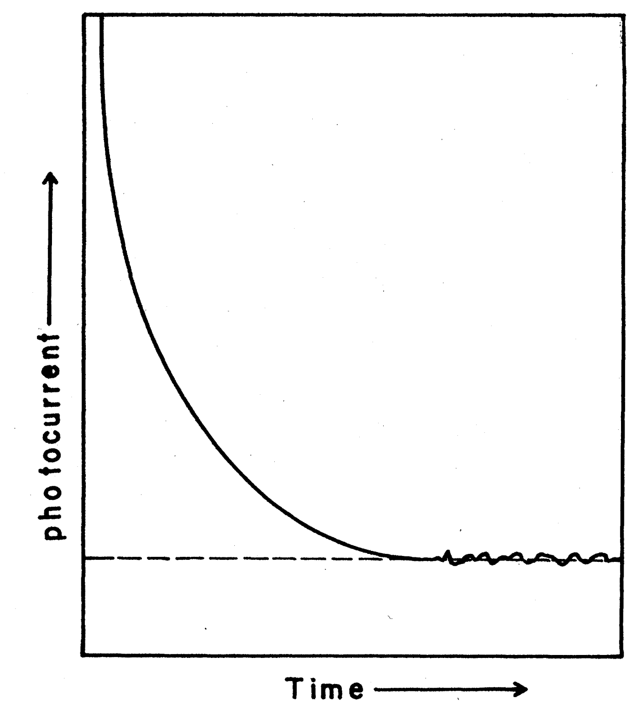

FIG. 2. One form of the relaxation curve in a photoconductor. Fluctuations about the equilibrium value may be a consequence of Volterra's equation.

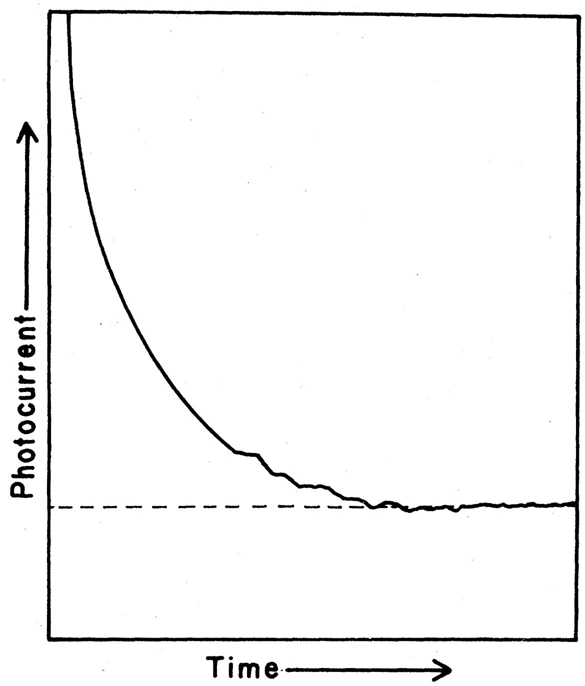

FIG. 3. A typical form of the relaxation curve expected from the present theory. Fluctuations start before reaching the equilibrium value.

equations.[15] The origin of the noise mentioned in the present case is different from that obtained in Volterra's[11] system. Firstly, the two systems are different. Volterra's system can be written as[15]

$$\frac{dN_i}{dt} = K_i N_i + \beta_1^{-1} \sum_{j=1}^{n} a_{ij} N_i N_j \ , \quad (12)$$

while there is an additional term, namely, $\sum_{j=1}^{n} c_j N_j$, in the present system. Moreover, Volterra's equation suggests that there exist fluctuations at the equilibrium value, whereas the present analysis shows that these fluctuations, caused by the different trajectories (originating from the same point in phase space), appear near the equilibrium value. The difference between both types could be explained with the help of Figs. 2 and 3. Further study is needed to improve an understanding of the treatment of the equilibrium noise near and at the mathematical equilibrium point.

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