RAPID COMMUNICATIONS

PHYSICAL REVIEW B VOLUME 31, NUMBER 12 15 JUNE 1985

Long-range order and the fractional quantum Hall effect

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(Received 25 March 1985)

It is shown that the wave function proposed by Tao and Thouless to explain the fractional quantization of the Hall current implies long-range correlations between the electrons that are very different from the properties expected from Laughlin's wave function. Calculations for a small number of electrons have shown that Laughlin's wave function is an excellent approximation to the true ground state, so it must be concluded that the Tao-Thouless wave function is not a good approximation. There is some discussion of the question of broken symmetry.

I. INTRODUCTION

Fractional quantization of the Hall conductance, observed in high-mobility electron layers,[1] has been explained as an effect of the Coulomb interaction between electrons. In the simplest case, where one-third of e^2/h is observed for the conductance, it is supposed that there is a ground state for a one-third-full Landau level which is especially low in energy, and which is free to move with the magnetic flux rather than pinned to the substrate. If this is the case, the value $e^2/3h$ should be observed. The Wigner solid satisfies neither of these two conditions—one-third occupation is not energetically favorable, and it should be pinned to the substrate—so alternative ground states for a two-dimensional electron system in a high magnetic field have been sought.

One promising suggestion was made by Laughlin.[2] In the symmetric gauge (with vector potential in the tangential direction) a wave function is in the manifold of degenerate ground states if it is of the form

$$f(z)\exp(-\tfrac{1}{4}|z|^2/l^2) \ , \tag{1.1}$$

where f is a polynomial in $z=x+iy$ and l the magnetic length whose square is $\hbar/eB$. Laughlin's wave function is of the form

$$\prod_{j\,(<i)}(z_i-z_j)^p\exp\left[-\tfrac{1}{4}\sum_i |z_i|^2/l^2\right] \ , \tag{1.2}$$

where p must be an odd integer for particles obeying Fermi statistics. For $p=1$ the electrons fill a disk centered on the origin, and for $p=3$ the disk is one-third full. This wave function is homogeneous in space, and the Coulomb energy is lowered (for $p\neq 1$) by the correlation between the electrons. In fact, as Laughlin observed, all correlations between electrons in this trial ground state are identical to those for a classical gas with logarithmic repulsions. The wave function bears a superficial resemblance to the many-body wave functions introduced many years ago by Bijl,[3] and later by Jastrow,[4] but the analogy is not particularly helpful. The circular geometry is not essential, and a conformal transformation of the variables z_i can be made to adapt it to other geometries. A very elegant reformulation of the theory to spherical geometry has been obtained by Haldane.[5]

An alternative suggestion was made by Tao and Thouless.[6] We suggested that the ground state could involve a preferential occupation of a regular subset of the degenerate Landau levels. To be specific, we took the Landau gauge (vector potential in the y direction) and, for the one-third quantization, suggested that every third level would be occupied in the unperturbed ground state, with the two intervening levels empty. Thus, there would be something like a one-dimensional solid in the space of Landau levels. We showed how the Coulomb interaction could favor such a ground state, and we showed that it would give a homogeneous density which would not be pinned by the substrate potential. Later work by Tao[7] showed that the wave function could be generalized to more complicated fractional occupation.

It was not initially clear whether these were two alternative ways of describing the same situation or if they were descriptions of two different states. It is the purpose of this paper to show unambiguously that the two wave functions are completely different. It is also argued that the evidence is strongly against the Tao-Thouless state as a correct description of interacting two-dimensional electrons in a strong magnetic field.

There is a discussion at the end of the paper on the question of broken symmetry which has been raised by Anderson[8] and by Tao and Wu.[9]

II. PAIR-CORRELATION FUNCTION AND IMAGES OF THE EXCHANGE HOLE

In a normal fermion system it is well known that the pair-correlation function $\rho(\mathbf{r}_1,\mathbf{r}_2)$ exhibits an "exchange hole" around $\mathbf{r}_2=\mathbf{r}_1$, so that the integral of $\rho(\mathbf{r}_1,\mathbf{r}_2)/\rho_0^2$ over $\mathbf{r}_2$ in any large volume including $\mathbf{r}_1$, where ρ_0 is the average density, is -1. A converse effect is found for nondegenerate bosons. We found, initially by direct calculation, that the Tao-Thouless[6] state with one-third occupancy has an exchange hole of one-third the strength. The reason for this is not hard to find. Occupancy of one-third of the available Landau levels, say those levels for which the y component of momentum is a multiple of $6\pi/L_2$, where L_2 is the dimension of the system in the y direction, is equivalent to replacing the system by one of dimension $L_2/3$, and then making a periodic continuation of this contracted system.

The result is that the exchange hole develops two images separated by $\pm L_2/3$ in the y direction, and each has one-third of the expected strength.

When corrections to the unperturbed wave function are taken into account these distant images of the exchange hole are likely to be reduced in strength, but there is no reason why they should disappear altogether. In fact, they would seem to be an inevitable symptom of the type of order considered by Tao and Thouless.

Some difficulties with the dielectric function of this state found by Toyoda, Gudmundsson, and Takahashi[10] may be related to this problem.

There is no reason at all to expect the Laughlin wave function to have this type of long-range order. Since the correlation functions can be related to those of a problem in classical statistical mechanics, this type of order seems very unlikely.

III. ABSENCE OF LONG-RANGE ORDER IN THE LAUGHLIN WAVE FUNCTION

Although the correspondence between the correlation functions for the Laughlin wave function and for the classical one-component plasma make long-range order in real space very unlikely, it seems possible that there might be some order hidden in the space of quantum states. To explore that possibility, a careful study was made of the Laughlin wave functions for up to eight particles in two different geometries. No sign of any long-range order was discovered.

The method used to search for long-range order was the method suggested by Yang[11] based on the Dirac density matrix; it is a generalization of the method developed for bosons by Penrose and Onsager.[12] Long-range order is displayed by an eigenvalue of the appropriate density matrix which is proportional to the number of particles in the system instead of being of order unity.

For example, to study the "off-diagonal long-range order" found in superfluid fermion systems it is possible to calculate the density matrix

$$M_{m_1,m_2}(n)=\langle a^\dagger_{n+m_1}a^\dagger_{n-m_1}a_{n-m_2}a_{n+m_2}\rangle \quad , \tag{3.1}$$

and then, for fixed n, find the eigenvalues. A Cooper pair with a "wave number" $2n$ will be displayed by a large eigenvalue of $M(n)$. This calculation was carried out both for Haldane's spherical geometry,[5] where the largest eigenvalues for four through eight electrons are 0.314, 0.271, 0.246, 0.230, and 0.220, so that the largest eigenvalue decreases rather than increases with the number of electrons. Similarly, for the Laughlin wave function on a square with periodic boundary conditions in one direction[13] the largest eigenvalues for four through eight electrons are 0.351, 0.235, 0.340, 0.267, and 0.260, so that, again, the largest eigenvalue does not increase. There is, therefore, no indication of a superfluid type of order in the Laughlin wave function.

The type of order found in a solid can be revealed by combining one creation and one annihilation operator together rather than by combining two creation or two annihilation operators. An appropriate matrix to study is

$$N_{m_1,m_2}=\langle (a^\dagger_{m_1}a_{m_1}-\rho_0)(a^\dagger_{m_2}a_{m_2}-\rho_0)\rangle \quad , \tag{3.2}$$

where ρ_0 is the average occupancy of a level. For the Tao-Thouless state, this has an eigenvalue $2N/3$ for N particles. For the Laughlin state in spherical geometry, the largest eigenvalues have the values 0.667, 0.649, 0.595, 0.596, and 0.555 as N goes from four to eight, while for square geometry the largest eigenvalues are 0.939, 0.931, 0.900, 0.772, and 0.728. Again, there is no sign of any long-range order in the Laughlin state.

IV. BROKEN SYMMETRY

The question of broken symmetry has been discussed by Anderson[8] and by Tao and Wu.[9] The argument of Tao and Wu is essentially an adaptation of Laughlin's argument for the integer quantization of conductance.[14] We suppose the electrons are arranged on the surface of a long cylinder, and the vector potential around the cylinder can be changed by changing the flux in a solenoid along the axis of a cylinder. A change of flux by h/e changes the gauge in such a way that there is a new state which is identical to the original ground state apart from phase factors. However, if the conductance is $e^2/3h$, it is necessary to change the flux by $3h/e$ to get back from the original ground state to a state which is identical, apart from phase factors and the transfer of one electron from one end of the system to the other. Therefore, the ground state must go into two different states when the flux is changed by h/e or $2h/e$, and there is a threefold discrete broken symmetry.

It is not clear that this argument is really sufficient to establish the existence of a broken symmetry such as occurs in the low-temperature phase of a continuous phase transition. Since we are concerned with the ground state rather than with a transition at nonzero temperature, the closest analogy might be to the antiferromagnetic Heisenberg-Ising chain with the S_zS_z interaction stronger than the $S_xS_x+S_yS_y$ term. This has a Néel-like ground state with two possible sublattice magnetizations. In the true ground state both sublattice magnetizations occur, but the two are separated by intermediate configurations of very low amplitude, and a very small perturbation can single out one or the other Néel state. The Tao-Thouless ground state would be quite like this, but there can be no mixing at all between the three different ground states because they have a different total momentum. The three states are those with wave number $6\pi n/L_2$, $2\pi(3n-1)/L_2$, and $2\pi(3n-1)/L_2$ occupied and their total momentum differs by hN/L_2.

This feature makes it difficult to discuss whether or not the Laughlin wave function on a cylinder has broken symmetry. When the flux is changed by h/e the ground state is mapped into a new state which (in the old gauge) would differ in momentum by hN/L_2, but this hardly gives evidence for a broken symmetry—there are wave functions with differing values of the total momentum.

V. DISCUSSION

The work reported here shows clearly that the Tao-Thouless wave function[6] differs significantly from the Laughlin wave function,[2] and the Laughlin wave function does not have any of the long-range order expected for the Tao-Thouless wave function. They are, therefore, different, and not alternative formulations of the same theory. One can, therefore, ask which, if either, of the two theories describes interacting electrons in a strong magnetic field.

There are two strong points in favor of the Laughlin theory. The first is that the overlap of the Laughlin wave function with the exact ground state for a small number of electrons is remarkably large.[15,16] The second is that the odd denominators for fractional quantization follow very naturally from Laughlin's theory. There is little to be said in favor of the Tao-Thouless wave function to counterbalance these arguments and I do not think modifications such as those suggested by Giuliani and Quinn[17] will change the situation. I think the time has come to abandon the Tao-Thouless theory.

ACKNOWLEDGMENTS

I have benefited from numerous discussions with Dr. R. Tao, but have no reason to believe that he accepts either the arguments presented here or the conclusion drawn from them. This work was supported in part by the Royal Society of London, and by the U.S. National Science Foundation under Grant No. DMR-83-19301.

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