RAPID COMMUNICATIONS

PHYSICAL REVIEW B VOLUME 31, NUMBER 11 1 JUNE 1985

Criticality of the $D=2$ quantum Heisenberg ferromagnet with quenched random anisotropy

Ananias M. Mariz and Constantino Tsallis

Centro Brasileiro de Pesquisas Físicas/CNPq, Rua Xavier Sigaud 150, 22290 Rio de Janeiro, Rio de Janeiro, Brazil and Departamento de Fisica, Universidade Federal do Rio Grande do Norte, 59000 Natal, Rio Grande do Norte, Brazil*

(Received 13 February 1985)

We consider the square-lattice spin-$\frac{1}{2}$ anisotropic Heisenberg ferromagnet with interactions whose symmetry can independently (quenched model) and randomly be of two competing types, namely, the isotropic Heisenberg type and the Ising type. Within a real-space renormalization-group framework, we perform a quite precise numerical calculation of the critical frontier, and establish its main asymptotic behaviors. We also characterize the relevant universality classes, through the analysis of the correlation-length critical exponent.

In recent years, several attempts have been made to study critical properties of magnetic systems characterized by interactions belonging to competing symmetries (detailed theoretical and experimental information can be, respectively, found in Refs. 1–3, and references therein). A particularly interesting case is that where spins on a regular lattice might be coupled through uniaxial, planar, or spherical interactions, whose respective prototypes are the Ising, isotropic *XY*, and isotropic Heisenberg models. Such situations have already been experimentally encountered in antiferromagnetic systems like $Fe_{1-x}Co_xBr_2$ (Ref. 4) (Ising-*XY* competition) and $Rb_2Co_xMn_{1-x}$ (Refs. 5 and 6) (Ising-Heisenberg competition). From the theoretical standpoint, Ising-Heisenberg mixtures have been studied, for quenched random-site systems, within effective field frameworks,[7] and, for $D=2$ quenched random-bond systems, with high-temperature series techniques.[2,8] For this second case, a *continuous* variation of the susceptibility critical exponent γ with concentration was obtained. As pointed out by Pekalski himself,[2,8] this result is clearly unsatisfactory; indeed, symmetry arguments strongly suggest that the universality class corresponding to the system under analysis should be, almost everywhere in the critical frontier, that of the Ising model.

In the present paper we study the phase diagram and universality classes of the quenched random-bond spin-$\frac{1}{2}$ anisotropic Heisenberg ferromagnet in square lattice, each bond of which being either an isotropic Heisenberg interaction or an Ising-like one. The formalism we use is a real-space renormalization-group (RG) one, which has recently been developed[9,10] for quantum spin systems, and whose performance has proved to be quite reliable (both qualitatively and quantitatively for square lattice).

We consider the following dimensionless Hamiltonian:

$$\mathscr{H}=\sum_{\langle i,j\rangle} K_{ij}[\sigma_i^z\sigma_j^z+(1-\Delta_{ij})(\sigma_i^x\sigma_j^x+\sigma_i^y\sigma_j^y)] \quad , \tag{1}$$

where $\langle i,j\rangle$ denotes first neighbors on a square lattice, the σ's are the Pauli operators, $K_{ij}\equiv J_{ij}/k_BT>0$ (J_{ij} is the coupling constant) is the same for all bonds, and $\Delta_{ij}\epsilon[0,1]$ is the random anisotropy parameter. For the limiting value $\Delta_{ij}=1(\Delta_{ij}=0)$ we recover the Ising (isotropic Heisenberg) model. The randomness of the problem is described by the following probability law:

$$P(K_{ij},\Delta_{ij})=[p\delta(\Delta_{ij}-\Delta)+(1-p)\delta(\Delta_{ij})]\delta(K_{ij}-K) \quad , \tag{2}$$

with $0\leqslant p\leqslant 1$, $0\leqslant\Delta\leqslant 1$, and $K\equiv J/k_BT>0$. The particular case $\Delta=1$ corresponds to the Ising-Heisenberg mixture analyzed in Refs. 2 and 8.

To construct the RG we follow along the lines of Refs. 9 and 10, renormalizing the cluster of Fig. 1(a) into that of Fig. 1(b) (the linear scale factor being consequently $b=2$); both clusters are self-dual, therefore particularly convenient for the square lattice.

We associate the binary distribution (2) with each one of the five bonds of the cluster of Fig. 1(a). Consequently, 2^5 different configurations are possible (some of them being topologically equivalent). Each configuration is characterized by the set $(\{K_{ij}^{(l)}\},\{\Delta_{ij}^{(l)}\})$ with $l=1,2,\ldots,5$. With each configuration we associate $K_H(\{K_{ij}^{(l)}\},\{\Delta_{ij}^{(l)}\})$ and $\Delta_H(\{K_{ij}^{(l)}\},\{\Delta_{ij}^{(l)}\})$ by imposing[9,10]

$$e^{\mathscr{H}'_{12}}=\operatorname*{Tr}_{3,4} e^{\mathscr{H}_{1234}} \quad , \tag{3}$$

where $\mathscr{H}_{1234}$ and $\mathscr{H}'_{12}$ are the Hamiltonians corresponding to the clusters of Figs. 1(a) and 1(b), respectively. These two Hamiltonians are precisely of the same type indicated in Eq. (1) (i.e., *the present RG generates no new types of terms)*. Although imposition (3) is a very natural one, its operational implementation is rather complex as it involves the computational treatment of 16×16 matrix (associated with $\mathscr{H}_{1234}$); practical details on the procedure can be found in Refs. 9 and 10.

The renormalized parameters K_{ij} and Δ_{ij} are now associated with a distribution law P_H which is no longer binary. It

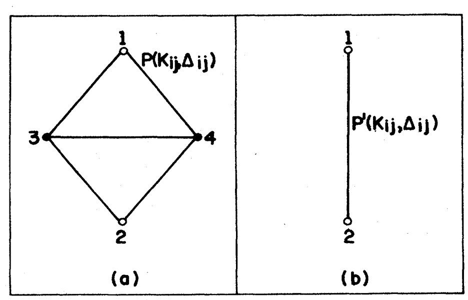

FIG. 1. (a), (b) Self-dual two-terminal clusters. The terminal and internal nodes are, respectively, denoted by ○ and ●.

has, in fact, 14 different δ's, and is given by

$$P_H(K_{ij},\Delta_{ij}) = \int \prod_{l=1}^{5} [dK_{ij}^{(l)} d\Delta_{ij}^{(l)} P(K_{ij}^{(l)},\Delta_{ij}^{(l)})] \times \delta(K_{ij}-K_H)\delta(\Delta_{ij}-\Delta_H) \ . \quad (4)$$

Under successive renormalizations the distribution law becomes more and more complex. It is, in principle, possible to keep track of its evolution up to an eventual stabilization, but, following along the lines of previous similar theories (e.g., Ref. 11), we shall instead approximate it by the following *binary* one:

$$P'(K_{ij},\Delta_{ij}) = [p'\delta(\Delta_{ij}-\Delta') + (1-p')\delta(\Delta_{ij})]\delta(K_{ij}-K') \ , \quad (5)$$

where p', K', and Δ' have to be found as functions of p, K, and Δ. To do this we impose that the main averages are preserved through renormalization. More specifically, we demand

$$\langle K_{ij}\rangle_{P'} = \langle K_{ij}\rangle_{P_H} \equiv g_1(p,K,\Delta) \ , \quad (6)$$

$$\langle \Delta_{ij}\rangle_{P'} = \langle \Delta_{ij}\rangle_{P_H} \equiv g_2(p,K,\Delta) \ , \quad (7)$$

$$\langle \Delta_{ij}^2\rangle_{P'} = \langle \Delta_{ij}^2\rangle_{P_H} \equiv g_3(p,K,\Delta) \ , \quad (8)$$

where $\langle \cdots \rangle$ denotes the standard mean values. While Eqs. (6) and (7) are quite natural choices, Eq. (8) has been adopted in order to decouple p and Δ. The set of Eqs. (6)–(8) immediately yield

$$K' = g_1 \ , \quad (9)$$

$$\Delta' = g_3/g_2 \ , \quad (10)$$

$$p' = g_2/\Delta' \ , \quad (11)$$

which constitute the *RG* recursive relations which close the formalism. Iteration [in the $(p,1/K,\Delta)$ space, for instance] provides the paraferromagnetic critical surface [see Figs. 2(a) and 2(b) for selected cuts of this surface], as well as the set of universality classes. For $p=1$ we recover the results obtained in Ref. 9. When p and/or Δ increase, the lower symmetry (Ising) becomes dominant; consequently, the critical temperature is expected to increase, as exhibited in Fig. 2. In the neighborhood of $p=\Delta=1$, we obtain the following asymptotic behaviors:

$$\frac{T_c(p=1,\Delta)-T_c(p,\Delta)}{T_c(p=1,\Delta)} \sim A(\Delta)(1-p), \quad (p\to 1) \ , \quad (12)$$

and

$$\frac{T_c(p,\Delta=1)-T_c(p,\Delta)}{T_c(p,\Delta=1)} \sim B(p)(1-\Delta)+C(p)(1-\Delta)^2 \ , \quad (\Delta\to 1) \ , \quad (13)$$

where $A(\Delta)$, $B(p)$, and $C(p)$ are shown in Figs. 3 and 4. Numerical difficulties prevented us from a reliable description of the $T\to 0$ asymptotic behaviors.

Two nontrivial fixed points belong to the critical surface, namely, the isotropic Heisenberg one [at $(p,k_BT/J,\Delta) = (1,0,0)$], and the Ising one [at $(p,k_BT/J,\Delta) = (1,2.269\ldots,1)$]; both are located at the *exact* values.[12,13] These points characterize the unique two universality classes of this problem; indeed, the RG flow shows that the universality class is that of the Ising (isotropic Heisenberg) model for all points of the critical surface at finite (vanishing) temperature. These results disagree with those obtained (for the antiferromagnetic system) by high-

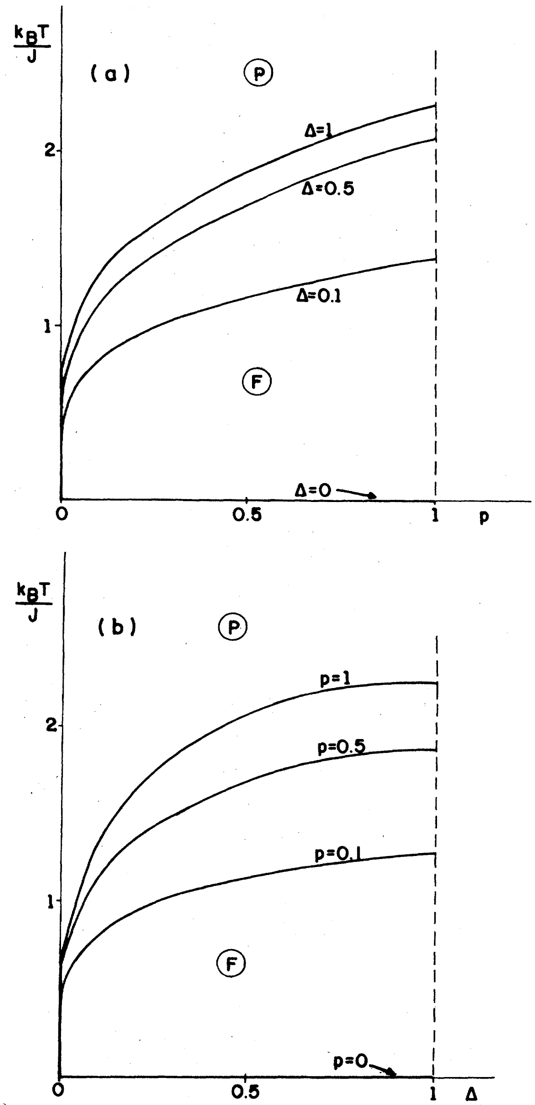

FIG. 2. (a) Cuts of the critical frontier for selected values of Δ; (b) cuts of the critical frontier for selected values of p. P and F are, respectively, the paramagnetic and ferromagnetic phases.

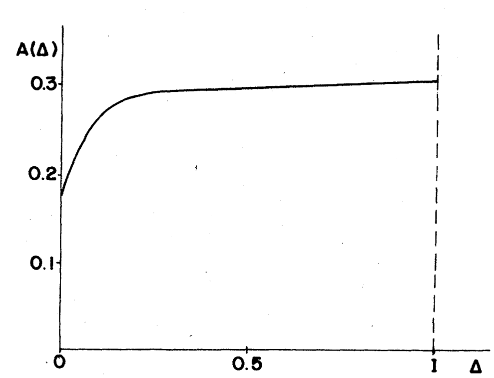

FIG. 3. Δ dependence of the asymptotic coefficient $A(\Delta)$ given by Eq. (12) [$A(1)\simeq 0.32$ and $A(0)\simeq 0.18$].

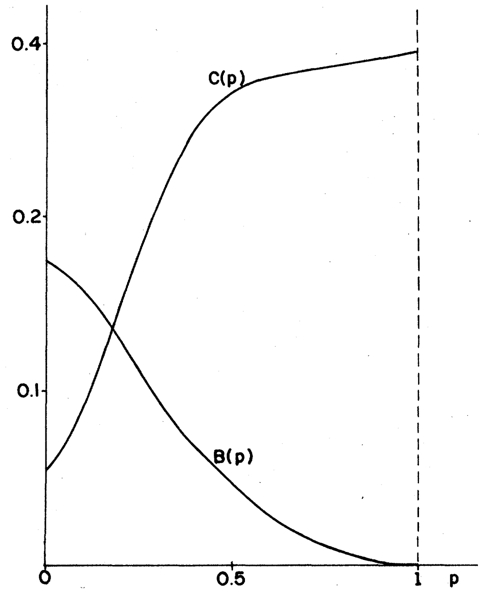

FIG. 4. p dependence of the coefficients $B(p)$ and $C(p)$ given by Eq. (13) $[B(1) \simeq 0, C(1) \simeq 0.29$, and $B(0) \simeq 0.17$, $C(0) \simeq 0.05]$.

temperature series,[2,8] and confirm the symmetry-based intuitive expectation.

The correlation-length critical exponents are given by

$$\nu_i = \ln b / \ln \lambda_i, \quad (i = T, \Delta) \ , \tag{14}$$

where λ_i is the relevant eigenvalue $(\lambda_i \geq 1)$ of the Jacobian matrix $\partial(p', K', \Delta')/\partial(p, K, \Delta)$ calculated at the corresponding fixed point. We obtain $\nu_T \simeq 1.15$ (the exact value equals one[14]) for the Ising fixed point, and $\nu_T = \infty$ (which reproduces the exact value[15]) and $\nu_\Delta \simeq 1.22$ (we found no other value in the literature for comparison) for the isotropic Heisenberg fixed point.

If we consider the hierarchical lattice generated by the recursive application of the graph of Fig. 1(a), the present results can alternatively be seen as *exact* for such system, *as long as the system is classical* (i.e., $\Delta = 1$ and $p = 1$). This is not necessarily true for quantum systems[16] (i.e., $\Delta \neq 1$ and/or $p \neq 1$). However, previous works[9,10] show that the approximation involved is a good one for the critical behavior of the system (also, for a detailed comparison with the usual Migdal-Kadanoff approach, see Ref. 10). Summarizing, we believe that the present approximation of the critical surface is (possibly for both the square and the hierarchical lattices) a numerically quite reliable one.

We acknowledge fruitful discussions with A. O. Caride; one of us (A.M.M.) also acknowledges useful remarks from C. A. P. Galvão.

*Permanent address for both authors.

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