p-wave superconductivity in cubic metals

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Group theory is used to classify p-wave superconducting states in a cubic crystal with spin-orbit coupling. All possible stable states are obtained based on the most general form of the Ginzburg-Landau free energy. Within the weak-coupling theory all stable states have real order parameters. These real states are even under time-reversal symmetry. However, in general, states with complex order parameters are also possible. In these complex states time-reversal symmetry is broken. This broken symmetry may show up through the existence of a Josephson current between a complex p-wave state and a usual s-wave superconductor. The form of the pairing in a magnetic field is also discussed.

I. INTRODUCTION

Since the discovery of superconductivity in heavy-electron systems such as $CeCu_2Si_2$,[1] UBe_{13},[2] and UPt_3,[3] there has been strong interest in the nature of their superconducting states. In particular an analysis of the specific heat of UBe_{13} below T_c led Ott and co-workers[4] to identify it as the first metal with a p-wave superconducting state. The analogy to ^{3}He was stressed by Anderson[5] and independently in Ref. 4. Stimulated by this analogy, the present authors and Rice *et al.*[6] have developed a generalization to finite temperatures of the Brinkman-Rice[7] theory of almost-localized electron systems and obtained a good description of the entropy (specific heat) in the normal state of UBe_{13}. By applying the theory of almost-localized electron systems to the heavy-electron systems, Valls and Tešanović[8] have obtained reasonable values for their transition temperatures. Others[9–13] however have argued in favor of a conventional s-wave pairing due to the electron-phonon interaction although these are clearly different to conventional metals in that the Coulomb interaction is not reduced due to the mismatch of energy scales and general considerations favor strong interactions between the quasiparticles due to spin fluctuations. In this work we do not however discuss the possible origin of p-wave pairing but instead concentrate on exploring the consequence of such pairing.

Although the analogy to ^{3}He has played a vital role in the understanding of the anomalous properties of the heavy-electron superconductors, there are also new aspects peculiar to the latter. First of all, these superconductors are crystals not liquids. In this paper we consider only cubic symmetry with an inversion center (O_h group), appropriate to UBe_{13}. Spin-orbit coupling is another important difference which has to be included especially because of the U ions.

The complexity of the order parameter in p-wave superconductivity allows various phases as equilibrium states.[14,15] In Ref. 4 an Anderson-Brinkman-Morel state was proposed for UBe_{13}. A polar state has been proposed for UPt_3.[16,17] Recent measurement[18] of the relaxation rate T_1^{-1}, of ^{9}Be nuclei in UBe_{13} were interpreted as favoring a polar state in this material also. But a definitive classification of the p-wave state requires an analysis of the symmetry classifications in the presence of cubic symmetry and spin-orbit coupling.

A classification of the most symmetric superconducting phases has been given by Volovik and Gor'kov[19] based on the examination of allowed subgroups of the symmetry group of the normal phases. A different approach was taken by Anderson[20] who examined the possible symmetries of the pairing order parameter near the transition temperature. There are some significant differences between their results as we will discuss below. In this paper we extend Anderson's approach by considering the form of the quartic terms in the Ginzburg-Landau (GL) expansion of the free energy. This restricts the forms the order parameter can take and we discuss these forms in detail. In agreement with previous authors, we find that a line of zeros of the order parameter is not allowed.

We also consider the influence of the spin-splitting terms introduced by a magnetic field. In particular we show that if the highest value of T_c belongs to a three-dimensional representation, there is a linear splitting.

II. SYMMETRY PROPERTIES

A. One-particle eigenstates

In the presence of spin-orbit coupling, the only symmetry operations are time reversal ($T=-i\sigma_y K$) and proper (R) and improper rotations (PR) which form a point group of the crystal. A natural basis functions for states near the Fermi surface can be set up in the following way.[21] First we divide the Brillouin zone into N parts (N is the number of elements of the point group). By choos-

ing one of the N parts, the star of the point k_1 is expressed by $k=gk_1$ where g is an element of the point group ($g=R$ or PR) and k_1 belongs to the first $1/N$ part of the Brillouin zone.

The one-particle eigenstate at k_1 is a spinor state,

$$\phi_{k_1\alpha}=V^{-1/2}e^{ik_1r}[U_{k_1}(r)\,|\uparrow\rangle+V_{k_1}(r)\,|\downarrow\rangle]\ . \tag{1}$$

The inversion and time reversal generate two states

$$\phi_{-k_1\alpha}=P\phi_{k_1\alpha}\ , \tag{2}$$

$$\phi_{-k_1\beta}=T\phi_{k_1\alpha}\ , \tag{3}$$

which belong to point $-k_1$ and are mutually orthogonal. Successive operation of P and T generates a state,

$$\phi_{k_1\beta}=PT\phi_{k_1\alpha}\ , \tag{4}$$

which is orthogonal to the original one. Thus we obtain doubly degenerate states at k_1 and $-k_1$. The Cooper pairing takes place among these four states. To avoid ambiguity we define $\phi_{k_1\alpha}$ as a state which is generated from a pure spin-up state by adiabatically switching on the spin-orbit coupling.

At a point k belonging to the star of k_1 in a different segment of the Brillouin zone, there is a rotation R which connects k with either a point k_1 through $k=Rk_1$ or a point $-k_1$ through $k=-Rk_1$. Then we can define $\phi_{k\alpha}$ and $\phi_{k\beta}$ by

$$O_R\phi_{k_1\gamma}=\sum_{\gamma'}[D^{(1/2)}(R)]_{\gamma'\gamma}\phi_{k,\gamma'}\quad(\gamma,\gamma'=\alpha\text{ or }\beta)\ , \tag{5}$$

where O_R is the total rotation operator and $D^{(1/2)}(R)$ is the rotation matrix for spin $\frac{1}{2}$. In the case of $k=-Rk_1$, $\phi_{k_1\gamma}$ should be replaced by $\phi_{-k_1\gamma}$.

B. Symmetry properties of the gap function

In terms of the pseudospin (α,β), the gap function is defined by

$$\Delta\gamma_1\gamma_2(k)=\begin{bmatrix}\Delta_{\alpha\alpha}(k)\ , & \Delta_{\alpha\beta}(k)\\ \Delta_{\beta\alpha}(k)\ , & \Delta_{\beta\beta}(k)\end{bmatrix}$$

$$=-\sum_{k',\gamma_3\gamma_4}V_{\gamma_2\gamma_1;\gamma_3\gamma_4}(k,k')\langle a_{k'\gamma_3}a_{-k'\gamma_4}\rangle\ , \tag{6}$$

where $V_{\gamma_1\gamma_2;\gamma_3\gamma_4}(k,k')$ is a matrix element of the pairing interaction

$$H_{\text{pair}}=\sum_{\substack{k,k'\\ \gamma_1\gamma_2\gamma_3\gamma_4}}V_{\gamma_1\gamma_2;\gamma_3\gamma_4}(k,k')a^{+}_{-k\gamma_1}a^{+}_{k\gamma_2}a_{k'\gamma_3}a_{-k'\gamma_4}\ . \tag{7}$$

There are several symmetry relations. First, from the anticommutation of fermions we have

$$\Delta_{\gamma_1\gamma_2}(k)=-\Delta_{\gamma_2\gamma_1}(-k)\ . \tag{8}$$

The inversion symmetry is expressed by

$$P\Delta_{\gamma_1\gamma_2}(k)=\Delta_{\gamma_1\gamma_2}(-k)\ , \tag{9}$$

and the time reversal symmetry by

$$T\Delta_{\gamma_1\gamma_2}(k)=\begin{bmatrix}\Delta^*_{\beta\beta}(-k)\ , & -\Delta^*_{\beta\alpha}(-k)\\ -\Delta^*_{\alpha\beta}(-k)\ , & \Delta^*_{\alpha\alpha}(-k)\end{bmatrix}\ . \tag{10}$$

Finally a rotation R is represented by

$$R\Delta_{\gamma_1\gamma_2}(k)=\sum_{\gamma_1'\gamma_2'}[D^{(1/2)}(R)]_{\gamma_1'\gamma_1}\times[D^{(1/2)}(R)]_{\gamma_2'\gamma_2}\Delta_{\gamma_1'\gamma_2'}(Rk) \tag{11}$$

from the definition of the pseudospin, Eq. (5)

In this paper we are interested in an odd-parity state (the p-wave state is the simplest one),

$$P\Delta_{\gamma_1\gamma_2}(k)=-\Delta_{\gamma_1\gamma_2}(k)\ .$$

From Eqs. (8) and (9), the gap function is symmetric $[\Delta_{\gamma_1\gamma_2}(k)=\Delta_{\gamma_2\gamma_1}(k)]$ for the odd-parity state and conveniently expressed by the vector notation

$$\Delta_{\gamma_1\gamma_2}(k)=i\mathbf{d}(k)\cdot(\tau\tau_y)_{\gamma_1\gamma_2}\ , \tag{12}$$

where τ_x, τ_y, τ_z are the Pauli matrices in the pseudospin space (α,β). Now the rotation property of Eq. (11) is rewritten as

$$Rd_l(k)=\sum_m[D^{(1)}(R)]_{ml}d_m(Rk)\quad(l,m=x,y,z)\ , \tag{13}$$

where $D^{(1)}(R)$ is the rotation matrix for spin 1.

III. GL FREE ENERGY AND STABLE STATES

A. Irreducible representations and T_c

The transition temperature to superconductivity is determined by the largest eigenvalue of the homogeneous integral equation,

$$-\sum_{\gamma_3\gamma_4}N(0)\langle V_{\gamma_2\gamma_1;\gamma_3\gamma_4}(k,k')\Delta_{\gamma_3\gamma_4}(k')\rangle_{k'}\equiv\omega\Delta_{\gamma_1\gamma_2}(k)\ , \tag{14}$$

where $N(0)$ is the density of states at the Fermi energy and the symbol $\langle\ \rangle_k$ denotes the average over the Fermi surface. Without spin-orbit coupling, the eigenvalue problem is independent of the spin indices and the spatial (k) part of the gap function is classified according to the irreducible representations of the cubic group. Our basic assumptions in this paper are: (1) The biggest eigenvalue belongs to the T_1 representation with eigenfunctions $\{\psi_x(k),\psi_y(k),\psi_z(k)\}$ which transform like a vector and (2) the biggest positive eigenvalue is well separated from the others. The first assumption means that the superconducting state we are considering is nothing but the p-wave one. We use the normalization $\sum_l\langle\psi_l(k)\psi_l(k)\rangle_k=1$ and in the simplest case of a spherical Fermi surface, $\psi_l(k)=\hat{\mathbf{k}}_l=k_l/k_F$.

Because of the second assumption, the effect of the spin-orbit coupling can be studied by using the eigenfunctions $\psi_l(k)$

$$\mathbf{d}(k)=\sum_{l,m}d_l^m\hat{\mathbf{l}}\psi_m(k)\ . \tag{15}$$

From the rotation property of Eq. (13) it is easily seen that $(\hat{\mathbf{x}},\hat{\mathbf{y}},\hat{\mathbf{z}})$ are also basis functions of the T_1 representation. Therefore the gap function belongs to a product representation $T_1\times T_1$. This product representation is decomposed as[22]

$$T_1\times T_1=A_1+E+T_1+T_2 .$$

Eigenfunctions for each representation are easily obtained with the aid of Clebsch-Gordan coefficients:

$$\begin{aligned}
A_1:&\quad \mathbf{d}(A_1)=\frac{1}{\sqrt{3}}(\hat{\mathbf{x}}\psi_x+\hat{\mathbf{y}}\psi_y+\hat{\mathbf{z}}\psi_z) ,\\
E:&\quad \mathbf{d}(E;u)=\frac{1}{\sqrt{6}}(2\hat{\mathbf{z}}\psi_z-\hat{\mathbf{x}}\psi_x-\hat{\mathbf{y}}\psi_y) ,\\
&\quad \mathbf{d}(E;v)=\frac{1}{\sqrt{2}}(\hat{\mathbf{x}}\psi_x-\hat{\mathbf{y}}\psi_y) ,\\
T_1:&\quad \mathbf{d}(T_1;x)=\frac{1}{\sqrt{2}}(\hat{\mathbf{y}}\psi_z-\hat{\mathbf{z}}\psi_y) ,\\
&\quad \mathbf{d}(T_1;y)=\frac{1}{\sqrt{2}}(\hat{\mathbf{z}}\psi_x-\hat{\mathbf{x}}\psi_z) ,\\
&\quad \mathbf{d}(T_1;z)=\frac{1}{\sqrt{2}}(\hat{\mathbf{x}}\psi_y-\hat{\mathbf{y}}\psi_x) ,\\
T_2:&\quad \mathbf{d}(T_2;\xi)=\frac{1}{\sqrt{2}}(\hat{\mathbf{y}}\psi_z+\hat{\mathbf{z}}\psi_y) ,\\
&\quad \mathbf{d}(T_2;\eta)=\frac{1}{\sqrt{2}}(\hat{\mathbf{z}}\psi_x+\hat{\mathbf{x}}\psi_z) ,\\
&\quad \mathbf{d}(T_2l\zeta)=\frac{1}{\sqrt{2}}(\hat{\mathbf{x}}\psi_y+\hat{\mathbf{y}}\psi_x) .
\end{aligned}\tag{16}$$

This classification and the eigenfunctions are in complete agreement with Anderson's results.[20]

In general we can expect that a different irreducible representation has a different eigenvalue ω by which the transition temperature is determined through

$$T_c=1.14\epsilon_c\exp(-1/\omega) ,$$

where ϵ_c is a cutoff energy. In the nine-dimensional space of $T_1\times T_1$ the pairing interaction can be rewritten by using effective spin- and orbital-momentum operators. Explicit expressions for the eigenvalues in terms of the effective pairing interaction are given in the Appendix.

B. GL free energy

With the eigenfunctions shown by Eqs. (16), the gap function can be expanded as

$$\mathbf{d}(k)=\sum_{\Gamma,\gamma}\lambda(\Gamma;\gamma)\mathbf{d}(\Gamma;\gamma) ,\tag{17}$$

where Γ specifies the irreducible representations and γ specifies its eigenfunctions (e.g., $\gamma=u$, v for $\Gamma=E$). The free energy is a function of $\lambda(\Gamma;\gamma)$. Near T_c (GL region) the free energy is expandable in terms of $\lambda(\Gamma;\gamma)$. Because a different irreducible representation has a different T_c, we may use one relevant irreducible representation which gives the highest (i.e., the actual) T_c.

Although it is straightforward to apply group theory to the GL free energy, it is more convenient to use the weak-coupling theory. In the weak-coupling theory, the GL free energy is given by[14,15]

$$\begin{aligned}F=\tfrac{1}{2}N(0)[&-a\langle \mathrm{Tr}\Delta^+(k)\Delta(k)\rangle_k\\&+b\langle \mathrm{Tr}\Delta^+(k)\Delta(k)\Delta^+(k)\Delta(k)\rangle_k] ,\end{aligned}\tag{18}$$

with $a=(T_c-T)/T_c$ and $B=\frac{7}{16}(\beta_c/\pi)^2\zeta(3)$. The second-order term is trivially given by

$$\langle \mathrm{Tr}\Delta^+(k)\Delta(k)\rangle_k=\tfrac{2}{3}\sum_{\gamma}|\lambda(\Gamma;\gamma)|^2 .\tag{19}$$

The fourth-order terms are shown in Table I. Based on the group theory we can prove that the tabulated fourth-order terms exhaust all fourth-order invariants. When we go beyond the weak-coupling theory the coefficients of these invariants (η_i) are independent parameters.

TABLE I. Fourth-order invariants in the GL free energy. $\lambda=(\lambda_x,\lambda_y,\lambda_z)$ for T_1 and $(\lambda_\xi,\lambda_\eta,\lambda_\zeta)$ for T_2. In the third column, coefficients given by the weak-coupling theory are shown. For an isotropic system $\langle\psi_x^4\rangle=\frac{1}{5}$ and $\langle\psi_x^2\psi_y^2\rangle=\frac{1}{15}$.

$A_1\,\lvert\lambda\rvert^4$	$\eta_1=\frac{2}{3}(\langle\psi_x^4\rangle+2\langle\psi_x^2\psi_y^2\rangle)$
$E\ (\lvert\lambda_u\rvert^2+\lvert\lambda_v\rvert^2)^2$	$\eta_1=\langle\psi_x^4\rangle+\langle\psi_x^2\psi_y^2\rangle$
$(\lambda_u^*\lambda_v-\lambda_v^*\lambda_u)^2$	$\eta_2=\frac{1}{3}(\langle\psi_x^4\rangle-7\langle\psi_x^2\psi_y^2\rangle)$
$T_1(\boldsymbol{\lambda}^*\cdot\boldsymbol{\lambda})^2$	$\eta_1=\frac{3}{2}(\langle\psi_x^4\rangle+\langle\psi_x^2\psi_y^2\rangle)$
$(\boldsymbol{\lambda}^*\cdot\boldsymbol{\lambda}^*)(\boldsymbol{\lambda}\cdot\boldsymbol{\lambda})$	$\eta_2=-\frac{1}{2}(\langle\psi_x^4\rangle+\langle\psi_x^2\psi_y^2\rangle)$
$\sum\lvert\lambda_x\rvert^2\lvert\lambda_y\rvert^2$	$\eta_3=-(\langle\psi_x^4\rangle-3\langle\psi_x^2\psi_y^2\rangle)$
$T_2(\boldsymbol{\lambda}^*\cdot\boldsymbol{\lambda})$	$\eta_1=\frac{3}{2}(\langle\psi_x^4\rangle+\langle\psi_x^2\psi_y^2\rangle)$
$(\boldsymbol{\lambda}^*\cdot\boldsymbol{\lambda}^*)(\boldsymbol{\lambda}\cdot\boldsymbol{\lambda})$	$\eta_2=-\frac{1}{2}(\langle\psi_x^2\rangle+\langle\psi_x^2\psi_y^2\rangle)$
$\sum\lvert\lambda_\xi\rvert^2\lvert\lambda_\eta\rvert^2$	$\eta_3=-(\langle\psi_x^4\rangle-3\langle\psi_x^2\psi_y^2\rangle)$

C. Stable states

One remark is in order before discussing equilibrium states for each representation. For any representation, η_1 in Table I should be positive to have a second-order phase transition. Of course, η_1 given by the weak-coupling theory satisfies this condition and $\eta_1>0$ is assumed in the following discussions.

A_1: In this one-dimensional representation, the stable state is trivial and given by the eigenfunction of Eq. (16) itself.

E: (a) $\eta_2<0$, (λ_u,λ_v) should be real except for a common phase factor, $(\lambda_u,\lambda_v)=\lambda(\cos\theta,\sin\theta)$. The continuous degree of freedom of θ remains in the fourth order of the GL free energy. In the sixth order there are two invariants for the real solution, $(\lambda_u^2+\lambda_v^2)^3$ and $(\lambda_u^2-\lambda_v^2)[\frac{1}{4}(\lambda_u^2+\lambda_v^2)^2-4\lambda_u^2\lambda_v^2]$. The second invariant lifts the continuous degeneracy. If its coefficient is positive, a state of $\mathbf{d}(E,v)$ type is stabilized. Equivalent domains are obtained by putting $\theta=0,\pm\frac{1}{3}\pi$. For a negative coefficient, the equilibrium states are of $d(E,u)$ type $(\theta=\frac{1}{2}\pi,\pm\frac{1}{6}\pi)$. (b) $\eta_2>0$. A complex solution, e.g., $(\lambda_u,\lambda_v)=\lambda/\sqrt{2}(1,i)$, gives a stable state.

T_1: It is convenient to use a vector notation,

$$\boldsymbol{\lambda}=(\lambda_x,\lambda_y,\lambda_z) .\tag{20}$$

If we define $\boldsymbol{\lambda}=\boldsymbol{\lambda}_1+i\boldsymbol{\lambda}_2$ with real vectors $\boldsymbol{\lambda}_1$ and $\boldsymbol{\lambda}_2$, the second fourth-order invariant is rewritten as

$$[(\boldsymbol{\lambda}_1\cdot\boldsymbol{\lambda}_1)-(\boldsymbol{\lambda}_2\cdot\boldsymbol{\lambda}_2)]^2+4(\boldsymbol{\lambda}_1\cdot\boldsymbol{\lambda}_2)^2 .$$

(a) $\eta_2<0$. A real solution $(\boldsymbol{\lambda}_2=0)$ minimizes the second term. The third invariant gives an anisotropy energy for $\boldsymbol{\lambda}_1$. (a.1) $\eta_3>0$. $\boldsymbol{\lambda}$ points along one of crystallographic axis, e.g., $\boldsymbol{\lambda}=\lambda(1,0,0)$. (a.2) $\eta_3<0$. $\boldsymbol{\lambda}$ points along one of threefold axes, e.g., $\boldsymbol{\lambda}=(1/\sqrt{3})\lambda(1,1,1)$. (b) $\eta_2>0$. A

TABLE II. Stable states of the p-wave superconductor in cubic metals. In the last column the symmetry axes whose intersections with the Fermi surface give zeros of the gap are shown. In the case of $\eta_2<0$ for E representation a stable state is determined by the sixth-order term. $(C_1,C_2,C_3)=[1/\sqrt{2}+i(1/\sqrt{6}),\ -1/\sqrt{2}+i(1/\sqrt{6}),\ -i[(2/3)^{1/2}]$.

A_1		$\frac{1}{\sqrt{3}}(\hat{\mathbf{x}}\psi_x+\hat{\mathbf{y}}\omega_y+\hat{\mathbf{z}}\psi_z)$	none
E	$\eta_2<0$	$\frac{1}{\sqrt{2}}(\hat{\mathbf{x}}\psi_x-\hat{\mathbf{y}}\psi_y)$	[0,0,1]
		$\frac{1}{\sqrt{6}}(2\hat{\mathbf{z}}\psi_z-\hat{\mathbf{x}}\psi_x-\hat{\mathbf{y}}\psi_y)$	none
	$\eta_2>0$	$\frac{1}{\sqrt{3}}(\hat{\mathbf{z}}\psi_z+e^{-(2/3)\pi i}\hat{\mathbf{x}}\psi_x+e^{(2/3)\pi i}\hat{\mathbf{y}}\psi_y)$	$[\pm1,\pm1,\pm1]$
	$\eta_2<\frac{1}{4}\eta_3$		
T_1	$\eta_3>0$	$\frac{1}{\sqrt{2}}(\hat{\mathbf{y}}\psi_z-\hat{\mathbf{z}}\psi_y)$	[1,0,0]
	$\eta_2<0$		
	$\eta_3<0$	$\frac{1}{\sqrt{6}}[\hat{\mathbf{x}}(\psi_y-\psi_z)+\hat{\mathbf{y}}(\psi_z-\psi_x)+\hat{\mathbf{z}}(\psi_x-\psi_y)]$	[1,1,1]
	$\eta_2>0$		
	$\eta_3<0$	$\frac{1}{2}[\hat{\mathbf{x}}(C_2\psi_z-C_3\psi_y)+\hat{\mathbf{y}}(C_3\psi_x-C_1\psi_z)+\hat{\mathbf{z}}(C_1\psi_y-C_2\psi_x)]$	[1,1,1]
	$\eta_2>\frac{1}{4}\eta_3$		
	$\eta_3>0$	$\frac{1}{2}[\hat{\mathbf{x}}(\psi_y+i\psi_z)-(\hat{\mathbf{y}}+i\hat{\mathbf{z}})\psi_x]$	[1,0,0]
	$\eta_2<\frac{1}{4}\eta_3$		
T_2	$\eta_3>0$	$\frac{1}{\sqrt{2}}(\hat{\mathbf{y}}\psi_z+\hat{\mathbf{z}}\psi_y)$	[1,0,0,]
	$\eta_2<0$		
	$\eta_3<0$	$\frac{1}{\sqrt{6}}[\hat{\mathbf{x}}(\psi_y+\psi_z)+\hat{\mathbf{y}}(\psi_z+\psi_x)+\hat{\mathbf{z}}(\psi_x+\psi_y)]$	none
	$\eta_2>0$		
	$\eta_3<0$	$\frac{1}{2}[\hat{\mathbf{x}}(C_2\psi_z+C_3\psi_y)+\hat{\mathbf{y}}(C_3\psi_x+C_1\psi_z)+\hat{\mathbf{z}}(C_1\psi_y+C_2\psi_x)]$	[1,1,1]
	$\eta_2>\frac{1}{4}\eta_3$		
	$\eta_3>0$	$\frac{1}{2}[\hat{\mathbf{x}}(\psi_y-i\psi_z)+(\hat{\mathbf{y}}-i\hat{\mathbf{z}})\psi_x]$	[1,0,0]

complex solution which satisfies $(\boldsymbol{\lambda}_1\cdot\boldsymbol{\lambda}_1)=(\boldsymbol{\lambda}_2\cdot\boldsymbol{\lambda}_2)$ and $(\boldsymbol{\lambda}_1\cdot\boldsymbol{\lambda}_2)=0$ minimizes the second term. The third term gives an anisotropy energy for an angular momentum vector $\hat{\mathbf{l}}=(\boldsymbol{\lambda}_1\cdot\boldsymbol{\lambda}_2)/(\boldsymbol{\lambda}_1\cdot\boldsymbol{\lambda}_1)$. (b.1) $\eta_3<0$. $\hat{\mathbf{l}}$ points along one of threefold axes, e.g.,

$$\boldsymbol{\lambda}=\frac{\lambda}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}+i\frac{1}{\sqrt{6}},-\frac{1}{\sqrt{2}}+i\frac{1}{\sqrt{6}},-i\left(\tfrac{2}{3}\right)^{1/2}\right].$$

(b.2) $\eta_3>0$. $\hat{\mathbf{l}}$ points along one of crystallographic axes, e.g.,

$$\boldsymbol{\lambda}=\frac{1}{\sqrt{2}}\lambda(0,1,i)\ .$$

For the states (a.1) and (b.2), the η_2 term and the η_3 term compete to give a boundary between them at $\eta_2=\frac{1}{4}\eta_3$.

T_2: Now we define

$$\boldsymbol{\lambda}=(\lambda_\xi,\lambda_\eta,\lambda_\zeta)\ . \tag{21}$$

Then we can repeat exactly the same arguments as for the T_1 representation.

All stable states thus obtained are shown in Table II. Weak-coupling theory gives $\eta_2<0$ for any representation. Therefore a real solution is expected within the weak-coupling theory. In some of the stable states the gap vanishes at points of intersections of symmetry lines with the Fermi surface. These symmetry lines are also shown in Table II. There are no states which have lines of zeros on the Fermi surface. For E, T_1, and T_2 representations, the equilibrium superconducting phases have equivalent domains. For example, in the case of (b), for the E representation there are two domains corresponding to $(\lambda_u,\lambda_v)=(1/\sqrt{2})\lambda(1,\pm i)$.

Now we compare the present results with the phases obtained by Volovik and Gor'kov.[19] Their $O\times R$, $S=1$ and $O(D_2)$, $S=1$ are nothing but our A_1 and case (b) of E. The phases of (b.2) of T_1 and T_2 correspond to their $D_4(E)$, $S=1$, with special choice of $f_1=-f_2$ and $f_1=f_2$, respectively [see Table of Ref. 20)]. If there is a splitting between the T_1 and T_2 representations [see Eq. (A4)], f_1 and f_2 cannot be arbitrary cubic functions, but there is a definite relation between the two.

IV. PAIRING IN A MAGNETIC FIELD (REF. 23)

In the problem of pairing in a magnetic field, there are two aspects. One is a diamagnetic effect due to orbital motion of electrons[24] and the other is a paramagnetic ef-

fect due to electron spins. In this section we will discuss the latter effect. Near H_{c2}, which is mainly determined by the former effect, spatial variation of the magnetic field is very slow and can be neglected.

We will concentrate on a linear effect of the magnetic field $\mathbf{H}$, since a quadratic effect may be very small. To the leading (second) order of the GL free energy, the selection rule for existence of the linear effect is very simple. Let us assume the order parameter near T_c belongs to one of the representations, Eqs. (16), Γ say. Since the uniform magnetic field behaves as a T_1 representation, it is necessary to have a T_1 representation in the product representation ($\Gamma\times\Gamma$). From product-decomposition rules[22] for the cubic group, we conclude that a linear effect exists only for the T_1 and T_2 representations. Again it is convenient to use the vector notation $\boldsymbol{\lambda}$, Eq. (20) for T_1 and Eq. (21) for T_2. The invariant, linear in H and quadratic in λ, is

$$i\sum_{l,m,n}\epsilon_{lmn}\lambda_l^*\lambda_m H_n\ , \tag{22}$$

where ϵ_{lmn} is the permutation symbol.

In the presence of the magnetic field, the eigenvalue or T_c further splits into

$$\tilde{\omega}=a, a\pm|\mathbf{h}|\ , \tag{23}$$

with $a=(T_c-T)/T_c$ and $\mathbf{h}=g\mathbf{H}$, where g is the coefficient of the invariant (22) in the GL free energy. We define a triad of unit vectors $(\hat{\mathbf{n}}_1\hat{\mathbf{n}}_2,\hat{\mathbf{h}})$ with $\hat{\mathbf{h}}=\mathbf{h}/|\mathbf{h}|$ and $\hat{\mathbf{n}}_1\times\hat{\mathbf{n}}_2=\hat{\mathbf{h}}$. Then the eigenvectors are given by

$$\begin{aligned}
&\boldsymbol{\lambda}_+=\frac{1}{\sqrt{2}}(\hat{\mathbf{n}}_1+i\hat{\mathbf{n}}_2)\quad(\tilde{\omega}=a+|h|)\ ,\\
&\boldsymbol{\lambda}_-=\frac{1}{\sqrt{2}}(\hat{\mathbf{n}}_1-i\hat{\mathbf{n}}_2)\quad(\tilde{\omega}=a-|h|)\ ,\\
&\boldsymbol{\lambda}_0=\hat{\mathbf{h}}\quad(\tilde{\omega}=a)\ .
\end{aligned} \tag{24}$$

The eigenvector for the maximum eigenvalue is complex. In a magnetic field, time-reversal symmetry is broken. So it is natural to have a complex state as an equilibrium state. The linear change of the eigenvalue results in an enhancement of H_{c2} from the value determined by the orbital effect. In this context it is worth mentioning that the H_{c2} of UBe_{13} is anomalously large[25]

For ^{3}He, it is well known that in a magnetic field there is a second continuous phase transition below T_c, A transition. In solids, anisotropy plays a crucial role. We expand the order parameter $\boldsymbol{\lambda}$ in terms of the eigenfunctions.

$$\boldsymbol{\lambda}=\alpha_+\boldsymbol{\lambda}_+ +\alpha_0\boldsymbol{\lambda}_0+\alpha_-\boldsymbol{\lambda}_-\ . \tag{25}$$

Below T_c, α_+ has a finite value and we can assume real α_+ without loss of generality. The cubic anisotropy produces linear coupling for $\boldsymbol{\lambda}_0$ and $\boldsymbol{\lambda}_-$ components,

$$-\frac{\eta_3}{2}\alpha_+^3\left\{\mathrm{Re}(\alpha_-)[(\cos^4\psi+\sin^4\psi)(1-\cos^4\theta)-\sin^4\theta]+\mathrm{Im}(\alpha_-)\tfrac{1}{2}\sin 4\psi\sin^2\theta\cos\theta\right.$$
$$\left.+\,\mathrm{Re}(\alpha_0)\frac{1}{2\sqrt{2}}\sin 4\psi\sin^3\theta+\mathrm{Im}(\alpha_0)\frac{1}{\sqrt{2}}\sin 2\theta(\sin^2\psi-\sin^2\theta+\cos^2\theta\sin^2\psi\right\}\ , \tag{26}$$

with $\hat{\mathbf{h}}=(\sin\theta\sin\psi,\sin\theta\cos\psi,\cos\theta)$. It is easily seen that the linear-coupling term vanishes when and only when the magnetic field is along a high-symmetry line ([1,0,0], [1,1,1] etc.). Thus, although T_c for T_1 or T_2 representation splits further due to the magnetic field, the lower transition is smeared out by the linear coupling except for the special orientation of the magnetic field. On the other hand, the A transition in ^{3}He is always possible, since there is no anisotropy energy in a liquid.

V. CONCLUSIONS

We have studied p-wave superconductivity in cubic metals taking also spin-orbit coupling into account. In the second order of the GL free energy, an important consequence is the splitting of T_c according to A_1, E, T_1, and T_2 irreducible representations as already pointed out by Anderson, Volovik, and Gor'kov. By considering higher-order terms, all possible stable states are enumerated. Some of the stable states have points where the gap vanishes but there is never a line of zeros.

In the weak-coupling theory, all stable states have real order parameters except for a trivial phase factor. But, in general, states with complex order parameters are possible for the E, T_1, and T_2 representations. In these complex states, time-reversal symmetry is broken. This fact may be important when we consider Josephson-junction experiments between s- and p-wave superconductors. Since inversion symmetry is broken in an experimental geometry, we can expect parity-breaking matrix elements for tunneling. A usual s-wave superconducting state is odd under time-reversal symmetry. Since a real p-wave state is even under time-reversal symmetry, there is no Josephson current in this case. On the other hand for a complex p-wave state there can be a Josephson current. Note that without spin-orbit coupling the Josephson current vanishes simply because of rotational symmetry in spin space; there is no mixing between a singlet order parameter and a triplet one.[26]

We can expect a qualitatively different temperature dependence of H_{c2} according to the symmetry of the superconducting state. If a state belonging to the T_1 or T_2 representation is stable at $H=0$, then the linear term Eq. (23) will counteract the usual diamagnetic contribution to $dH_{c2}(T)/dT$ and may lead to anomalously large value of dH_{c2}/dT. On the other hand, if the stable state at $H=0$ belongs to the E or A_1 representation, there is no linear effect for a weak field. Only at the field strength at which the change of the eigenvalue, Eq. (23), is comparable with the difference of eigenvalues at $H=0$, Eq. (A4),

can we expect a significant paramagnetic effect through mixing of T_1 or T_2 components. In such a case $H_{c2}(T)$ may have an upward turn. In the case of UBe_{13} such an anomaly has not been reported to our knowledge but the very large value of $dH_{c2}(T)/dT$ [$=-440$ kOe/K (Ref. 25)] favors a T_1 or T_2 representation. A definitive test would be the observation of the two transitions with H oriented along a symmetry axis as discussed in the preceding section or the orientational diamagnetic shift predicted by Gor'kov.[24]

Note added in proof. We have recently received a more detailed manuscript from the author of Ref. (19), G. E. Volovik and L. P. Gor'kov (unpublished).

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APPENDIX

Within the subspace of $T_1\times T_1$, where $\hat{\mathbf{x}},\hat{\mathbf{y}},\hat{\mathbf{z}}$, and ψ_x,ψ_y,ψ_z are the basis functions for the spin and the orbital part, respectively, we can introduce spin operators S_l by

$$S_l\hat{\mathbf{m}}=i\epsilon_{lmn}\hat{\mathbf{n}}\ , \tag{A1}$$

and angular momentum operators L_l by

$$\langle L_l(k,k')\psi_m(k')\rangle_{k'}=i\epsilon_{lmn}\psi_n(k)\ . \tag{A2}$$

In the presence of spin-orbit coupling the pairing interaction of Eq. (7) can be parametrized by these operators as

$$V=V_0+V_1\mathbf{S}\cdot\mathbf{L}+V_2(\mathbf{S}\cdot\mathbf{L})^2+V_3\sum_{x,y,z}S_x^2L_x^2\ . \tag{A3}$$

The eigenvalues for the irreducible representations are:

$$\begin{aligned} &A_1\text{:}\ \ V_0-2V_1+4V_2+2V_3\ ,\\ &E\text{:}\ \ V_0+V_1+V_2+2V_3\ ,\\ &T_1\text{:}\ \ V_0-V_1+V_2+V_3\ ,\\ &T_2\text{:}\ \ V_0+V_1+V_2+V_3\ . \end{aligned} \tag{A4}$$

In ^{3}He, a dipole-dipole interaction is the origin of the coupling between the spin and the orbital degrees of freedom. In such a case, $V_1=V_3=0$ and T_1 and T_2 representations are degenerate and the Anderson-Brinkman-Morel state can remain as a stable state.

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