

Localization and electron-electron interaction effects in thin Pt wires

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(Received 25 June 1984; revised manuscript received 7 November 1984)

We have studied the low-temperature electrical properties of Pt wires with cross-sectional areas as small as $1.7 \times 10^{-12} \text{ cm}^2$. As the temperature is lowered, we find a resistance increase, ΔR , due to localization and electron-electron interaction effects. The dependence of ΔR on temperature and on the cross-sectional area of the sample is the same as that found in previous studies of thin wires. When these results are compared with those for wires composed of other materials, the dependence of ΔR on resistivity observed previously for wires with higher resistivities is found to agree well with the present results. However, this behavior is different from the resistivity dependence of ΔR which has recently been found for wires of lower resistivity.

I. INTRODUCTION

It has been known for some time that the nature of the electronic states in a disordered system depends strongly on the spatial dimensionality.¹⁻³ In particular, it has been shown that all of the states in a one-dimensional disordered system are spatially localized.²⁻⁴ This prediction did not appear to be accessible to experiment until Thouless⁵ showed that it should also apply to thin (i.e., small diameter) wires at low temperatures. In this case localization is manifest as an increase in the resistance as the temperature is reduced. A number of experimental studies of thin wires have been reported in the past several years.⁶⁻¹² These results indicate that the resistance of a thin wire does indeed increase at low temperatures. The relative magnitude of the resistance increase, $\Delta R/R_0$, where ΔR is the resistance increase and R_0 is the (temperature-independent) impurity resistance, has been found to vary in a manner which is at least qualitatively consistent with the predictions of localization theory. Meanwhile, it has been demonstrated¹³ that electron-electron interactions in the presence of disorder can give rise to behavior, in particular a resistance increase, which is very similar to that predicted by localization theory. It seems quite likely that both mechanisms, localization and interactions, are important in thin wires, and indeed, one goal of the experiments is to determine the relative importance of the two effects.

For wires composed of high-resistivity alloys, experiments have shown⁶⁻¹⁰ that $\Delta R/R_0$ varies approximately linearly with the impurity resistivity of the alloy, ρ_e . This result is inconsistent with *both* localization and interaction theories, at least in their present forms. The purpose of the experiments described in this paper was to extend the measurements to lower-resistivity wires, in order to see if the discrepancy persists in this case. As we will see, it does, at least for the resistivity we have studied here. However, work on wires of even lower resistivities than those studied in this experiment, which was reported after our work was completed,^{11,14} seems to indicate that the resistivity dependence of $\Delta R/R_0$ is different for these very-low-resistivity wires.

A brief review of the pertinent theory is given in Sec. II. Section III contains a description of the experimental technique, and the results are presented in Sec. IV. These results are compared with those of other workers and with the theory in Sec. V. A preliminary account of some of our results has been given previously.¹⁵

II. THEORY

It has been known for many years that all states in a disordered one-dimensional system are localized.²⁻⁴ However, the application of this result to thin wires, together with the behavior at finite temperatures, have received attention only relatively recently. We will not review all of the theoretical work here, but simply state the results which we will need in this paper. First, all states in a thin wire are localized, with the "length" of the state, the localization length, L_{loc} , being equal to the length of wire which has an impurity resistance¹⁶⁻¹⁸ of $2\pi\hbar/e^2 \approx 25.8 \text{ k}\Omega$. At absolute zero such a wire will have a resistance which varies exponentially with its length.^{5,16} At finite temperatures, the resistance will be given by¹⁷

$$R = R_0 + \Delta R \quad , \quad (1)$$

with

$$\frac{\Delta R}{R_0} = \frac{L_i}{L_{\text{loc}}} \quad . \quad (2)$$

Here L_i is the distance which an electron diffuses between inelastic collisions, also called the inelastic mean-free path, and is given by

$$L_i = (D\tau_i)^{1/2} \quad , \quad (3)$$

where D is the electron diffusion constant (due to elastic collisions), and τ_i is the inelastic scattering time. We should note that the prediction (2) applies only at relatively high temperatures, where $\Delta R/R_0$ is small. This is the case applicable to the experiments which will be described in this paper. Since L_{loc} is related to the resistivity and cross-sectional area of the wire, and D is also related to the resistivity, (2) may be rewritten as¹⁰

$$\frac{\Delta R}{R_0} = \frac{\rho_e L_i}{AR_T} = \frac{v_F}{eAR_T} \left(\frac{2m\rho_e\tau_i}{3n} \right)^{1/2} \quad (4)$$

Here, we have used the usual relations for a free electron gas,¹⁹ where v_F is the Fermi velocity, n is the electron density, m is the mass, e is the charge of an electron, ρ_e is the impurity resistivity, A is the cross-sectional area of the wire, and R_T is a constant¹⁸ equal to $2\pi\hbar/e^2 \approx 25.8$ k Ω . From (4) we see that provided τ_i is independent of both the resistivity and the cross-sectional area, $\Delta R/R_0$ should vary as $(\rho_e)^{1/2}$, and also be inversely proportional to the cross-sectional area.

As mentioned in the Introduction, it has been shown¹³ that electron-electron interactions in a disordered system will lead to behavior very similar to that caused by localization. This theory predicts²⁰

$$\frac{\Delta R}{R_0} = \frac{\sqrt{2}v_F}{eAR_T} \left(2 - \frac{3}{4}F \right) \left(\frac{2m\rho_e\hbar}{3nk_B T} \right)^{1/2}, \quad (5)$$

where k_B is Boltzmann's constant, and F is a screening factor whose value lies between 0 and 1. Note that this theory does *not* involve the localization length, L_{loc} , as such, but that a combination of the constants which enter the denominator of (5) turn out to be equal to L_{loc} , and this is how the factor of R_T arises. The prediction of interaction theory, (5), is very similar in form to (4). For our purposes, the most important aspects of (4) and (5) are that $\Delta R/R_0$ is predicted to vary as $(\rho_e)^{1/2}$, and as A^{-1} .

III. EXPERIMENTAL TECHNIQUE

The samples studied in this work were fabricated with a method involving substrate step techniques, in which a wire is formed in the corner of a step which has been ion milled into a substrate. This method has been discussed in detail elsewhere.^{6,9,21,22} Here we only note that, as in previous work, a Commonwealth Scientific model 2-30 ion gun was used, and that the final milling was performed at a 45° angle, so that the smallest wires had a (right) triangular cross section. Both glass and sapphire substrates were employed, although the yield of good wires was much lower with the latter, presumably because the ion milling rate of sapphire is less favorable for the fabrication process than that of glass.

The Pt films from which the wires were made were deposited by ion beam sputtering, using the ion gun noted above. The configuration for sputtering is shown in Fig. 1. The Ar ion beam was incident on a Pt foil at 45° and the sputtered Pt was "collected" on the substrate, which was oriented as shown in Fig. 1. The substrate was oriented in this way (as opposed to being normal to the Pt "beam") in order to ensure that the step in the substrate was coated with Pt. The Pt deposition rate²³ was ≈ 1.7 Å/s. Transmission electron microscope studies²² of films lifted off of their substrates showed that the grain size was ≈ 75 Å, and the films (on substrates) were found to be electrically continuous for thicknesses greater than about 25 Å. The wires were made from films which were 100–200 Å thick, and had resistivities of 40 ± 6 $\mu\Omega$ cm at room temperature. The low-temperature resistivity was

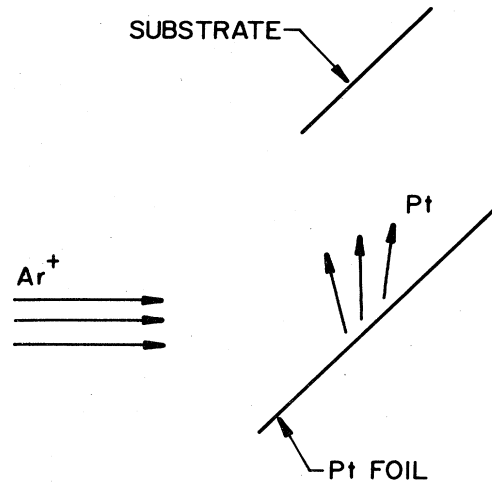


FIG. 1. Schematic of the set-up used to ion beam sputter Pt.

32 ± 5 $\mu\Omega$ cm. The wires and films had very similar resistance ratios (i.e., ratios of room-temperature to low-temperature resistance), which indicates that they had similar resistivities. Both the films and wires were quite stable, and could be stored at room temperature with no special precautions.

In our experiments, the cross-sectional areas of the wires were varied in two different ways. First, the size of the substrate step was varied with the thickness of the Pt film held fixed. The cross sections of the wires obtained in this way are discussed in Ref. 9. The smallest wires were triangular while the large wires were much wider than they were thick. The second way the cross-sectional area was varied was by changing the thickness of the Pt film. In our work two different film thicknesses were employed, 100 and 200 Å.

The measurements of resistance as a function of temperature were made using essentially the same techniques as have been described elsewhere.⁹ Two cryostats were employed. A ⁴He cryostat was used for measurements in the range 1.2–15 K. In this apparatus the sample was mounted on a Cu block which was enclosed in a vacuum can, which was in turn immersed in liquid ⁴He. The desired temperature was attained with the use of a heater or by lowering the pressure over the liquid bath. The temperature was measured using a calibrated Ge resistance thermometer. Measurements at temperatures below 1.2 K were performed using a ³He cryostat which has been described elsewhere.^{24,9}

The sample resistances were measured using standard low-frequency bridge techniques, in which a ratio-transformer was the variable element of the bridge.^{25,22,26} In all cases, measurements were initially made as a function of the sample current to determine the importance of Joule heating. For low currents the resistance varied approximately quadratically with the current, as expected for Joule heating. The measurements presented in this paper were all made with the current sufficiently small that these heating effects were negligible. For the lowest temperatures and the smallest wires this required currents of order 10^{-10} A.

In a separate series of experiments, we have studied the behavior of Pt films prepared in the manner described above. These measurements²⁷ showed that the films behaved as expected for two-dimensional disordered systems.²⁸ That is, they exhibited a resistance rise at low temperatures which varied as the logarithm of the temperature, and was proportional to the sheet resistance of the film. These results also indicated that other effects, such as the Kondo effect, were negligible. Since our wires were made from essentially the same films, this should be true for the wires as well.

IV. EXPERIMENTAL RESULTS

Some typical results for the resistance as a function of temperature for several Pt wires are shown in Fig. 2. These wires were made from 200-Å films; similar results were obtained for samples made from 100-Å films. We also note (see in addition Fig. 4 below) that the results in Fig. 2 are replotted in Fig. 3, where it is seen that the low-temperature resistance rise varies approximately as $T^{-1/2}$. This is the same temperature dependence as found previously,⁶⁻¹⁰ and suggests that either (4) is appropriate and that τ_i varies as T^{-1} , that (5) alone is applicable, or that some combination of these possibilities is realized.

Figure 4 shows the slopes of the low-temperature portions of the curves in Fig. 3 as a function of A^{-1} , where we also show data for a number of other samples. Here we see that the results for wires made from 100-Å and 200-Å films fall on two distinct curves. These curves can be understood as follows. First, consider the behavior in the limit $A^{-1} \rightarrow 0$. Here both curves have a nonzero intercept; that is, the resistance rise is nonzero when $A^{-1} = 0$, and this intercept is different for the two cases. This can be easily understood when one recalls that in the limit $A^{-1} \rightarrow 0$ our samples become very wide; indeed, they are essentially films in this case.⁹ In fact the data at $A^{-1} = 0$

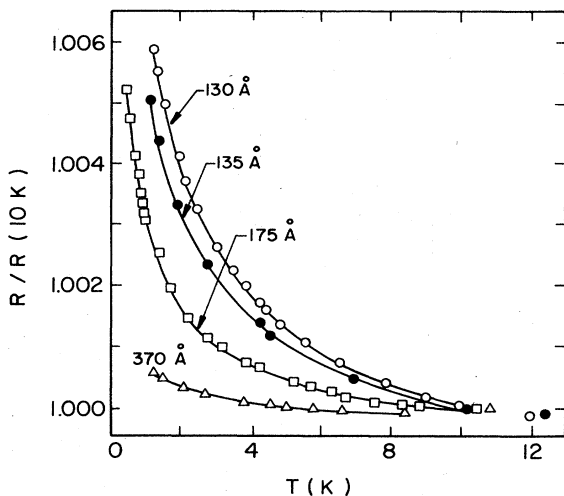


FIG. 2. Resistance as function of temperature for several Pt wires made from 200-Å films. The values of \sqrt{A} for the different wires are indicated in the figure.

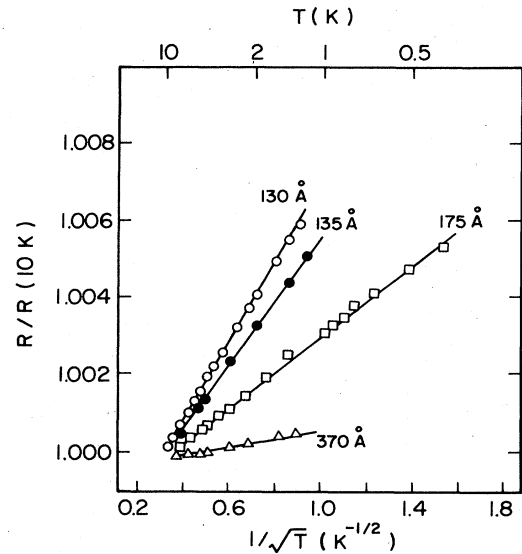


FIG. 3. The same data as shown in Fig. 2, but here plotted as a function of $T^{-1/2}$.

in Fig. 4 are from measurements on films.^{27,29} Since the different sets of samples have different Pt film thicknesses (and hence also different sheet resistances) they have²⁸ different behavior at $A^{-1} = 0$, and this is why there are different intercepts in Fig. 4. Now, when A is large (but not infinite), the behavior will be intermediate between one and two dimensional; i.e., ΔR will consist of the two-dimensional contribution like that seen at $A^{-1} = 0$, along

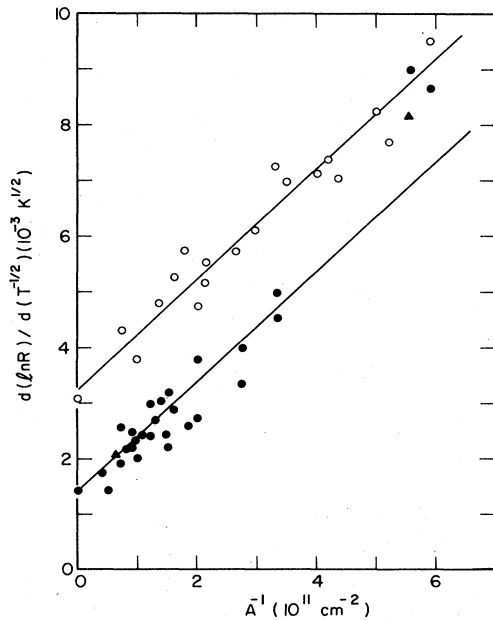


FIG. 4. Limiting low-temperature slopes (Ref. 29) of the curves in Fig. 3 as a function of A^{-1} . The open symbols correspond to measurements on wires made from 100-Å films, while the closed symbols correspond to wires made from 200-Å films. The circles correspond to samples on glass substrates, while the triangles correspond to samples on sapphire substrates.

with a one-dimensional contribution. From (4) and (5), and from previous work,⁹ we expect the one-dimensional contribution to vary as A^{-1} . As can be seen from Fig. 4, the results for large A (i.e., small A^{-1}) suggest that the one- and two-dimensional contributions are, to within our uncertainties, simply additive. So far as we know, there has been no theoretical work which predicts this, but such behavior, which is consistent with previous experimental results, seems reasonable. Thus, the behavior for large A which is shown in Fig. 4 can be understood as a type of dimensional crossover from two- to one-dimensional behavior, as the samples become narrower; i.e., as A^{-1} becomes larger. Eventually, as A is made smaller, the sample cross section will become triangular. In this case, the two-dimensional contribution should vanish, and we would expect ΔR to again be proportional to A^{-1} , but now with an intercept of zero. The point at which this small A behavior is reached will depend upon the thickness of the films from which the wires are made. For 200-Å films simple geometrical considerations show that this limit will be reached when $A \approx 4 \times 10^{-12} \text{ cm}^2$ ($A^{-1} = 2.5 \times 10^{11} \text{ cm}^{-2}$), which is seen to be in reasonable agreement with the point in Fig. 4 where the deviation from the limiting large A behavior occurs. More precisely, this is roughly the value of A at which the results for wires made from 200-Å films begin to deviate from the limiting line (drawn in Fig. 4) observed for large A . Note also that for the wires made from 100-Å films similar deviations would be expected, but at a smaller value of A . In this case the deviations should occur at $A \approx 1 \times 10^{-12} \text{ cm}^2$, and hence they are not visible in Fig. 4. Thus, we conclude that the behavior observed in Fig. 4 is quite consistent with expectations based on the known sample cross sections together with reasonable assumptions concerning the cross over from two- to one-dimensional behavior.

V. DISCUSSION AND COMPARISON WITH PREVIOUS WORK

Perhaps the most direct way to compare the results for different materials is to consider them as a function of ρ_e , since this quantity appears directly in (4) and (5). A plot of $\Delta R/R_0$ as a function of ρ_e for the high-resistivity materials which have been studied^{6-10,30} is shown in Fig. 5. We see that our results for Pt agree very well with a simple linear extrapolation of the previous results for higher ρ_e . If we interpret this result, namely that $\Delta R/R_0$ is linearly proportional to ρ_e , in terms of localization theory, then from (4) we see that the inelastic mean-free path, L_i , must be independent of ρ_e . This in turn implies that $\tau_i \approx D^{-1}$; i.e., that τ_i becomes larger as the material becomes more disordered. However, as noted previously,^{9,10} there is no known scattering mechanism for which τ_i varies in this way and also varies as T^{-1} , as is needed to explain the temperature dependence of the resistance.^{31,32} It should also be noted that this result, namely that $\Delta R/R_0$ varies linearly with ρ_e , is not consistent with interaction theory (5).

The behavior for low resistivities^{11,14} is shown in Fig. 6. Except for the AuPd data of White *et al.*,³³ there again seems to be a smooth variation as a function of ρ_e . How-

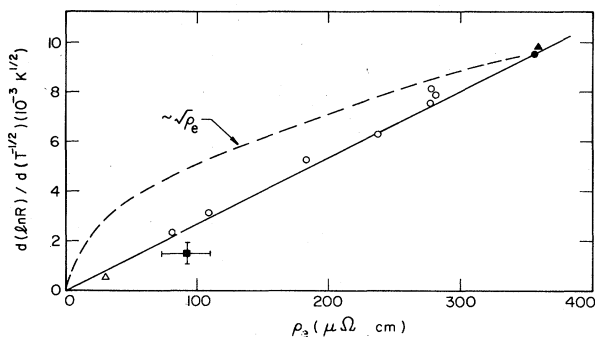


Fig. 5. Limiting low-temperature slope, $d(\ln R)/d(T^{-1/2})$ as a function of ρ_e for $\sqrt{A} = 500 \text{ \AA}$, for wires made of various materials. The circles (open and closed) and the closed square correspond to AuPd wires (Refs. 6, 9, and 10), the closed triangle to WRe wires (Refs. 7 and 8), and the open triangle to the Pt wires studied in this work. Typical uncertainties are indicated. The straight line is simply a guide to the eye. The dashed curve shows a $(\rho_e)^{1/2}$ dependence for comparison.

ever, in this range the variation does not appear to be linear in ρ_e . As discussed by White *et al.*¹¹ the data in this range are more consistent with a $(\rho_e)^{1/2}$ form, which is in agreement with both theories, (4) (if τ_i is assumed to be independent of ρ_e) and (5). For comparison, the dashed curve in Fig. 6 illustrates a $(\rho_e)^{1/2}$ variation. We should note that White *et al.*¹¹ and also Giordano¹⁰ have emphasized that the sort of analysis we have used here may be misleading because we have ignored the possible material dependence of parameters such as v_F , n , etc., which enter (4) and (5).³⁴ Unfortunately, it is very difficult to estimate these parameters reliably. Even if one could, it is doubtful that the theories, (4) and (5), which assume a simple spherical Fermi surface, etc., could be trusted to

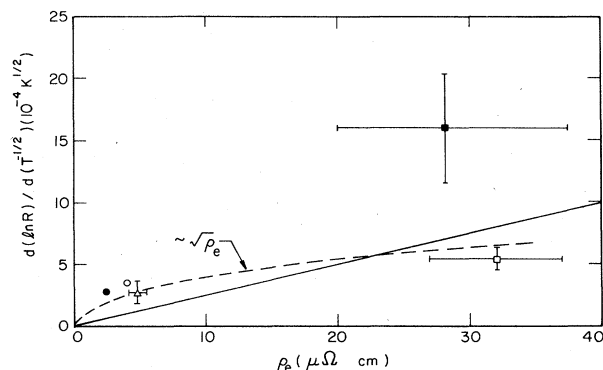


Fig. 6. Limiting low-temperature slope, $d(\ln R)/d(T^{-1/2})$ as a function of ρ_e for $\sqrt{A} = 500 \text{ \AA}$, for wires made of various materials. The open square corresponds to the Pt wires studied in this work, the closed square to the AuPd wires studied by White *et al.* (Ref. 11), the closed circle to Cu wires (Ref. 11), the open circle to Ni wires (Ref. 11), and the open triangle to Au wires (Ref. 14). Typical uncertainties are indicated. This is the same plot as in Fig. 5 except that here we show only data for low-resistivity samples. The solid line is an extrapolation of the solid line in Fig. 5, while the dashed line illustrates a $(\rho_e)^{1/2}$ dependence for comparison.

accurately predict such "subtle" variations. In any event, it seems extremely unlikely that variations in v_F , etc., would be both large enough and of just the form needed to account for the linear variation with ρ_e which is seen at high ρ_e (Fig. 5). The different dependencies on ρ_e seen in Figs. 5 and 6 therefore suggest a changeover in the behavior in the neighborhood of $\rho_e \approx 30 \mu\Omega \text{ cm}$. The origin of this change is not clear. If we assume that the behavior is dominated by electron-electron interaction effects, then one possibility is that the assumption that the elastic mean-free path is long compared to the Fermi wavelength, which is used in the derivation of (5), is no longer valid. The breakdown of this condition is certainly expected as ρ_e is made larger, but it is not yet known how much of an effect this will have on (5); that is, the size of the corrections to (5) have not yet been calculated. If, on the other hand, we assume that the behavior is dominated by localization, then it is possible that the change we are observing is due to a change in the behavior of τ_i . It is conceivable that for high resistivities τ_i varies linearly with ρ_e , as would be required to explain the overall resistivity dependence of $\Delta R/R_0$, but that a different scattering mechanism, for which τ_i is independent of ρ_e , dominates for low resistivities. A problem with this explanation is that, as noted above, no known scattering mechanism yields a scattering time which varies both linearly with ρ_e and as T^{-1} . A third possibility is that the cross-over is due to a change from behavior dominated by interactions to behavior dominated by localization. Crucial to this possibility is the manner in which the two mechanisms combine, and unfortunately this problem has not yet been addressed quantitatively by the theory for the one-dimensional case.

In conclusion, we have studied the behavior of thin Pt wires at low temperatures. They exhibit behavior very similar to that seen previously in wires made from other materials. Combining our results with those of other workers indicates that the resistivity dependence of localization and electron-electron interaction effects in thin wires depends on the impurity resistivity of the wire. The origin of this dependence is not clear at this time.

Note added. After this paper was submitted for publication, we learned of work by Prober and co-workers³⁵ in which the effect of spin-orbit scattering on the resistance and magnetoresistance of thin wires was considered, both experimentally and theoretically. It is worthwhile to discuss their results in the context of our experiments. While we have not discussed it in this paper, it turns out that the theory predicts that in the presence of a magnetic field the effects of localization and electron-electron interactions will be quite different. Hence, magnetoresistance measurements provide a powerful way of determining the relative importance of the two mechanisms, a prime goal of the present work. Work of this type in two dimensions (i.e., with thin films) has been very successful.³⁶ Prober and co-workers have extended these kinds of studies to thin Al wires, and have also successfully separated the contributions of localizations and interactions. They have concluded that the theory correctly describes the experiments (at least their experiments), *provided* that the effects of spin-orbit coupling are taken into account, and it is in-

teresting to consider if this might also be the case for our experiments. Unfortunately, magnetoresistance measurements have not yet been performed with our Pt wires, but we can still consider the behavior in the absence of a magnetic field. There are several different possible cases to consider, depending on the values of various parameters. If we assume that localization is important, then we have two possibilities. First, if we assume that, as was found by Prober *et al.* for Al wires, the inelastic scattering is due to electron-phonon scattering and dirty-limit electron-electron scattering, then we would expect the temperature dependence of the resistance [see (4)] to be much stronger than observed in Fig. 3.³⁷ Here the effect of spin-orbit scattering would be, according to the theory, to change the sign of the localization contribution to ΔR , but we would still expect to see the very strong temperature dependence, and large magnitude of ΔR characteristic of the above scattering processes, and this is clearly not seen experimentally. A second possibility is that localization is important, but that the inelastic scattering is due to some disorder related scattering process (such as electron—two-level-system scattering), and is therefore quite short. Again we would expect the spin-orbit scattering to make the localization contribution to ΔR negative, so to explain the positive ΔR which is observed, we need to assume the importance of electron-electron interactions. If we now presume that the contributions of the two mechanisms simply add (although as noted above, as far as we know there is no detailed theoretical justification of this presumption), then we still cannot explain the fact that ΔR depends linearly on ρ_e (see Fig. 5). One last possibility is that for some reason the localization contribution to ΔR is negligible (as might be the case if magnetic scattering is very large³⁵), in which case the measured ΔR would be due solely to interactions. However, this still does not account for the linear dependence of ΔR on ρ_e .

In short, the recent work of Prober and co-workers does not really affect any of the conclusions which can be drawn from our results (compare the discussion here with that in Sec. V). Namely, our findings cannot be explained by the existing theories of localization or electron-electron interactions, or any simple combination of the two. The separation of the two effects in systems like the Al wires studied by Prober *et al.* is possible only if the theory is capable of explaining the observed results, which as we have seen, is not the case for our experiments.

ACKNOWLEDGMENTS

We thank W. D. Williams for permission to show his results in Fig. 6 prior to publication. We also thank D. E. Beutler and W. D. Williams for helpful discussions, and M. D. Feuer and D. E. Prober for providing the mask from which the wires were made. This work was supported in part by the Research Corporation, by a fellowship from the Alfred P. Sloan Foundation (to N. G.), and by the National Science Foundation under Grant No. DMR79-06716. The use of central facilities supported by the National Science Foundation—Materials Research Laboratory program through Grant No. DMR80-20249 is also gratefully acknowledged.

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- ²⁸E. Abrahams, P. W. Anderson, D. C. Licciardello, and T. V. Ramakrishnan, *Phys. Rev. Lett.* **42**, 673 (1979). See also R. C. Dynes, *Physica* **109&110B**, 1857 (1982).
- ²⁹We note that strictly speaking the films do not follow a $T^{-1/2}$ dependence at low temperatures. For the sake of comparison with the results for the wires, we have obtained the slope plotted in Fig. 4 by "forcing" the films to fit this dependence. Since the logarithmic dependence displayed by the films is not too different from a $T^{-1/2}$ variation, the ambiguity in this procedure is negligible for our purposes.
- ³⁰Here we have not included the recent results of Westervelt and co-workers on drawn Pt wires. Despite their small size, these wires appear to be exhibiting three-dimensional behavior (see Ref. 12).
- ³¹It is important to note that the measurements of phase slip centers by Chaudhari and co-workers [P. Chaudhari, A. N. Broers, C. C. Chi, R. Laibowitz, E. Spiller, and J. Viggiano, *Phys. Rev. Lett.* **45**, 930 (1980); H. Raffy, R. B. Laibowitz, P. Chaudhari, and S. Maekawa, *Phys. Rev. B* **28**, 6607 (1983)] indicate that τ_i is quite short in these systems. In fact the measured values of τ_i are consistent with (4) in conjunction with the measured $\Delta R/R_0$. This strongly suggests that localization is playing an important role in these systems, but the actual inelastic scattering mechanism is still not known.
- ³²The arguments concerning the behavior of a disorder related inelastic scattering mechanism have been given elsewhere (Ref. 10), but it is worth repeating them here. The fact that τ_i seems to be much shorter than expected based on mechanisms known for pure metals suggests that some sort of disorder induced or related scattering mechanism is playing an important role. In this case we would expect τ_i to increase as ρ_e is made smaller, since in the clean limit this new mechanism should not be important. However, the experiments suggest that τ_i is decreasing as the material is made cleaner, and it is difficult to see how any disorder related mechanism could yield this sort of behavior. However, we should note that recent work on electron-two-level-system scattering [A. D. Stone, J. D. Joannopoulos, and D. J. Thouless (unpublished); A. D. Stone, thesis, Massachusetts Institute of Technology] suggests that these very qualitative arguments concerning the dependence of τ_i on the amount of disorder could well be incorrect. The problem of inelastic scattering in these systems has also been discussed by Gefen and Schön [Y. Gefen and G. Schön, *Phys. Rev. B* **30**, 7323 (1984)]; they find an electron-phonon scattering time which varies as T^2 , which is not consistent with our results. We should also note that recent experimental work on a two-dimensional system, $\text{In}_2\text{O}_{3-x}$ films [Z. Ovadyahu, *J. Phys. C* **16**, L845 (1983); *Phys. Rev. Lett.* **52**, 569 (1984)], has also found an inelastic scattering length which is independent of ρ_e , just as we have observed in Fig. 5.
- ³³The data from Ref. 11 for AuPd appears to be well off the common curve in Fig. 6. The reason for this is not clear, but it is possible that the value for ρ_e in this case is not reliable. Other workers (Refs. 9 and 22) have prepared AuPd films in a similar manner, and have found much larger values of ρ_e . It is thus possible that the value of ρ_e quoted in Ref. 11 is in error, and hence that the result for AuPd cannot be accurately placed in Fig. 6. A specific problem which may have arisen

in the preparation of the AuPd in Ref. 11 is that the two metals were simply coevaporated from the same boat. Since they have different melting temperatures, this procedure may not have yielded a homogeneous film.

³⁴Indeed, in their analysis White *et al.* (Ref. 11) attempted to estimate these parameters from other measurements.

³⁵P. Santhanam, S. Wind, and D. E. Prober, *Phys. Rev. Lett.* **53**, 1179 (1984); in *Proceedings of the 17th International Conference on Low Temperature Physics*, edited by U. Eckern, A. Schmid, W. Weber, and H. Wuhl (North-Holland, New York, 1984), p. 495; D. E. Prober, S. Wind, and P. Santhanam, in *Proceedings of the International Conference*

on Localization, Interactions, and Transport in Impure Metals, Braunschweig, 1984 (unpublished).

³⁶See for example, G. Bergmann, *Phys. Rep.* **107**, 1 (1984).

³⁷In discussing dirty-limit electron-electron scattering one must be careful to recall that the temperature dependence of this scattering time depends on dimensionality. Given the size of our wires, one would expect this process to be one dimensional. If so, then this scattering time should depend on the cross-sectional area of the sample, and thus yield a prediction for ΔR [see (4)] which has a stronger area dependence than observed experimentally.