Spin-polarization instability in a tilted magnetic field of a two-dimensional electron gas with filled Landau levels

Gabriele F. Giuliani
Department of Physics, Purdue University, West Lafayette, Indiana 47907

J. J. Quinn
Department of Physics, Brown University, Providence, Rhode Island 02912
(Received 14 December 1984)

We have investigated the stability of a two-dimensional electron gas with two filled Landau levels (of opposite spin) in the high-field limit. The Zeeman energy can be increased by adding a component of the magnetic field parallel to the surface. The lowest-lying excitations can be described in terms of singlet and triplet excitons. Taking interaction effects into account, we have found that at a critical Zeeman energy (smaller than the cyclotron energy), there is a first-order transition to a fully spin-polarized state in which two Landau levels of equal spin are filled. Exotic intermediate states of the spin-density-wave type have been found not to occur in the simple case of nondegenerate bands.

INTRODUCTION

In the presence of a sufficiently large magnetic field the carriers of a two-dimensional electron gas will populate only the lowest Landau, lowest spin level. The occurrence of the fractional quantum Hall effect[1] when this level is partially occupied has led to a great deal of interest in the effect of electron-electron interactions[2,3] on the properties of the system. The fractional quantum Hall regime is particularly challenging because all of the single-particle states are degenerate in the absence of electron-electron interactions. If the cyclotron energy $\hbar\omega_c$ is sufficiently large, then the Coulomb energy e^2/l, where $l=(\hbar c/eB)^{1/2}$ is the magnetic length, is the only relevant energy scale in the problem. Because of this, no small parameter exists with which one can construct a perturbation expansion.

In the situation where Landau levels are either completely filled or completely empty, the situation can be considerably simpler. The low-lying excitations consist of electron-hole (e-h) pairs, and there are two types of e-h pair excitations. The singlet e-h pair occurs when a carrier is promoted to a higher Landau level without a change in spin; the triplet excitation involves the promotion of a carrier to the opposite spin state of either the same Landau level or a higher one. Because the excited electron and the hole left behind in the lower energy level interact with one another, they can form a bound state (exciton). The binding energy of singlet and triplet excitons in two-dimensional systems in a strong magnetic field has been studied by a number of authors.[4] The ratio of the Coulomb energy to the cyclotron energy can act as the small parameter for a perturbation expansion.

The present work is motivated by the consideration of systems in which the spin splitting is of the same order of magnitude as the cyclotron splitting. Then if both spin states of a given Landau level are fully occupied and the next Landau level is empty, the lowest energy excitation results from promoting an electron from the occupied upper spin state of the filled Landau level to the lower spin state of the next Landau level. In the absence of electron-electron interactions the energy of such an excitation would be $\epsilon\equiv\hbar(\omega_c-\omega_s)$, where ω_s is the spin-resonance frequency. Because the cyclotron frequency ω_c depends on the component of magnetic field normal to the surface, while the spin-resonance frequency depends on the magnitude of $\mathbf{B}$, this excitation can be made arbitrarily small (in fact, it can be negative in which case the lower spin states of both Landau levels are occupied while the upper spin states are empty). When electron-hole interactions are included, the triplet exciton binding energy can exceed ϵ and one might expect a spin-density-wave (SDW) instability.[5] What actually occurs is that before ϵ becomes smaller than the triplet exciton binding energy, the electron-electron interactions cause a paramagnetic to ferromagnetic phase transition.[6]

EXCITONS

Before proceeding to investigate the phase transition, it is worth reviewing the evaluation of the singlet and triplet excitons.

The energy of the excitonic excitations of a two-dimensional electron gas in a strong perpendicular magnetic field can be evaluated in closed form making use of the formalism developed in Ref. 3 in connection with the fractional quantum Hall-effect problem.

The Hamiltonian for a two-dimensional electron gas in the presence of a dc magnetic field $\mathbf{B}$ can be written as $H=H_0+H_I$, where

$$H_0=\sum_{n,k,\sigma}[\hbar\omega_c(n+\tfrac{1}{2})+\sigma\epsilon_Z]c^\dagger_{n,k,\sigma}c_{n,k,\sigma}\ , \tag{1}$$

and

$$H_I=\sum_{\substack{n,n',m,m',\\ k,k',q,\\ \sigma,\sigma'}} V_{nm,n'm'}(k',k;q) \times c^\dagger_{n,k+q,\sigma}c^\dagger_{n',k'-q,\sigma'}c_{m',k',\sigma'}c_{m,k,\sigma} \ . \tag{2}$$

In these equations σ takes on the values ± 1, and ϵ_Z is the Zeeman energy ($2\epsilon_Z=\hbar\omega_s$). The operator $c^\dagger_{n,k,\sigma}$ creates an electron of spin $\sigma=\frac{1}{2}$ in the single-particle state,

$$\psi_{n,k}(r)=L^{-1/2}e^{iky}u_n(x+l^2k) \ . \tag{3}$$

Here L is the length of the sample, $k=(2\pi/L)$ times an integer, $u_n(x)$ is the nth eigenfunction of the simple harmonic oscillator, and $l=(\hbar c/eB_z)^{1/2}$ is the magnetic length. For any given Landau-level index n there are $N_L=L^2/2\pi l^2$ such states labeled by the wave vector k. These N_L states are degenerate solutions of the noninteracting problem. The matrix element $V_{nm,n'm'}(k',k;q)$ is given by

$$V_{nm,n'm'}(k',k;q)=\int d^2\mathbf{r}\,d^2\mathbf{r}'\psi^*_{n,k+q}(r)\psi^*_{n',k'-q}(r')V(|\mathbf{r}-\mathbf{r}'|)\psi_{m',k'}(r')\psi_{m,k}(r) \ , \tag{4}$$

where $V(|\mathbf{r}-\mathbf{r}'|)$ is the electron-electron interaction. The sum appearing in Eq. (2) is over all values of n, n', m, m', k, k', q, σ, and σ'. In writing down these equations we have taken the two-dimensional electron gas to lie in the plane $z=0$, and have used the Landau gauge $\mathbf{A}=(0,B_zx,0)$ for the vector potential causing the normal component of the magnetic field.

For the sake of simplicity we concentrate on the situation in which the filling factor ν, defined as the ratio of the number of electrons N to the Landau-level degeneracy N_L, is equal to two, so that the two spin states of the $n=0$ Landau level are the only occupied states. This situation is sketched in Fig. 1. The elementary excitations which we consider are the singlet exciton and the triplet exciton. The former is generated by promoting an electron from the filled $n=0$ Landau level to the same spin state of the $n=1$ Landau level and then "turning on" the many-particle interactions. The latter is generated when the spin of the electron is flipped in the process of promotion to the next Landau level.

Because we consider the cyclotron energy $\hbar\omega_c$ to be much larger than the Coulomb energy e^2l^{-1}, a simple perturbation theory can be constructed in powers of $e^2l^{-1}/\hbar\omega_c$. To first order in this parameter one need only consider intermediate states containing a single exciton. Then, the exciton energy consists of three parts:[4] (i) the "kinetic" energy, i.e., the excitation energy in the absence of electron-electron interactions, (ii) the exchange energy of the particle and hole, and (iii) the electron-hole binding energy. For the singlet exciton the kinetic energy is $\hbar\omega_c$, while for the triplet it is $\hbar(\omega_c\mp\omega_s)$.

The exchange-matrix element E_x^{nm} of an electron in the nth Landau level interacting with an electron of the same spin in the mth Landau level is $-v_{nm}(lq,0)$ where

$$v_{nm}(lq,l(p'-q-q))=V_{nn,mm}(p',p;q) \ .$$

Useful expressions are

$$v_{00}(x,y)=\frac{e^2}{L}\int_{-\infty}^{\infty}dz(z^2+x^2)^{-1/2}\exp[-\tfrac{1}{2}(z^2-2izy+x^2)] \ , \tag{5}$$

$$v_{10}(x,y)=\frac{e^2}{L}\int_{-\infty}^{\infty}dz(z^2+x^2)^{-1/2}\left[1-\frac{x^2}{2}-\frac{z^2}{2}\right]\exp-[\tfrac{1}{2}(z^2-2izy+x^2)] \ , \tag{6}$$

$$v_{11}(x,y)=\frac{e^2}{L}\int_{-\infty}^{\infty}dz(z^2+x^2)^{-1/2}\left[1-\frac{x^2}{2}-\frac{z^2}{2}\right]^2\exp[-\tfrac{1}{2}(z^2-2izy+x^2)] \ . \tag{7}$$

In particular if both electrons are in the $n=0$ level we have

$$E_x^{00}(q)=-v_{00}(lq,0) =\frac{-e^2}{L}e^{-(ql/2)^2}K_0(q^2l^2/2) \ , \tag{8}$$

where K_0 is a modified Bessel function.[7] If the two electrons are one in the $n=0$ and one in the $n=1$ level the result is

$$E_x^{01}(q)=-v_{01}(lq,0) =\frac{-2\sqrt{\pi}e^2}{L}e^{-(ql/2)^2}W_{1/2,1/2}(q^2l^2/2) \ , \tag{9}$$

where $W_{1/2,1/2}$ is a Whittaker function.[7] The exchange energy of an electron in the nth Landau level interacting with electrons of the same spin in the filled mth Landau level is

$$\epsilon_x^{nm}=-\sum_p v_{nm}(p,0) \ . \tag{10}$$

The exchange energy ϵ_x^{00} is equal to $(\pi/2)^{1/2}e^2l^{-1}$, and $\epsilon_x^{10}=\frac{1}{2}\epsilon_x^{00}=\frac{2}{3}\epsilon_x^{11}$. Promoting an electron from the full $n=0$ to the otherwise empty $n=1$ Landau level gives rise to a net change $\epsilon_x^{00}-\epsilon_x^{01}=\frac{1}{2}\epsilon_x^{00}$ in exchange energy.

The electron-hole binding energy results from solving the following integral equation for the electron-hole vertex function $\Gamma_{n\sigma,m\sigma'}(p'-p,q,\omega)$ in terms of the irreducible vertex function $\gamma_{n\sigma,m\sigma'}(p'-p,q)$, and the Green's functions $G^{e,h}_{n,\sigma}(p,\omega)$,

$$\Gamma(p'-p,q,\omega)=\gamma(p'-p,q)+i\sum_{p'',\omega''}\gamma(p''-p,q)\Gamma(p''-p',q,\omega)G^h(p'',\omega'')G^e(p''+q,\omega+\omega'') . \quad (11)$$

This integral equation is shown schematically in Fig. 2(a). The corresponding expansion for $\gamma_{n\sigma,m\sigma'}$ is graphically represented in Fig. 2(b). In Eq. (11) appropriate Landau-level and spin indices are understood. It is clear that within the Landau gauge the various contributions to $\gamma_{n\sigma,m\sigma'}$ are given by the matrix elements $v_{nm}(lp,lq)$ introduced above.

The fact that we are restricting the calculation to the single-exciton approximation results in considerable simplification. For an electron and hole of opposite sign Eq. (11) generates only the ladder graphs because the unscreened interaction is instantaneous. For an electron and a hole of the same spin the first diagram of Fig. 2(b) corresponds to the RPA for $\gamma_{n\sigma,m\sigma'}(p'-p,q)$. For the case of the triplet exciton the RPA diagram does not occur and $\gamma(p'-p,q)$ is in the present case equal to $v_{10}(p'-p,q)$. The single-particle Green's functions are given by

$$G_h^{e,h}(p,\omega)=(\omega-\epsilon_{n,\sigma}^{e,h}\pm i\delta)^{-1} , \quad (12)$$

where $\epsilon_{n,\sigma}^{e,h}$ is the spin-dependent Hartree-Fock single-particle or single-hole energy in the nth Landau level including the exchange contribution and the $\pm i\delta$ refers to electron or hole states. The solution to Eq. (11) can readily be obtained by introducing $\widetilde{\Gamma}(k,q,\omega)$ the Fourier transform of $\Gamma(p'-p,q,\omega)$ with respect to the variable $p'-p$. The poles of $\widetilde{\Gamma}(k,q,\omega)$ are the exciton energies.[3] It is easily found that if the electron and the hole are, respectively, in the nth and mth Landau level the resulting exciton energy can be simply written as

$$E_{t,s}^{nm}(R^2)=\Delta_{n\sigma,m\sigma'}-\widetilde{\gamma}_{n\sigma,m\sigma}(R^2) , \quad (13)$$

where $\Delta_{n\sigma,m\sigma'}=\epsilon_{n,\sigma}^{e}-\epsilon_{m,\sigma'}^{h}$ is the energy of the noninteracting electron-hole pair, and $\widetilde{\gamma}_{n\sigma,m\sigma'}$ is the Fourier transform with respect to the variable $p'-p$ of the appropriate irreducible vertex function. The labels t and s stand for triplet and singlet. In Eq. (13) we have made explicit that $\widetilde{\gamma}_{n\sigma,m\sigma'}$ (and therefore $E_{t,s}^{nm}$) is solely a function of $R^2=l^2(k^2+q^2)$, a quantity that can be interpreted in terms of the exciton size.[3,6] It is interesting to notice that this property is an explicit proof of the independence of our analysis upon the particular choice of the gauge [Eq. (3)].

The exciton energies of Eq. (13) can be readily evaluated in closed form and the results agree with those given by various authors.[4]

SDW VERSUS FERROMAGNETIC INSTABILITY

The case of particular interest in this work is that of the triplet exciton whose "kinetic" energy is $\epsilon\equiv\hbar(\omega_c-\omega_s)$. We find that the energy of this exciton as a function of R^2 is given by[6]

$$E_t^{01}(R^2)=\epsilon+[\tfrac{1}{2}-\mu(R^2)]\epsilon_x^{00} , \quad (14)$$

with

$$\mu(x)=\tfrac{1}{2}e^{-x}[(1+2x)I_0(x)-2xI_1(x)] . \quad (15)$$

In this equation $I_n(x)$ is the modified Bessel function of order n. It is clear that $E_t^{01}(R^2)$ becomes negative when $\epsilon/\epsilon_x^{00}<\mu(R^2)-\frac{1}{2}$. The maximum value of $\mu(x)$ is $\mu_{\max}\simeq 0.573$, so this corresponds to a positive value of $\epsilon\equiv\hbar(\omega_c-\omega_s)$. If this inequality is satisfied, the binding energy of the triplet exciton is larger than the sum of the "kinetic" and exchange energies, and an instability must occur.

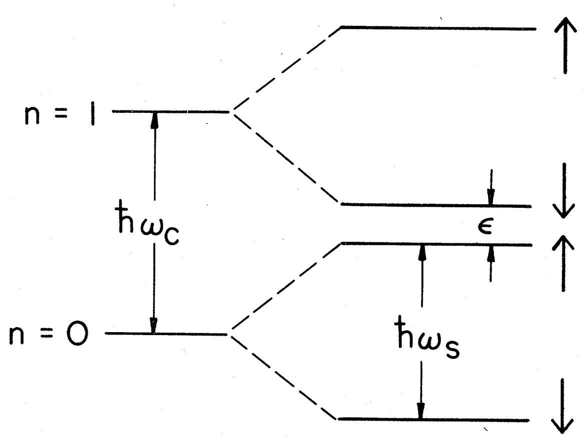

FIG. 1. Schematic of the energy levels: $\hbar\omega_c$ is the Landau-level separation, whereas $\hbar\omega_s$ is the spin splitting. Here $\hbar\omega_c$ and $\hbar\omega_s$ are comparable in magnitude.

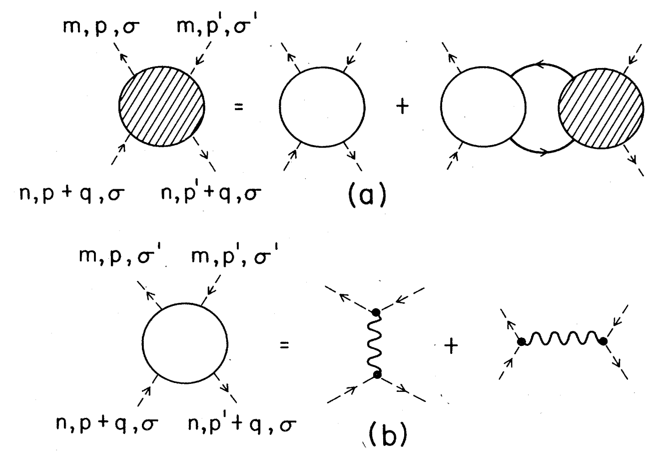

FIG. 2. (a) Diagrammatic representation of the Bethe-Salpeter equation for the vertex function of interacting electron-hole pairs of a two-dimensional electron gas in a magnetic field. Here we use the asymmetric Landau gauge representation in which the noninteracting electronic states are labeled by means of an integer Landau index n, one of the components of the wave vector, and the spin projection. (b) Perturbative contributions to the electron-hole irreducible vertex function to be used in the Bethe-Salpeter equation. The second term is absent in the triplet case.

At first glance one might expect a spin-density-wave instability[5] with the value of the SDW wave vector Q_{SDW} determined by the location of the maximum of $\mu(x)$, i.e., $Q_{\mathrm{SDW}} \simeq 1.2 l^{-1}$. In order to investigate the behavior of the system for values of $e^2 l^{-1}$ large enough to cause this instability, we introduce new operators which are linear combinations of $c_{nk\sigma}$ for $|nk\sigma\rangle$ equal to $|0,k,\downarrow\rangle$ and $|l,k+Q,\downarrow\rangle$. Because these are the only two Landau levels which are modified in the new ground state, we make the following simplification in notation: c_k stands for $c_{0k\uparrow}$ and a_k stands for $c_{1k\downarrow}$. The Hamiltonian can be written $H = H_0 + V$ where

$$H_0 = N_L(\tfrac{1}{2}\hbar\omega_c - \epsilon_Z) + \sum_k [(\tfrac{1}{2}\hbar\omega_c + \epsilon_Z)c_k^\dagger c_k + (\tfrac{3}{2}\hbar\omega_c - \epsilon_Z)a_k^\dagger a_k] \tag{16}$$

and

$$V = -\tfrac{1}{2}N_L\epsilon_x^{00} - \epsilon_x^{10}\sum_k a_k^\dagger a_k + \tfrac{1}{2}\sum_{pp'q}\{v_{00}(q,p'-p-q)c_{p+q}^\dagger c_{p'-q}^\dagger c_{p'}c_p + v_{11}(q,p'-p-q)a_{p+q}^\dagger a_{p'-q}^\dagger a_{p'}a_p + v_{10}(q,p'-p-q)[c_{p+q}^\dagger a_{p'-q}^\dagger a_{p'}c_p + a_{p+q}^\dagger c_{p'q}^\dagger c_{p'}a_p]\} . \tag{17}$$

The three terms in H_0 correspond to the kinetic energies of the $0\downarrow$ Landau level, the $0\uparrow$, and the $1\downarrow$ levels. The potential energy has five terms: The first is the exchange energy of the electrons in the $0\downarrow$ level interacting among themselves. The second is the exchange energy due to the particles in the $1\downarrow$ level interacting with those in the $0\downarrow$ level. The final three terms are the interactions of the $0\uparrow$ particles among themselves, the $1\downarrow$ particles among themselves, and finally the interaction of the $0\uparrow$ and the $1\downarrow$ particles with one another. In writing down this approximation we assume that the $0\downarrow$ level is full (contains N_L electron) and always remains full. The $1\uparrow$ level and all higher levels are empty and always remain empty. Only the $0\uparrow$ and $1\downarrow$ levels enter the dynamics.

We make a Bogoliubov-Valatin transformation to new operators[8] α_p and β_k defined by

$$c_p = \cos\theta_p\alpha_p + \sin\theta_p\beta_p , \tag{18}$$

$$a_{p+Q} = -\sin\theta_p\alpha_p + \cos\theta_p\beta_p . \tag{19}$$

We express the Hamiltonian in terms of the operators α_p and β_p and their Hermitian conjugates.[9] We then apply the Hartree-Fock approximation assuming that the linear combination corresponding to the state α_p is the lower energy state and therefore the occupied state. That is, we assume that $\langle\alpha_p^\dagger\alpha_p\rangle = N_p$ is finite while $\langle\beta_p^\dagger\beta_p\rangle = 0$ where the angular brackets denote ground-state expectation value. After assuming that $v_{nm}(0,p'-p)=0$ due to charge neutrality we find that

$$\langle H\rangle_{\mathrm{HF}} = N_L(\tfrac{1}{2}\hbar\omega_c - \epsilon_Z - \tfrac{1}{2}\epsilon_x^{00}) + (\tfrac{1}{2}\hbar\omega_c + \epsilon_Z)\sum_k \cos^2\theta_k + (\tfrac{3}{2}\hbar\omega_c - \epsilon_Z - \epsilon_x^{10})\sum_k \sin^2\theta_k - \tfrac{1}{4}\sum_{k,k'} v_{10}(k'-k,Q)\sin 2\theta_k \sin(2\theta_{k'}) - \tfrac{1}{2}\sum_{k,k'} v_{00}(k'-k,0)\cos^2\theta_k\cos^2\theta_{k'} - \tfrac{1}{2}\sum_{k,k'} v_{11}(k'-k,0)\sin^2\theta_k\sin^2\theta_{k'} . \tag{20}$$

Because all the single-particle states are degenerate in the absence of electron-electron interactions, we expect that with periodic boundary conditions $\cos\theta_k$ must be independent of k. In that case Eq. (20) simplifies to

$$N_L^{-1}\langle H\rangle_{\mathrm{HF}} = \tfrac{1}{2}\hbar\omega_c - \epsilon_Z - \tfrac{1}{2}\epsilon_x^{00} + (\tfrac{1}{2}\hbar\omega_c + \epsilon_z)\cos^2\theta + (\tfrac{3}{2}\hbar\omega_c - \epsilon_Z - \epsilon_x^{10})\sin^2\theta - \tfrac{1}{4}\epsilon_x^{00}\mu(l^2Q^2)\sin^2(2\theta) - \tfrac{1}{2}\epsilon_x^{00}\cos^4\theta - \tfrac{3}{8}\epsilon_x^{00}\sin^4\theta . \tag{21}$$

The extreme of $\langle H\rangle_{\mathrm{HF}}$ as a function of θ must satisfy the equation

$$\sin(2\theta)(a - b\sin^2\theta) = 0 , \tag{22}$$

where

$$a = \epsilon + \epsilon_x^{00}[\tfrac{1}{2} - \mu(l^2Q^2)] \tag{23}$$

and

$$b = 2\epsilon_x^{00}[\tfrac{7}{8} - \mu(l^2Q^2)] . \tag{24}$$

There are three possible solutions to Eq. (22): $\theta=0$, $\theta=\pi/2$, and $\theta=\theta^*$ where $\sin^2\theta^* = a/b$. The solution $\theta=0$ corresponds to $\alpha_k = c_k$ and gives the paramagnetic state. This state is a stable Hartree-Fock solution $(\partial^2\langle H\rangle_{\mathrm{HF}}/\partial^2\theta > 0)$ if $\epsilon/\epsilon_x^{00} > \mu_{\max} - \tfrac{1}{2} \approx 0.073$. This is exactly the condition we found for the triplet exciton instability, so that our starting paramagnetic state is a stable Hartree-Fock solution when the triplet exciton energy $E_t^{01}(R^2)$ is positive.

The solution $\theta=\pi/2$ corresponds to $\alpha_p = a_{p+Q}$ giving a ferromagnetic ground state (i.e., the $n=0$ and $n=1$ spin-down states are both occupied while the $0\uparrow$ state is empty). This extremum corresponds to a stable Hartree-Fock solution if $\epsilon/\epsilon_x^{00} < \tfrac{5}{4} - \mu_{\max} \sim 0.667$. The energy per particle (remember we have $2N_L$ particles) is

$$E_{\text{para}}=\tfrac{1}{2}(\hbar\omega_c-\epsilon_x^{00})\ , \tag{25}$$

$$E_{\text{ferro}}=\hbar\omega_c-\epsilon_Z-\tfrac{11}{16}\epsilon_x^{00}\ . \tag{26}$$

These two energies are equal at

$$\epsilon/\epsilon_x^{00}\equiv(\hbar\omega_c-2\epsilon_Z)/\epsilon_x^{00}=\tfrac{3}{8}\ .$$

What about the solution $\theta=\theta^*$ which corresponds to a spin-density-wave state? This solution occurs when $|a|<|b|$ and a and b have the same sign. These conditions are satisfied if $\mu_{\max}-\frac{1}{2}<\epsilon/\epsilon_x^{00}<\frac{5}{4}-\mu_{\max}$, the region where both the paramagnetic and ferromagnetic solutions are minima as functions of θ. This means that θ^* is always a maximum energy solution and hence unstable.

DISCUSSION

As shown in the previous section, in this simple situation the SDW state we expected never occurs.[6,10] When the energy of the triplet exciton of the paramagnetic state vanishes, the paramagnetic state becomes unstable. However, before that occurs a paramagnetic to ferromagnetic phase transition will preempt such an excitonic instability.[11] It is apparent that if we had started with the stable ferromagnetic state and calculated the energy of the "triplet" exciton resulting from promoting a $1\downarrow$ electron to an unoccupied $0\uparrow$ state, we would find that the energy of the exciton vanished when $\epsilon/\epsilon_x^{00}>\frac{5}{4}-\mu_{\max}$. This would signal the instability of the ferromagnetic state.

The paramagnetic to ferromagnetic transition occurs at $\epsilon/\epsilon_x^{00}=\frac{3}{8}$. This can be seen simply by writing the total energy. For the paramagnetic state a $0\downarrow$ particle has energy $\frac{1}{2}\hbar\omega_c-\epsilon_Z-\epsilon_x^{00}$ while $0\uparrow$ particle has energy $\frac{1}{2}\hbar\omega_c+\epsilon_Z-\epsilon_x^{00}$. The total Hartree-Fock energy is the sum of the "kinetic" and half the exchange energies of the individual particles

$$E_P=N_L\left[\frac{\hbar\omega_c}{2}+\epsilon_Z-\tfrac{1}{2}\epsilon_x^{00}\right]+N_L\left[\frac{\hbar\omega_c}{2}-\epsilon_Z-\tfrac{1}{2}\epsilon_x^{00}\right],$$

so that $E_P=N_L(\hbar\omega_c-\epsilon_x^{00})$. For the ferromagnetic state a $0\downarrow$ particle has energy $\frac{1}{2}\hbar\omega_c-\epsilon_Z-\epsilon_x^{00}-\epsilon_x^{01}$, while a $1\downarrow$ particle has energy $\frac{3}{2}\hbar\omega_c-\epsilon_Z-\epsilon_x^{11}-\epsilon_x^{01}$. Adding the kinetic and half the exchange energies gives

$$E_F=N_L(2\hbar\omega_c-2\epsilon_Z-\tfrac{1}{2}\epsilon_x^{00}-\tfrac{1}{2}\epsilon_x^{11}-\epsilon_x^{01})$$
$$=N_L(2\hbar\omega_c-2\epsilon_Z-\tfrac{11}{16}\epsilon_x^{00})\ .$$

By equating these we see that $E_F=E_P$ at $\epsilon/\epsilon_x^{00}=\frac{3}{8}$, just as we showed after Eq. (26).

It might be possible to observe the transition discussed in this paper by measuring the magnetic susceptibility in a field whose z component is held fixed (to keep filling factor $\nu=2$) and whose component parallel to the surface is varied. The de Haas–van Alphen effect has recently been studied in two-dimensional systems,[12] so that the magnetization itself is large enough to be detected. Structure in cyclotron and spin resonance should reflect the singlet and triplet exciton spectrum through the memory function[13] or its spin equivalent. Because the exciton energies are different for the paramagnetic and ferromagnetic states, the phase transition might also be observed by this technique.

ACKNOWLEDGMENTS

This work was supported in part by the National Science Foundation Grant No. DMR-81-2069. J.J.Q. would like to acknowledge the hospitality of colleagues at the IBM Thomas J. Watson Research Center, where some of the initial work on the problem was started. G.F.G. wishes to acknowledge the hospitality of the Aspen Center for Physics where part of this work was carried out.

[1] D. C. Tsui, H. L. Stormer, and A. C. Gossard, Phys. Rev. Lett. **48**, 1559 (1982).

[2] R. B. Laughlin, Phys. Rev. Lett. **50**, 1395 (1983); Surf. Sci. **142**, 163 (1984); R. Tao and D. J. Thouless, Phys. Rev. B **28**, 1142 (1983); B. I. Halperin, Helv. Phys. Acta **56**, 75 (1983); Phys. Rev. Lett. **52**, 2390 (1984); F. D. M. Haldane, *ibid.* **51**, 605 (1983); J. J. Quinn, Bull. Am. Phys. Soc. **29**, 428 (1984); G. F. Giuliani and J. J. Quinn, *ibid.* **29**, 428 (1984).

[3] G. F. Giuliani and J. J. Quinn, Phys. Rev. B **31**, 3451 (1985).

[4] Yu. A. Bychkov, S. V. Iordanskii, and G. M. Eliashberg, Pis'ma Zh. Eksp. Teor. Fiz. **33**, 152 (1981) [Sov. Phys.—JETP Lett. **33**, 143 (1981)], and references quoted therein; C. Kallin and B. I. Halperin, Phys. Rev. B **30**, 5655 (1984); G. F. Giuliani and J. J. Quinn (unpublished).

[5] A. W. Overhauser, Phys. Rev. Lett. **4**, 462 (1960); Phys. Rev. **128**, 1437 (1962).

[6] G. F. Giuliani and J. J. Quinn, Solid State Commun. (to be published).

[7] I. S. Gradshtein and I. M. Ryzhik, *Tables of Integrals, Series and Products* (Academic, London, 1981).

[8] N. N. Bogoliubov, J. Exptl. Theoret. Phys. (U.S.S.R.) **34**, 58 (1958) [Sov. Phys.—JETP **7**, 41 (1958)]; J. G. Valatin, Nuovo Cimento **7**, 843 (1958).

[9] The most appropriate transformation is in this case of the excitonic type, in which the "new" operators are a linear superposition of the "old" ones which spans the entire range of the wave-vector label. It turns out, however, that in a *translationally invariant* system the simple transformation of Eqs. (18) and (19) is sufficient to describe the phenomenon.

[10] It should be noticed that in the case of silicon the present analysis does not apply in a straightforward fashion as the valley degeneracy, not accounted for in our analysis, might play a relevant role. A study of this specific problem is currently being carried out.

[11] See, for instance, the review by B. I. Halperin and T. M. Rice, in *Solid State Physics*, edited by H. Ehrenreich, F. Seitz, and D. Turnbull (Academic, New York, 1968), Vol. 21, p. 115.

[12] F. F. Fang and P. J. Stiles, Phys. Rev. **28**, 6962 (1983); Surf. Sci. **142**, 290 (1984); T. Haavasoia, H. L. Stormer, D. J. Bishop, V. Narayanamurti, A. C. Gossard, and W. Weigmann, *ibid.* **142**, 294 (1984).

[13] C. S. Ting, S. C. Ying, and J. J. Quinn, Phys. Rev. Lett. **37**, 215 (1976); Phys. Rev. B **16**, 5394 (1977); C. Kallin and B. I. Halperin, Phys. Rev. B **31**, 3635 (1985).