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Nonequilibrium superconductivity based on quasithermal phonon and quasiparticle distributions

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A new phenomenological model describing nonequilibrium superconductivity is proposed which is characterized by the introduction of a quasithermal distribution of phonons with energies of 0–∞. The model exhibits μ^*-model-like and T^*-model-like Δ vs n and μ^* vs n characteristics when the excess quasiparticle density n is low and high, respectively. The estimated critical density n_c for the onset of instability is much lower than that of the μ^* model. These results are in fairly good agreement with the experimental results reported by Willemsen and Gray, and Akoh and Kajimura.

Nonequilibrium superconductivity which develops under the intense injection of quasiparticles is of great interest in physics engineering. Many papers have been published dealing with the quasiparticle lifetime, instability near the multiple-gap state, and the proposal of functional superconductive devices.[1–5] As for theoretical works, many physical models have been proposed which describe the observed nonequilibrium phenomena.[6–9] However, no one has given a satisfactory explanation for the experimental results. The subject which has attracted the most attention is solving the instability appearing in the nonequilibrium superconductivity. In a recent paper, Iguchi and Konno proposed a new set of rate equations to describe more adequately the nonequilibrium quasiparticles and phonons. These were characterized by the addition of extra phonon generation via a relaxation of hot quasiparticles.[10] Their model was later pointed out by Chang and Chi to not give a stable inhomogeneous nonequilibrium state for quasiparticle injection above a critical value.[11]

An attempt is made in the present work to present a new phenomenological model for a more reasonable description of nonequilibrium superconductors. A new feature of this model is the introduction of a quasithermal distribution for phonons with energies of 0–∞. This can be compared to that of the conventional treatment where phonons with energies of 0–2Δ are completely ignored. Here, Δ is a superconductive gap parameter. Our present view is strongly supported by Chang's calculation[9] revealing that a large amount of relaxation phonons are created in the energy range of 0–2Δ during the injection of quasiparticles even when the injected quasiparticle energy is not equal to nor higher than 4Δ. Furthermore, it might be possible that the nonequilibrium phonon distribution tends to relax to the quasithermal equilibrium one via intense internal thermalization of the phonon system. Though we do not have conclusive evidence, these two facts suggest that it is much more reasonable to approximate the real phonon distribution by a quasithermal distribution over the energy range 0–∞, rather than one over the energy range 2Δ–∞ as in the conventional treatment. A further advantage of the present treatment is that one can more reasonably define the effective lattice temperature $T_{\rm eff}$ of the nonequilibrium superconductors than with the T^* model,[8] and simultaneously one can express the nonequilibrium quasiparticle distribution as

$$f(E)=\{1+\exp[(E-\mu_{\rm eff})/kT_{\rm eff}]\}^{-1} \ ,$$

where $\mu_{\rm eff}$ is an effective chemical potential.[7,12]

A set of rate equations for the quasiparticles and phonons is given by

$$\frac{dN_q}{dt}=I_0-RN_q^2+\frac{2}{\tau_B}N_\omega \ , \tag{1}$$

$$\frac{dN_\omega}{dt}=\frac{R}{2}N_q^2-\frac{N_\omega}{\tau_B}-\frac{N_{\omega 0}-N_{\omega To}}{\tau_{es}} \ , \tag{2}$$

where

$$N_q=4N(0)\int_\Delta^\infty f(E)\rho(E)\,dE \ , \tag{3}$$

$$N_\omega=AT_{\rm eff}^3\int_{X_G}^\infty \frac{x^2}{e^x-1}dx, \quad X_G=\frac{2\Delta}{kT_{\rm eff}} \ , \tag{4}$$

$$N_{\omega 0}=AT_{\rm eff}^3\int_0^\infty \frac{x^2}{e^x-1}dx \ , \tag{5}$$

$$N_{\omega T0}=AT^3\int_0^\infty \frac{x^2}{e^x-1}dx \ . \tag{6}$$

N_q and N_ω are, respectively, the quasiparticle number density and the phonon density with $\hbar\omega \geq 2\Delta$. $N_{\omega 0}$ and $N_{\omega T0}$ denote the total density of phonons in nonequilibrium and equilibrium states, respectively. The notation other than that above is the same as used in Refs. 6–8. Here, we concentrate our attention on Eq. (2). Time variation of the phonon number density N_ω ($\hbar\omega \geq 2\Delta$) is, as is well known, due partly to quasiparticle recombination and also partly to pair-particle breaking. For phonon escape from the nonequilibrium superconductor to a cold bath, we introduced the third term of Eq. (2) where the excess phonons are redistributed obeying a quasithermal distribution function with $T_{\rm eff}$. It should be noted that the phonon redistribution is performed on the basis of phonon number conservation, for simplicity, as Parker[8] did for an estimation of the effective temperature T^*. Moreover, in the present treatment, the excess phonons are alloted to the energy range from 0–∞ to build up the quasithermal distribution, instead of the partial energy range 2Δ–∞ as in Parker's treatment. Since the phonon distribution is peaked at rather low ener-

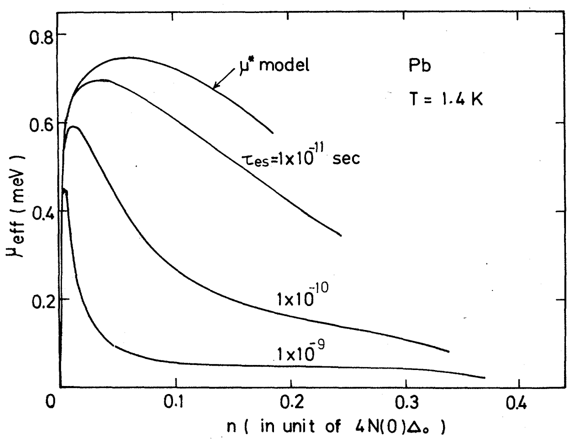

FIG. 1. Effective chemical potential μ_{eff} as a function of the excess quasiparticle density n. Solid curves indicate the superconductive state. Beyond the end point of the curve, the system falls into the normal state via a first-order phase transition.

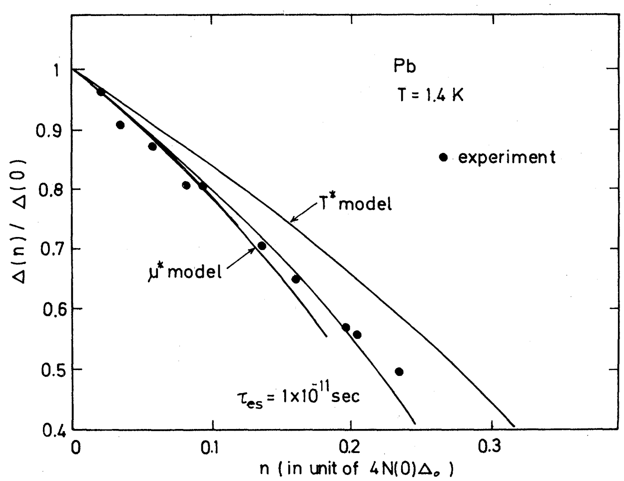

FIG. 3. Gap parameter Δ as a function of n. Curves derived from the μ^* and T^* models are also drawn. The experimental results shown by solid circles are reproduced from Ref. 13.

gy, the excess phonons with $\hbar\omega = 0 - 2\Delta$ make up a significant part of the escape term. This important process was not taken into account in the conventional model.

Combining Eqs. (1)–(6) with the self-consistent BCS equation

$$1 = N(0)V \int_0^{\hbar\omega_D} \frac{d\xi}{E} \tanh\left(\frac{E-\mu_{\text{eff}}}{2kT_{\text{eff}}}\right) , \tag{7}$$

one can easily obtain μ_{eff}, T_{eff}, Δ, and so on in terms of the normalized excess quasiparticle density n. Our calculation was carried out for pure lead at 1.4 K. The pair-breaking time τ_B of 3.4×10^{-11} sec was deduced from Kaplan's expression for the near-equilibrium state.[13]

From comparison of the curves in Figs. 1 and 2, it appears that the nonequilibrium behavior for small n can be almost exactly described by an increase only in μ_{eff} and not in T_{eff}. On the other hand, for n beyond some critical value depending on τ_{es}, a dramatic decrease in μ_{eff} as well as a steep increase in T_{eff} is seen. These results allow us to say that our proposal is a unified model which expresses the so-called μ^* and T^* models for small and large n, respectively. There is some experimental data due to Willemsen and Gray,[12] and Akoh and Kajimura,[14] which compare well to the present calculation. Willemsen and Gray reported a sudden decrease in μ^* and an increase in T^* against an increase in the injector voltage beyond 400 μV. Although their experiments were done on Al film, the qualitative agreement exists between our calculation and the experiments. Akoh and Kajimura found a shift of the Δ vs n curve from that of the μ^* model to the T^* model with increasing n. Their results are reproduced by the solid circles in Fig. 3 together with our calculated results, showing good agreement with the experimental results and the calculation for $\tau_{es} = 1\times10^{-11}$ sec.

Finally, we discuss the critical quasiparticle density n_c at which instability begins to occur. According to Chang and Scalapino,[15] a necessary condition for the occurrence of system instability is that $d^2F_T/dN_q^2 < 0$ where F_T is the system free energy at T_{eff}. In Fig. 4, the result of numerical calcu-

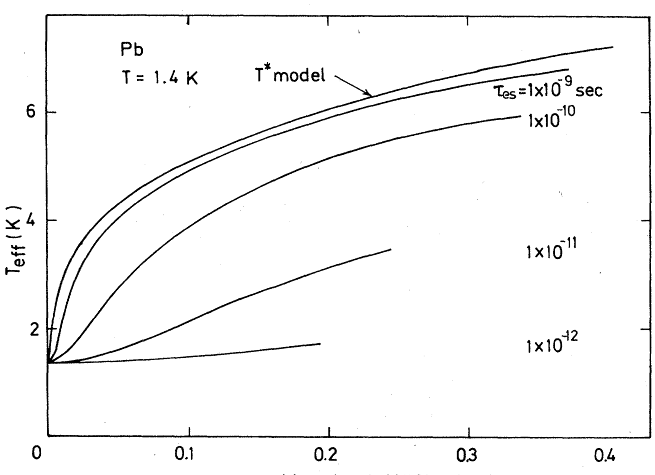

FIG. 2. Effective temperature T_{eff} as a function of n.

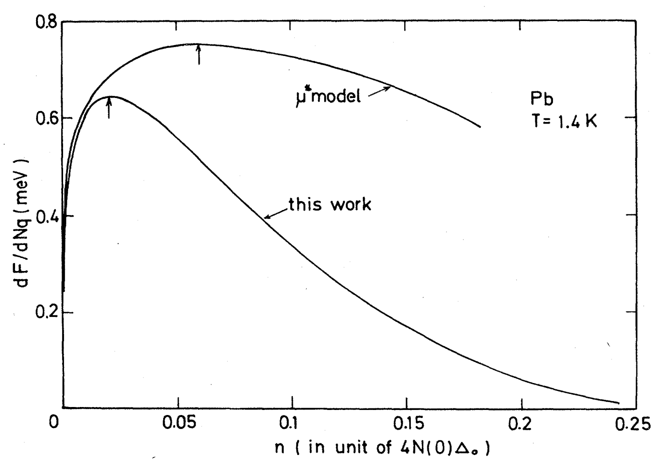

FIG. 4. dF_T/dN_q as a function of n. The calculation was performed for $\tau_{es} = 1\times10^{-11}$ sec. The peak of dF_T/dN_q is indicated by an arrow.

lations for dF_T/dN_q is depicted with a reference curve of the μ^* model. The quasiparticle densities n given by the arrows are the critical values n_c beyond which the superconducting system falls into instability. It is very interesting to see that the n_c estimated from the present model is much smaller than that obtained by the μ^* model. Besides, the value of n_c, 0.021, obtained by the new model is very close to the experimentally determined value (≤ 0.034) by Akoh and Kajimura.[16]

In conclusion, we have proposed a new model describing nonequilibrium superconductivity. The experimentally observed Δ vs n, μ^* vs n, and T^* vs n characteristics, and the critical quasiparticle density n_c for the onset of instability, are to a certain extent, explained semiquantitatively by this model.

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