Brief Reports

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Amplitude modulation of the supercurrent in Josephson junctions

Roman Sobolewski* and Charles V. Stancampiano†
Department of Electrical Engineering, University of Rochester, Rochester, New York 14627
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An analytical solution of the resistively shunted-junction-model equation for the case of a periodic amplitude modulation of the supercurrent is presented. The occurrence of current steps in the junction I-V characteristics at integral multiples of the modulation frequency is predicted. The height of the first current step is calculated and discussed in detail. The result is in very good agreement with the published numerical calculations.

The ac Josephson effect is usually demonstrated by applying, via a microwave field, a rapidly oscillating voltage which modulates the superconducting phase difference and leads to Shapiro steps[1] in the dc I-V characteristics. These steps appear at dc voltages corresponding to integral multiples of the microwave frequency.

It can be shown[2–4] that a similar effect [referred to as amplitude-modulation (AM) steps] should be observed if one modulates the amplitude of the Josephson supercurrent, I_0, rather than the phase. The theoretical feasibility of such an effect has been recently demonstrated by Perrin and Vanneste,[5] where the authors study the behavior of a superconducting film perturbed by a periodic laser drive. Earlier, Vanesste, Gilabert, Sibillot, and Ostrowsky[4] presented a detailed analysis of the experimental conditions under which AM steps could be observed. To achieve modulation of I_0 they selected illumination of the Josephson junction by a modulated laser beam as the most promising technique. However, to the best of our knowledge, no experimental results have yet been published, except a trial experiment performed on an optically induced weak link.[6]

As far as the theory of AM steps is concerned, there exist analog simulations[2] and numerical solutions[4] based on the simple resistively shunted junction (RSJ) model. In both cases a series of current steps in the dc I-V curve corresponding to integral multiples of the modulation frequency ($V = n\hbar\omega_m/2e$) have been predicted. The importance of numerical results is not to be denied, but analytical solutions are also desirable, since they provide a deeper physical insight into the process.

In this paper we present an analytical solution of the resistively shunted-junction-model equation for a junction irradiated by a sinusoidally modulated laser beam, and we discuss the results both in the "low"- and "high"-voltage regimes. We develop a treatment similar to that given by Stancampiano[7] for Josephson oscillators. The approach given in Ref. 7 perfectly fits our problem, because it combines the Josephson current and the rf drive into a single term, an amplitude-modulated version of the Josephson current. On the other hand, we show that, in the case of a weak perturbation, the effect of AM may be expressed by a formula identical to that for rf frequency modulation of the supercurrent.

We assume that the modulation frequency ω_m is smaller than the effective quasiparticle recombination rate $\tau_{\rm eff}^{-1}$. Also, the light intensity is small enough for the linear regime of the Owen and Scalapino[8] or Parker[9] theory to apply. The validity of this approach has been checked numerically by Faris, Chi, Cronemeyer, and Loy,[3] but a detailed discussion can be found in Refs. 4 and 5.

Under the above restrictions, the modulated light causes a similar modulation of the supercurrent. Thus, the supercurrent amplitude can be written in a standard form

$$I_0(t) = I_0(1-a) - I_0 a \sin(\omega_m t) \quad , \tag{1}$$

where a is the effective modulation amplitude. Introducing the modulation depth parameter m, as $m = a/(1-a)$, we obtain

$$I_0(t) = \langle I_0 \rangle [1 - m \sin(\omega_m t)] \quad , \tag{2}$$

where $\langle I_0 \rangle = I_0/(1+m)$ is the time average of the pair current amplitude, and represents the zero-order AM current step. Note that its dependence is completely different from the case of phase modulation. However, our assumption of a weak perturbation leads to $m \ll 1$ and to $\langle I_0 \rangle \approx I_0$. Therefore, for the sake of simplicity, we drop the average ($\langle\ \rangle$) in further considerations.

The RSJ model equation with modulation of the critical current included [Eq. (2)] has the form

$$I_{\rm dc} = I_0[1 - m \sin(\omega_m t)] \sin\phi + (\hbar/2eR_J)(d\phi/dt) \quad , \tag{3}$$

where $I_{\rm dc}$ is the drive current, ϕ is the phase difference across the junction, and R_J represents the shunt resistance.

Introducing the usual normalized units $i_{dc}=I_{dc}/I_0$, $\dot{\phi}=d\phi/dT$, $T=\omega_{cr}t$, $\omega_{cr}=2eI_0R_J/\hbar$, and $\Omega_m=\omega_m/\omega_{cr}$ we can rewrite (3) as

$$i_{dc}=[1-m\sin(\Omega_m T)]\sin\phi+\dot{\phi} \ . \tag{4}$$

For convenience in later calculations, we divide both sides of (4) by the term $(1-m\sin\Omega_m T)$ and take into account that $m \ll 1$, to give

$$i_{dc}[1+m\sin(\Omega_m T)]=\sin\phi+\dot{\phi}[1+m\sin(\Omega_m T)] \ . \tag{5}$$

Observe that the term $m\sin(\Omega_m T)$ which multiplies $\dot{\phi}$ is a purely sinusoidal modulation (it multiplies the voltage), and it cannot lead to any locking phenomenon required to produce a constant-voltage step.[7] Thus, we can ignore this term in further considerations and write Eq. (5) as

$$\dot{\phi}+[\sin\phi-mi_{dc}\sin(\Omega_m T)]=i_{dc} \ . \tag{6}$$

Equation (6) has the same form as Eq. (A4) in Ref. 7. Therefore, the amplitude modulation leads to the same locking effects as the phase modulation, and on the dc I-V curve we expect current steps at voltages corresponding to $n\hbar\omega_m/2e$ $(n=1,2,3,\ldots)$. This result is in exact agreement with the earlier numerical simulations.[2-4]

Now we calculate the amplitude of the first AM step. For this purpose we may follow the calculations presented in Ref. 7, noting that in our case the term mi_{dc} plays the role of the incident radiation amplitude (i_{rf} in Ref. 7), and Ω_m corresponds to Ω_{rf}.

Before proceeding further, we remark that Eq. (4) can also be written in the following form:

$$i_{dc}-\dot{\phi}=\sin\phi-(m/2)[\cos(\phi-\Omega_m T)-\cos(\phi+\Omega_m T)] \ , \tag{7}$$

and be solved without the weak-modulation assumption $(m \ll 1)$. In the high-voltage regime, around the induced step the cosine function of a phase difference α, defined as[7] $\alpha=\phi(T)-\Omega_m T$, is the only slowly varying function of time on the right-hand side of Eq. (7). Thus, we can average this equation over such a time interval that the fast oscillating terms will be eliminated (the slowly varying envelope approximation). The resultant equation, which describes the induced AM step, becomes

$$\dot{\alpha}-(m/2)\cos\alpha=i_{dc}-\Omega_m \ . \tag{8}$$

Using suitable transformations it can be put into the same form as the basic RSJ equation.

Under the weak-modulation approximation, even in the low-voltage regime, the shape of the dc I-V curve in the neighborhood of an AM step has the same shape as the RSJ model I-V curve and the full amplitude of the step (evaluated at $v_{dc}=\Omega_m$) is given by the maximum difference between the free-running and pulled Josephson frequency multiplied by twice the dynamic conductance of the I-V curve.[7] Thus, the normalized amplitude i_1 of the first AM step can be written as

$$i_1=mi_{dc}(i_{dc}-v_{dc})2(v_{dc}/i_{dc})=2v_{dc}(i_{dc}-v_{dc})m \ , \tag{9}$$

where $i_{dc}^2=v_{dc}^2+1$. Equation (9) can also be written more compactly as

$$i_1=A_1 m \ , \tag{10}$$

where $A_1\equiv 2v_{dc}(i_{dc}-v_{dc})$ is the first Fourier coefficient of the normalized current through the Josephson channel.[7] The calculated dependence of A_1 as a function of v_{dc} is shown in Fig. 1.

Equation (10) shows (see also Fig. 1) that, as opposed to the case of microwave-induced Shapiro steps,[1] the amplitude of the first AM step is a monotonic function of the fundamental component of the Josephson voltage, and is proportional to the modulation depth. This analytical result is in very good quantitative agreement with the published numerical findings.[2,4] We observe that, as plotted in Fig. 1, the dependence of A_1 on v_{dc} can be divided into two distinctive (high- and low-voltage) regions. In the high-voltage limit the curve saturates ($A_1 \rightarrow 1$), and as a result, Eq. (10) simplifies (in regular units) to

$$I_1=I_0 m \ . \tag{11}$$

Note that the same result can be obtained directly from Eq. (8). Thus, in this case it is not necessary to assume that $m \ll 1$, and the correct result should be written in the form $I_1=\langle I_0\rangle m$, the same as obtained by Vanneste *et al.*[4] for a constant-voltage-biased model.

The restriction that $\omega_m\tau_{eff} \ll 1$ makes the high-voltage regime difficult to realize experimentally (for details see Ref. 4) and for a gap-suppression type of AM experiments only the low-voltage limit seems feasible. In this case A_1 is the linear function of v_{dc} ($A_1 \approx 2v_{dc}$), and Eq. (10) predicts current step height given by

$$I_1=2m(\hbar\omega_m/2e)(1/R_J) \ . \tag{12}$$

I_1 is now independent of the value of the junction critical current and is proportional to the ratio of the step voltage to the junction resistance. Thus, low-resistance junctions (e.g., superconducting-normal-superconducting junctions) are preferred. Another interesting feature of Eq. (12) is that it coincides with the result obtained for weak, low-frequency phase modulation (see, e.g., Ref. 10). It means that, within the RSJ model, in the low-voltage regime, both amplitude and phase modulation lead to quantitatively the same step structure on the junction I-V characteristics.

Finally, we want to stress that we have chosen the model of a periodic gap suppression for the amplitude modulation of the supercurrent because of the extensive literature on

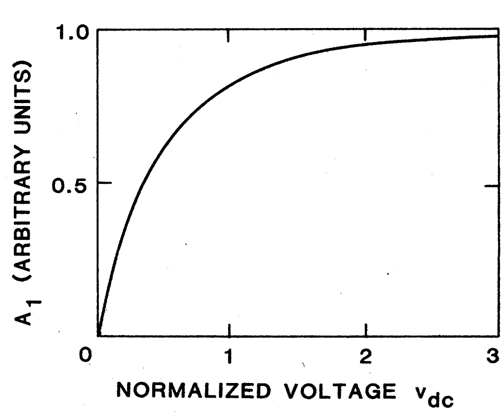

FIG. 1. Calculated dependence of A_1 as a function of v_{dc} (or, equivalently, Ω_m).

this subject and ongoing efforts to perform the experiment. However, our calculations are valid irrespective of the particular method of creating the modulated critical current [Eq. (2)]. Recent progress in fabrication of tunnel junctions with artificial, active barriers, and especially photosensitive barriers, makes it possible to modulate I_0 via changing the potential height of the barrier,[11] as opposed to changing the superconductive properties of electodes. In the former case, the junction response is free from the heating effect and is mainly limited by the relaxation time of the photoexcited carriers, which can be less than one picosecond with the proper choice of materials.

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*On leave of absence from the Institute of Physics, Polish Academy of Sciences, Aleja Lotników 32/46, PL-02668 Warszawa, Poland.
†Present address: Eastman Kodak Research Laboratory, Rochester, NY 14650.
[1]S. Shapiro, Phys. Rev. Lett. **11**, 80 (1963).
[2]H. Seifert, J. Low Temp. Phys. **37**, 595 (1979).
[3]S. M. Faris, C. C. Chi, D. C. Cronemeyer, and M. M. Loy, Bull. Am. Phys. Soc. **25**, 412 (1980).
[4]C. Vanneste, A. Gilabert, P. Sibillot, and D. B. Ostrowsky, J. Low Temp. Phys. **45**, 517 (1981).
[5]N. Perrin and C. Vanneste, Phys. Rev. B **28**, 5150 (1983).
[6]C. Vanneste, A. Gilabert, P. Sibillot, and D. B. Ostrowsky, Appl. Phys. Lett. **38**, 941 (1981).
[7]C. V. Stancampiano, IEEE Trans. Electron. Devices **ED-27**, 1934 (1980).
[8]C. S. Owen and D. J. Scalapino, Phys. Rev. Lett. **28**, 1559 (1972).
[9]W. H. Parker, Phys. Rev. B **12**, 3667 (1975).
[10]K. K. Likharev and B. T. Ulrich, *Systems with Josephson Junctions* (Moscow Univ. Press, Moscow, 1978) (in Russian).
[11]H. Akoh, T. Tokumoto, K. Kajimura, and H. Hayakawa, Physica B **107**, 731 (1981).