Critical current of thin superconducting wire with side branches

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When a transport current is injected into a thin superconducting wire with dangling superconducting side branches periodically attached to it, distance L apart, and the side branches are very much longer than the temperature-dependent coherence length, then the critical-current density is, according to the Ginzburg-Landau theory, larger than that of a wire without side branches, approaching the latter for $L/\xi \rightarrow \infty$ and becoming very much larger than the latter for $L/\xi \ll 1$.

Great interest in superconductive micronetworks has emerged[1–10] recently because of their potential application to percolation problems and practical circuits. Network equations, similar to Kirchhoff's laws, have been developed, and the mathematically manageable forms are expressed in terms of the linearized Ginzburg-Landau (GL) equations.[2,3] All of the hitherto published results, with the exception of Ref. 4, are applicable to second-order phase-transition regions because of linearization. It is found, in general, that the magnetic phase boundary between the superconductive (S) and normal (N) states is shifted to higher temperatures for closed structures with dangling side branches (without currents) compared to those without side branches. This is even true when the system is nonuniform.[10] A closed structure is a circuit which contains at least one loop for which fluxoid quantization has to be taken into account. Thus, phase coherence is *enhanced* near a second-order NS phase boundary.

However, when a transport current is applied to a wire with dangling side branches as shown in Fig. 1(a), the linearized GL equations do not apply (as is the case for a bare wire) and fluxoid quantization places no constraints on the system. One could argue that the presence of long side branches will enhance the modulus of the normalized order parameter $f(x)$ within a coherence length $\xi(t)$ near a node and therefore decrease the free energy relative to that of the bare wire. With that information alone, one would conjecture that a wire with side branches is likely to have a critical current which exceeds that of the bare wire. This conjecture is supported indirectly by the above-mentioned shift of the second-order phase boundary to higher temperatures. However, there is also a term involving the gradient of the order parameter to be considered which arises from the condition that the complex current must be conserved [the sum of the quantities $\psi^*(i\nabla + 2\pi A/\Phi_0)\psi$ entering a node must be zero]. The latter will increase the free energy and compete with the above-mentioned decrease in free energy. Thus, it is not obvious whether or not dangling side branches will appreciably enhance the critical current.

When the thickness of the wires is neglected, the open structure becomes quasi-one-dimensional. In that case, when the nonlinear GL equations are combined, the differential equation for f is

$$d^2f/dx^2 + (1-f^2-I^2/f^4)f = 0 \ , \qquad (1)$$

where

$$I \equiv m\xi J/\hbar e F_\infty^2 = 4\pi\lambda J/c\sqrt{2}H_c$$

is the normalized, dimensionless current density and J the current density in cgs Gaussian units. The curvilinear coordinate x is normalized by $\xi(t)$ in Eq. (1). For a long bare wire, one may assume that f does not depend on x, hence $f''=0$, and $I=f^2(1-f^2)^{1/2}$ which has a maximum at $f^2=\frac{2}{3}$. Thus, $I_c = 2/\sqrt{27}$, a well-known result. With side

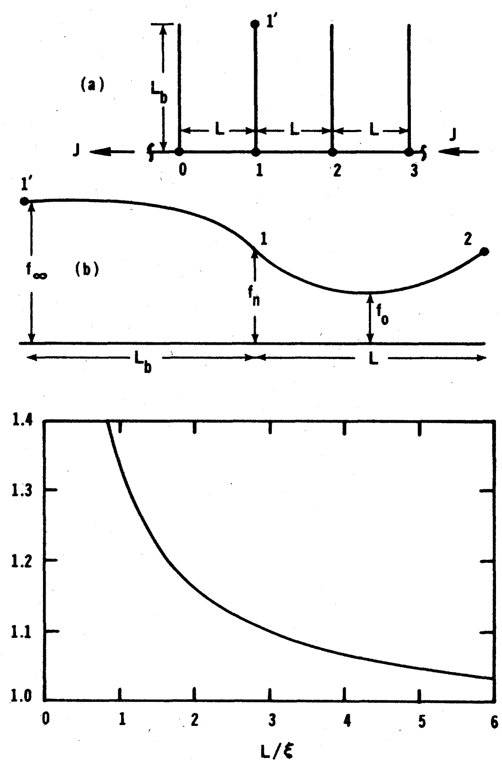

FIG. 1. Shown are (a) thin wire with dangling side branches of length L_b, (b) schematic of the order parameter along the branches 1′-1-2, and (c) critical-current density of a wire with long side branches normalized by that of a bare wire ($I_{CB} = 2/\sqrt{27}$) which can be carried by the above structure as a function of $L/\xi(t)$.

branches, f becomes a function of x. Then the first integral of Eq. (1) is

$$2f^2(df/dx)^2=(f_0^2-f^2)[f^2(2-f_0^2-f^2)-2I^2/f_0^2] \quad , \tag{2}$$

where f_0 is the value of $f(x)$ at some value $x=x_0$, where $df/dx=0$ [extremum of $f(x)$]. With the definition $f^2(x)=f_0^2+t^2(x)$, Eq. (2) becomes

$$2(dt/dx)^2=(t^2-R_1)(t^2-R_2) \tag{3}$$

with

$$R_{1,2}=(1-3f_0^2/2)\pm[(1-f_0^2/2)^2-2I^2/f_0^2]^{1/2} \quad . \tag{4}$$

When Eq. (3) is integrated, an incomplete elliptic integral of the first kind results whose solutions are inverse Jacobian elliptic functions which depend on R_1 and R_2. The value of $f(x)$ at a node n, f_n, is then obtained in terms of f_0 and I for a fixed value of the nodal distance L normalized by $\xi(t)$. Apart from the fact that f_0 can be interpreted as a minimum or maximum value of a periodic function, there are a number of different solutions of $t(x)$ possible. The function $t(x)$ can be proportional to $\text{cn}(u|m)$ or $\text{sn}(u|m)$, or $\text{sn}(u|m)/\text{cn}(u|m)\equiv\text{sc}(u|m)$ or $\text{sc}(u|m)\text{dn}(u|m)$, where u is the normalized distance between nodes and m is the parameter, both of which depend on R_1 and R_2. We found numerically, as explained below, that the type of solution which leads to the largest current, is obtained when R_2 is the complex conjugate of R_1 and is of the form

$$f_n^2=f_0^2+|R_1|\text{sc}^2(u|m)\text{dn}^2(u|m) \quad , \tag{5}$$

where $u=L(|R_1|/8)^{1/2}$, $m=[1-\text{Re}(-R_1)/|R_1|]/2$, and f_0 is a minimum. With L fixed, Eq. (5) and the corresponding other types of solutions are surfaces in the (f_0,f_n,I) space.

Complex current conservation requires[3] that at an arbitrary node n, the following condition must be satisfied:

$$\sum(i\partial\psi_n/\partial x+2\pi A\,\psi_n/\Phi_0)=0 \quad , \tag{6a}$$

where the sum is over all branches connected directly to node n and $\psi_n=f_n\exp(-i\phi_n)$ is the complex order parameter at this node (same material assumed throughout). From the real and imaginary parts of (6a) one finds that

$$\sum(d\phi_n/dx+2\pi A/\Phi_0)\equiv\sum q_n=0 \quad , \tag{6b}$$

$$\sum(df_n/dx)=0 \quad , \tag{6c}$$

where q_n is the superfluid velocity, and the derivative of f, with respect to x, is taken radially outward from the node. Equation (6b) is satisfied when the current in Eq. (1) is treated as a parameter. Equation (6c) at node n is $|df_n/dx|_{n,n'}=2|df_n/dx|_{n,n+1}$. When the branch length $L_b\rightarrow\infty$ then $f_\infty\rightarrow 1$ [see Fig. 1(b)] and the latter equation expressed in terms of Eq. (2) becomes

$$I^2=f_n^2[3f_n^4-6f_n^2+4f_0^2(2-f_0^2)-1]/[8(1-f_n^2/f_0^2)] \quad . \tag{7}$$

This is a surface in the (f_0,f_n,I) space similar to Eq. (5).

The intersection of the two surfaces satisfies Eqs. (1) and (6a). Therefore, Eqs. (5) and (7) lead to solutions of f_0 and f_n in terms of L and I. For numerical convenience, L and f_0 (minimum value of f) were used as input parameters, and f_n and I were obtained.

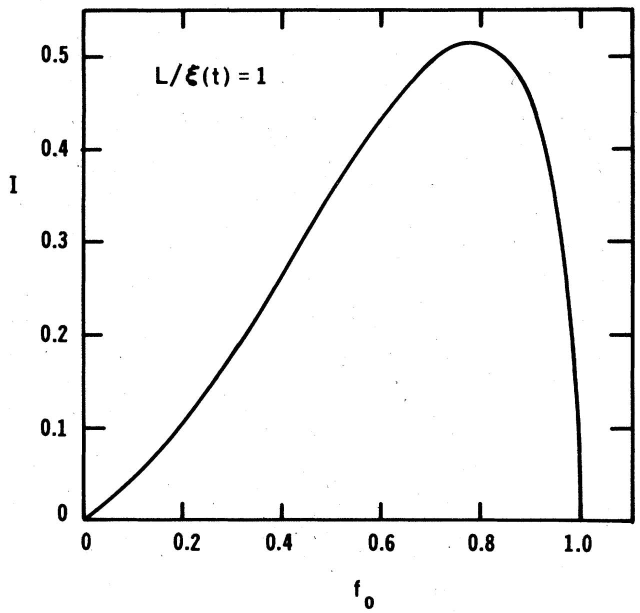

FIG. 2. Normalized current density as a function of the minimum order parameter f_0 for nodal spacings $L/\xi(t)=1$. Solutions of f for f_0 values near unity are of the $\text{sc}(u|m)$ type and for f_0 near zero are of the $\text{sn}(u|m)$ type. Near and at the current maximum f is of the $\text{sc}(u|m)\text{dn}(u|m)$ type [Eq. (5)].

Figure 1(c) shows the results of the largest (critical) current as a function of the nodal spacing L. For $L>>1$, the current density I approaches the value of the bare wire and for $L<<1$, the value of I becomes very large. Figure 2 shows I as a function of f_0 for $L=1$ for three different types of solutions. Solutions of $f(x)$ for f_0 values approaching zero are of the $\text{sn}(x)$ type, while for f_0 near unity $f(x)$ is of the $\text{sc}(x)$ type. For values of f_0 in the intermediate range $f(x)$ is of the $\text{sc}(x)\text{dn}(x)$ type. Solutions with f_0 values between the maximum of I and $f_0=1$ are stable, lowest-energy-type solutions. Solutions of one type change smoothly into another as I is changed.

A thin superconductive wire with long superconductive side branches has critical-current densities the values of which are larger than those of the bare wire. It is conjectured that a wire with finite length branches will have critical-current densities the values of which are between the above results and those of the bare wire. An essential part of the present solution is Eq. (6a), a condition which is stronger than ordinary current conservation. The latter is satisfied by Eq. (6b), but Eq. (6c) is an additional constraint, which the authors believe has not been tested explicitly by experiments. Critical-current measurements with and without side branches would constitute such a test.

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