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Solitons of the square-rectangular martensitic transformation

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The static solitons of the square-rectangular martensitic transformation are obtained with the use of higher-order elasticity theory; both first-order (rectangular-rectangular and square-rectangular solitons) and second-order (rectangular-rectangular only) phase transitions are treated. The shear and dilatational strains vanish in all cases. The deviatoric strains are calculated exactly, and the displacement vectors are obtained as power series, using the full (nonlinear) Lagrangian strain tensor. In the first-order case, the width of the rectangular-rectangular soliton diverges as the transition temperature is approached from below; the soliton splits gradually into two square-rectangular solitons, both of finite width, whose separation diverges at the transition temperature.

I. INTRODUCTION

The martensitic transformation and the related phenomena of shape memory, pseudoelasticity, and ferroelasticity are of considerable technological importance; the proceedings[1] of a recent conference are a good introduction to the literature. Until recently, theories of an important class of such transformations, the cubic-tetragonal transformation (examples: Nb_3Sn, V_3Si, In-Tl alloys), have been dominated by dislocation models. The soliton model of Barsch and Krumhansl,[2] which provides a dislocation-free explanation of the observed twinned regions in such materials in terms of higher-order elasticity theory, is therefore of much interest.

The present paper is concerned with the martensitic transformation in two dimensions, the high-temperature and low-temperature phases having square and rectangular symmetry (point groups $4mm$ and $2mm$), respectively. The phase transition is the two-dimensional analog of the cubic-tetragonal transition of the preceding paragraph; it is studied using higher-order elasticity theory, as in Refs. 2 and 3. Both first-order and second-order phase transitions are treated. Attention is restricted to the static solitons linking (a) a square region and a rectangular region and (b) two rectangular regions of different orientations.

The equation of motion of the displacement vector is obtained using higher-order elasticity theory and the full, nonlinear Lagrangian strain tensor. Exact solutions are obtained for the strains describing both rectangular-rectangular and square-rectangular solitons. The displacements are also obtained without the common approximation of linearizing the strain tensor; more precisely, power series for the displacements are obtained.

The rectangular-rectangular solitons for both first-order and second-order square-rectangular phase transitions are obtained analytically, at all temperatures below the transition, using the equation of motion of the preceding paragraph; both the strains and the displacements are obtained. Like the special-case tetragonal-tetragonal solutions found by Barsch and Krumhansl,[2] these solutions have shear and dilatational strains identically zero. As described in the abstract, interesting and unusual behavior is found in the first-order case as the transition is approached from below. This behavior has been found previously by Lajzerowicz[4] in a study of domain walls in ferroelectrics; in fact, the solutions obtained below for the strains [Eqs. (20) and (22)] are, apart from differences in notation, identical to those obtained by Lajzerowicz for the ferroelectric order parameter.

The square-rectangular soliton (both its strains and displacement vector) is found analytically in the first-order case (it does not occur in the second-order case). Again the shear and dilatational strains are identically zero.

Some of the results obtained in the first-order case parallel those obtained by Falk[3] who studied the corresponding one-dimensional model; the origin of the parallelism is the assumption that the strain in the two-dimensional problem considered here is a function of a single variable [see Eq. (11) below]. For both kinds of solitons, the boundaries are lines parallel to the [11] and $[1\bar{1}]$ directions.

II. ELASTIC ENERGY DENSITY

The starting point for a description of the square-rectangular martensitic transformation is the following Landau-theory expression for the elastic energy density:

$$\mathscr{F}=A_1e_1^2+D_1(\nabla e_1)^2+Ae_2^2+Be_2^4+Ce_2^6+D(\nabla e_2)^2 + A_3e_3^2+D_3(\nabla e_3)^2 \, , \tag{1}$$

where e_1, e_2, and e_3 are the dilatational, deviatoric, and shear strains, respectively:

$$e_1=(\eta_{11}+\eta_{22})/\sqrt{2} \, , \tag{2}$$

$$e_2=(\eta_{11}-\eta_{22})/\sqrt{2} \, , \tag{3}$$

$$e_3=\eta_{12}=\eta_{21} \, . \tag{4}$$

Here the η_{ij} are the components of the Lagrangian strain tensor (repeated-index convention here and in the following)

$$\eta_{ij}=\tfrac{1}{2}(u_{i,j}+u_{j,i}+u_{k,i}u_{k,j}) , \quad (5)$$

where the u_i are the components of the displacement vector relative to the reference state (the square state) which has all strains (e_1, e_2, and e_3) equal to zero. As usual, $u_{i,j}=\partial u_i/\partial x_j=\partial_j u_i$. The strains in the rectangular state are twofold degenerate: $e_1=0$, $e_2=\pm e_{20}$, $e_3=0$ with e_{20} a constant. In the square state, the x_1 and x_2 axes are mirror lines and the x_3 axis is a fourfold axis.

In a more complete treatment, many other terms would be included in Eq. (1). First of all, however, although e_1 is an invariant, there can be no term linear in e_1 (as discussed in Ref. 5). Omitted from Eq. (1) are terms such as $B_1e_1e_2^2$, $C_1e_1(\nabla e_2)^2$, etc., which couple e_1 and its derivatives to e_2 and its derivatives; these terms are necessary for a discussion of the volume (here area) change at a first-order martensitic transformation but are neglected here, as in previous work,[2] first because the volume change is usually small, and second in order to avoid unnecessary complications. Also omitted from Eq. (1) are terms coupling the deviatoric strain e_2 and its derivatives to the shear strain e_3 and its derivatives. This is done again for simplicity but for another reason as well; with such terms, a strain e_2 might induce a shear e_3, but it might not be obvious whether the shear was an essential part of the transformation or merely a consequence of the coupling terms. Terms coupling e_1 and its derivatives to e_3 and its derivatives are superfluous (and therefore omitted). Finally, terms of eighth and higher order in e_2 are omitted, again to avoid unnecessary complications. Equation (1) contains only those terms necessary for a qualitative understanding of the phase transitions.

The order parameter of the transition is the displacement **u** though of course only derivatives of **u** appear in the elastic energy density. The strains e_1, e_2, and e_3 are not independent variables and it is therefore not legitimate, even in the static case, to minimize the elastic energy with respect to variations in e_1, e_2, and e_3. The solutions obtained below have vanishing dilatational (e_1) and shear (e_3) strains but it is not obvious from Eq. (1) that such solutions are possible; in fact, shearless solutions are possible for only a few orientations of the domain walls relative to the axes.

As is customary in a Landau theory, it is assumed that the coefficient A in Eq. (1) has the temperature-dependent form $A=A_0(T-T_0)$, with T_0 some reference temperature, and that A_0 and the remaining seven Landau coefficients are independent of temperature; for stability, the coefficients A_1, D_1, D, A_3, and D_3 must be positive. With regard to the other coefficients (B and C), two cases must be distinguished, $B>0$ (in which case C can be taken as zero), and $B<0$ (in which case C must be >0); these cases yield second-order and first-order phase transitions, respectively. The eight Landau coefficients A_1 to D_3 are various combinations of second-, fourth-, and sixth-order elastic constants.

The above expression [Eq. (1)] for the elastic energy density is not the two-dimensional form of the expression given by Barsch and Krumhansl;[2] the important difference is that the high-temperature state here has square symmetry and therefore it is necessary to include the term Ce_2^6 in order to describe a first-order transition. In the cubic-tetragonal case,[2] there is a third-order invariant[5–7] and the Landau expansion can be terminated in fourth order.

As in Ref. 2, the Lagrangian density is ($i=1,2$)

$$\mathcal{L}=\tfrac{1}{2}\rho_0\dot{u}_i\dot{u}_i-\mathcal{F} , \quad (6)$$

where ρ_0 is the mass density, and the equation of motion is

$$\rho_0\ddot{u}_i=\partial_j\frac{\partial\mathcal{F}}{\partial u_{i,j}}-\partial_j\partial_k\frac{\partial\mathcal{F}}{\partial u_{i,jk}} . \quad (7)$$

For $\mathcal{F}$ as given by Eq. (1), Eq. (7) yields

$$\rho_0\ddot{u}_1=\partial_1\{(1+u_{1,1})G_1[e_1]+(1+u_{1,1})G_2[e_2]+u_{1,2}G_3[e_3]\}+\partial_2\{u_{1,2}G_1[e_1]-u_{1,2}G_2[e_2]+(1+u_{1,1})G_3[e_3]\} , \quad (8a)$$

$$\rho_0\ddot{u}_2=\partial_1\{u_{2,1}G_1[e_1]+u_{2,1}G_2[e_2]+(1+u_{2,2})G_3[e_3]\}+\partial_2\{(1+u_{2,2})G_1[e_1]-(1+u_{2,2})G_2[e_2]+u_{2,1}G_3[e_3]\} , \quad (8b)$$

where the functionals G_1, G_2, and G_3 are

$$G_1[e_1]=(-2D_1\nabla^2e_1+2A_1e_1)/\sqrt{2} , \quad (9a)$$

$$G_2[e_2]=(-2D\nabla^2e_2+2Ae_2+4Be_2^3+6Ce_2^5)/\sqrt{2} , \quad (9b)$$

$$G_3[e_3]=-D_3\nabla^2e_3+A_3e_3 . \quad (9c)$$

The full (nonlinear) Lagrangian strain tensor of Eq. (5) was used in the derivation of these results.

For static solitons, Eqs. (8) are satisfied if

$$e_1=0 , \quad (10a)$$

$$2D\nabla^2e_2=2Ae_2+4Be_2^3+6Ce_2^5 , \quad (10b)$$

$$e_3=0 . \quad (10c)$$

These are to be viewed as trial "solutions," subject to verification; again, the order parameter is the displacement vector **u**, and Eqs. (10) are not necessarily mutually compatible—in fact, they are self-contradictory in general. The procedure in the following is to assume $e_3=0$, solve Eq. (10b) subject to the restrictions $e_1=0$ and e_2 a function of the single variable x' given by

$$x'=x_1\cos\phi+x_2\sin\phi , \quad (11)$$

where ϕ is to be determined (x' is the coordinate normal to the interface), calculate the displacement vector **u**, and then verify that the shear indeed vanishes everywhere; it turns out that this is true if

$$\phi=\pi/4,\ 3\pi/4,\ 5\pi/4,\ \text{or}\ 7\pi/4 . \quad (12)$$

Equation (12) holds for both the square-rectangular and rectangular-rectangular solitons; perhaps there is a symmetry argument for this result. Of course the above procedure proves only that dilatationless and shearless solutions are possible and that they are at least metastable against development of a dilatation or a shear; whether these solutions yield the absolute minimum of the elastic energy is a difficult question whose answer would seem to require a full solution of Eqs. (8).

The vanishing of the dilatational strain e_1 in both the square and rectangular states means that the transition is area-preserving. The area is also preserved in the presence of solitons (since e_1 vanishes in the neighborhood of the solitons as well as in the asymptotic regions).

The displacement field is found as follows from the solution $e_2(x')$ of Eq. (10b). By integration, functions $u_1^{(1)}$ and $u_2^{(1)}$, each proportional to the strain e_{20} in the rectangular state, are found which satisfy

$$u_{1,1}^{(1)}=e_2(x')/\sqrt{2}\ , \tag{13a}$$

$$u_{2,2}^{(1)}=-u_{1,1}^{(1)}\ , \tag{13b}$$

$$u_{1,2}^{(1)}+u_{2,1}^{(1)}=0\ , \tag{13c}$$

so that $\eta_{11}+\eta_{22}\equiv\sqrt{2}e_1=0$, $\eta_{11}-\eta_{22}=\sqrt{2}e_2$, and $\eta_{12}\equiv e_3=0$, all three to first order in e_{20}; it is the last equation which gives the requirement $\phi=(\pi/4)\mathrm{mod}(\pi/2)$. Expanding u_1 and u_2 in powers of e_{20} (it appears that the expansion parameter is really $e_{20}/\sqrt{2}$), one determines the higher-order functions from the equations

$$u_{1,1}=u_{1,1}^{(1)}-\tfrac{1}{2}u_{1,1}^2-\tfrac{1}{2}u_{2,1}^2\ , \tag{14a}$$

$$u_{2,2}=u_{2,2}^{(1)}-\tfrac{1}{2}u_{1,2}^2-\tfrac{1}{2}u_{2,2}^2\ , \tag{14b}$$

which guarantee that $\eta_{11}+\eta_{22}\equiv\sqrt{2}e_1=0$ and $\eta_{11}-\eta_{22}=\sqrt{2}e_2$ to all orders in e_{20}; these equations might serve for numerical work as well. One finds easily ($i=1,2$; $j=1,2$)

$$u_{i,j}^{(2)}=-\tfrac{1}{2}e_2^2,\quad u_{i,j}^{(3)}=0\ ,$$
$$u_{i,j}^{(4)}=-\tfrac{1}{4}e_2^4,\quad u_{i,j}^{(5)}=0\ , \tag{15}$$

etc. Finally, the assumption $e_3=0$ is verified order by order. In this way, solutions for the displacement field are obtained which are consistent with both the full (nonlinear) Lagrangian strain tensor and the equation of motion.

III. SOLITONS

In this section the solitons linking (a) a square region and a rectangular region and (b) two rectangular regions are obtained. Both kinds of solitons have positive energies. The extension of the results to describe chains of square-rectangular or rectangular-rectangular solitons appears straightforward.

For the case of a second-order square-rectangular transition ($B>0$), the Landau coefficient C is chosen equal to zero; Eq. (10b) then reduces to the form familiar from the theories of superconductivity, magnetism, etc. The square state ($e_2=0$) is stable for $A>0$; the rectangular state, with strain $e_2=\pm e_{20}$ where

$$e_{20}=(-\tfrac{1}{2}A/B)^{1/2}\ , \tag{16}$$

is stable for $A<0$. There is no square-rectangular soliton. The rectangular-rectangular soliton has the familiar form ($A<0$)

$$e_2(x')=e_{20}\tanh(\kappa x') \tag{17}$$

where $\kappa=[-A/(2D)]^{1/2}$; the width parameter (κ^{-1}) of the soliton diverges as $A\to 0$, familiar behavior for a second-order transition. The well-known periodic solution is also easily obtained. As noted above, shearless solutions can be obtained only for $\phi=\pi/4\,\mathrm{mod}\,\pi/2$. For $\phi=\pi/4$, the displacement vector is $[x'=(x_1+x_2)/\sqrt{2}]$

$$\mathbf{u}(x_1,x_2)=(1,-1)e_{20}\kappa^{-1}\ln[\cosh(\kappa x')]-(1,1)e_{20}^2(\kappa\sqrt{2})^{-1}[\kappa x'-\tanh(\kappa x')]$$
$$-(1,1)e_{20}^4(\kappa 2\sqrt{2})^{-1}[\kappa x'-\tanh(\kappa x')-\tfrac{1}{3}\tanh^3(\kappa x')]+O(e_{20}^6/\kappa)\ . \tag{18}$$

The constants of integration have been chosen so that the displacement vanishes at the origin (and therefore along the entire line $x_1+x_2=0$ which is the center of the domain wall). Plots of unstrained and strained regions are given in Fig. 1; the domain wall is a twin boundary [see the discussion following Eq. (25) below].

The case of a first-order square-rectangular transition ($B<0$) is more interesting. The square state ($e_2=0$) is stable for $A>B^2/4C$ and the rectangular state, with $e_2=\pm e_{20}$, where

$$e_{20}=\{[-B+(B^2-3AC)^{1/2}]/3C\}^{1/2}\ , \tag{19}$$

is stable for $A<B^2/4C$. The square state is metastable for $0<A<B^2/4C$ and the rectangular state is metastable for $B^2/4C<A<B^2/3C$. The differential equation for e_2 has been studied thoroughly by Falk;[3] attention here is restricted to the case of a single domain wall.

The square-rectangular soliton exists only for the temperature defined by $A=B^2/4C$ (at which temperature the square and rectangular states have the same energy). For the case of square and rectangular regions in the limits $x'\to-\infty$ and $+\infty$, respectively, the strain is

$$e_2(x')=e_{20}(1+e^{-2\kappa x'})^{-1/2} \tag{20}$$

with $e_{20}=[\,|B\,|\,/2C]^{1/2}$ and $\kappa=\,|B\,|\,/(4CD)^{1/2}$; these definitions of e_{20} and κ are the limiting values of Eqs. (19) and (23) [hence the factor of 2 in the exponent in Eq. (20)]. The one-dimensional version of Eq. (20) was obtained by Falk;[3] Eq. (20) also occurs in the theory of ferroelectrics.[4] The displacement field for $\phi=\pi/4$ [that is $x'=(x_1+x_2)/\sqrt{2}$] is

$$\mathbf{u}(x_1,x_2)=(1,-1)e_{20}\kappa^{-1}\ln[(1+e^{2\kappa x'})^{1/2}+e^{\kappa x'}]-(1,1)e_{20}^2(2\kappa\sqrt{2})^{-1}\ln(1+e^{2\kappa x'})$$
$$-(1,1)e_{20}^4(4\kappa\sqrt{2})^{-1}[\ln(1+e^{2\kappa x'})-\tfrac{1}{2}-\tfrac{1}{2}\tanh(\kappa x')]+O(e_{20}^6/\kappa)\ . \tag{21}$$

The constants of integration have been chosen so that $\mathbf{u}$ vanishes in the limit $x'\to-\infty$. A plot of a distorted region is given in Fig. 2(a).

The rectangular-rectangular soliton exists for all $A<B^2/4C$ (in which temperature region the rectangular state is stable relative to the square state). The solution for the strain is

$$e_2(x')=e_{20}\sinh(\kappa x')/[\cosh^2(\kappa x')+\alpha]^{1/2}\ , \tag{22}$$

where e_{20} is given by Eq. (19) and κ and α by

$$\kappa=e_{20}[(B^2-3AC)^{1/2}/D]^{1/2}\ , \tag{23}$$

$$\alpha=[-B+(B^2-3AC)^{1/2}]/[B+2(B^2-3AC)^{1/2}]\ . \tag{24}$$

The one-dimensional version of Eq. (22) was obtained by Falk;[3] Eq. (22) also occurs in the theory of ferroelectrics.[4] The parameter α ranges from $\frac{1}{2}$ (as $A\to-\infty$) to ∞ (at $A=B^2/4C$), never taking on the value 0 which would reduce Eq. (22) to Eq. (17); the rectangular-rectangular solitons are therefore different for the two cases $B<0$ and $B>0$, the difference becoming pronounced near the transition temperature. As $A\to B^2/4C$ from below, the parameter κ approaches a finite nonzero value, but α diverges. The width of the soliton therefore diverges in this limit, very unusual behavior at a first-order transition. While strictly speaking the foregoing is true, it is nevertheless misleading. As Fig. 3 shows, as $A\to B^2/4C$ from below, the rectangular-rectangular soliton splits gradually into two square-rectangular solitons; this is allowed because the energy density in the central square region gradually approaches that in the rectangular regions as $A\to B^2/4C$. The width of each square-rectangular soliton is finite; it is their separation which diverges in the limit $A=B^2/4C$. The splitting is easily verified analytically: In Eq. (22), replace x' by $x'+x_0$, where $\exp(2\kappa x_0)=4\alpha$, and let $\alpha\to\infty$ to obtain Eq. (20). The divergence of the width of the soliton (but not the splitting of the soliton) was noted by Falk.[3] The splitting of the soliton appears to have been first noted by Lajzerowicz.[4]

The displacement (again with the constants chosen to give $\mathbf{u}=0$ at the origin) is, for $\phi=\pi/4$,

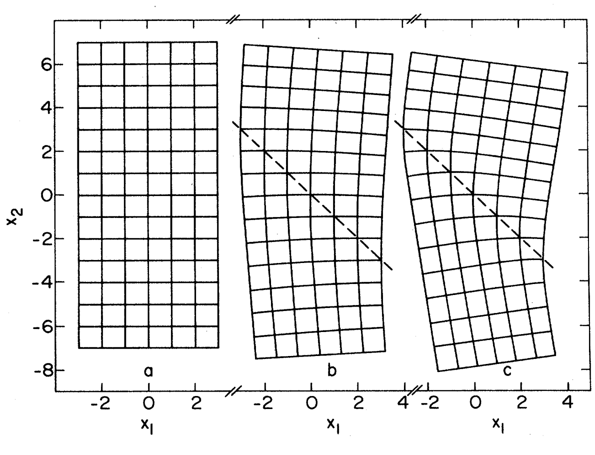

FIG. 1. Unstrained (a) and strained (b and c) regions for a rectangular-rectangular soliton in the case $B>0$. The parameters are $e_{20}=0.1$, $\kappa=0.5$ in b and $e_{20}=0.2$, $\kappa=1$ in c. One of the infinite number of sets of Landau parameters giving these values is $A=-0.1$ in b, $A=-0.4$ in c, and $B=5$, $C=0$, and $D=0.2$ in both. The dashed lines are the centers of the domain walls (the lines $x_1+x_2=0$ along which the strain e_2 vanishes). Here (and in Fig. 2 as well) large strains (unrealistically so) are required for display.

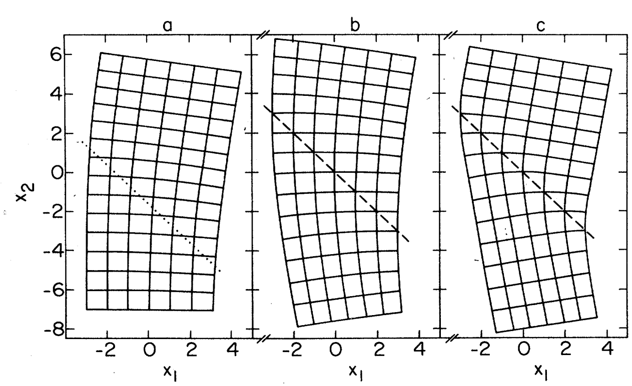

FIG. 2. a: Square-rectangular soliton for $e_{20}=0.2$, $\kappa=1$; the dotted line is the center of the wall $[e_2(x')=e_{20}/2]$. b and c: Rectangular-rectangular solitons for $e_{20}=0.201$, $\kappa=1.020$, $\alpha=51.2$ (corresponding to curve c in Fig. 3) and $e_{20}=0.231$, $\kappa=1.633$, $\alpha=2$ (corresponding to curve a in Fig. 3), respectively; the strain e_2 vanishes along the dashed lines. Possible sets of Landau parameters are $A=0.2$ in a, $A=0.196$ in b, $A=0$ in c, with $B=-10$, $C=125$, $D=0.2$ in all three parts.

$$\mathbf{u}(x_1,x_2)=(1,-1)\frac{e_{20}}{\kappa}\ln\left[\frac{\cosh(\kappa x')+[\cosh^2(\kappa x')+\alpha]^{1/2}}{1+(1+\alpha)^{1/2}}\right]$$
$$-(1,1)\frac{e_{20}^2}{\kappa\sqrt{2}}\left\{\kappa x'-\left[\frac{1+\alpha}{\alpha}\right]^{1/2}\operatorname{arctanh}\left[\left[\frac{\alpha}{1+\alpha}\right]^{1/2}\tanh(\kappa x')\right]\right\}$$
$$-(1,1)\frac{e_{20}^4}{2\kappa\sqrt{2}}\left\{\kappa x'-\frac{(2\alpha-1)}{2\alpha}\left[\frac{1+\alpha}{\alpha}\right]^{1/2}\operatorname{arctanh}\left[\left[\frac{\alpha}{1+\alpha}\right]^{1/2}\tanh(\kappa x')\right]\right.$$
$$\left.-\frac{(1+\alpha)}{4\alpha}\frac{\sinh(2\kappa x')}{\cosh^2(\kappa x')+\alpha}\right\}+O\left[\frac{e_{20}^6}{\kappa}\right]. \quad (25)$$

Figure 2 shows a rectangular-rectangular soliton at temperatures well below (part c) and just below (part b) the square-rectangular transition; in the latter, there is a reasonably sized square region between the two rectangular regions. Part a shows a square-rectangular soliton at the transition temperature.

The components of the displacement vectors for the rectangular-rectangular solitons in both cases [$B>0$, Eq. (18) and $B<0$, Eq. (25)] have the form

$$u_1(x_1,x_2)=f_e(x_1+x_2)-f_o(x_1+x_2)\ ,$$
$$u_2(x_1,x_2)=-f_e(x_1+x_2)-f_o(x_1+x_2)\ , \quad (26)$$

where f_e and f_o are respectively even and odd functions of their arguments. Hence the components of $\mathbf{u}$ at the points (x_1,x_2) and $(-x_2,-x_1)$, which are twin related with respect to the line $x_1+x_2=0$, satisfy

$$u_1(-x_2,-x_1)=-u_2(x_1,x_2)\ ,$$
$$u_2(-x_2,-x_1)=-u_1(x_1,x_2)\ ; \quad (27)$$

therefore the line $x_1+x_2=0$ is a twin boundary for both rectangular-rectangular solitons.

The other three possibilities for ϕ obviously also give twin boundaries. Consequently in the asymptotic region (well away from any boundary) there are four possible orientations of the rectangles.

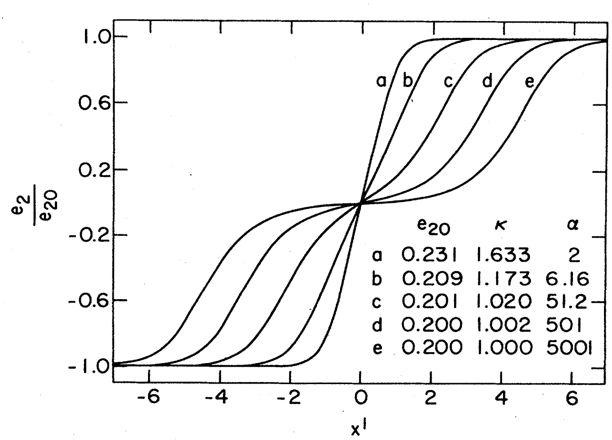

FIG. 3. Plots of the strain e_2 (normalized to the value e_{20}) vs $x'=(x_1+x_2)/\sqrt{2}$. A possible set of Landau parameters yielding the values of e_{20}, κ and α given is $A=0$, 0.16, 0.196, 0.1996, 0.199 96 for curves $a-e$, and $B=-10$, $C=125$, $D=0.2$.

IV. DISCUSSION

Barsch and Krumhansl[2] have shown (for the cubic-tetragonal transition) that higher-order elasticity theory provides a simple explanation of the twinned regions which are experimentally observed and which play such an important role in the explanation of the phenomena of shape memory, pseudoelasticity, and ferroelasticity; one expects soliton models such as theirs to be important in the future development of these fields. The soliton model appears to provide such a natural explanation of the twinned regions that the burden of proof may now lie with the dislocation model.

There remain, however, many important questions. In the theory of the cubic-tetragonal martensitic transformation a few of these are (1) the structure of the cubic-tetragonal soliton (at the transition temperature), (2) the structure of the tetragonal-tetragonal soliton for arbitrary temperatures below the transition (the Barsch-Krumhansl solution applies at only one temperature) and whether the soliton splits into two cubic-tetragonal solitons as the transition temperature is approached, and (3) the more macroscopic question of how the various tetragonal regions join with each other and with untransformed cubic material. It should be pointed out that the existence of solitons connecting regions which are asymptotically homogeneously strained is not obvious. Finally, there is the question of whether the soliton models provided by higher-order elasticity theory have applicability to other martensitic transformations.

The present paper has provided support for the soliton model by explicit solution of a simple model at all temperatures. In the process, the phenomenon of soliton splitting (which occurs also in the theory of ferroelectrics[4]) was found in the case of a first-order phase transition; the splitting here is only superficially similar to that in the theory of polyacetylene. On the technical side, it has been shown that the nonlinearity of the Lagrangian strain tensor need not prevent a complete calculation of the strains and the displacement. Whether these results will prove useful in the theory of the cubic-tetragonal and other martensitic transformations remains to be seen.

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