Nonlocal response of Josephson tunnel junctions to a focused laser beam

Jhy-Jiun Chang
Department of Physics and Astronomy, Wayne State University, Detroit, Michigan 48202
and Institute for Theoretical Physics, University of California, Santa Barbara, California 93106

C. H. Ho
Department of Physics and Astronomy, Wayne State University, Detroit, Michigan 48202

D. J. Scalapino
Institute for Theoretical Physics, University of California, Santa Barbara, California 93106
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The effect of a focused laser beam on the critical junction current of a one-dimensional Josephson tunnel junction is analyzed for both short and long junctions. The laser heating locally alters the critical current density and London penetration depth, giving rise to a change in the critical current which is proportional to the local current density of the irradiated region. In addition, the relative pair phase is modified in a nonlocal manner which gives rise to an additional contribution to the change in the critical current of the junction. For short junctions, this nonlocal phase modification contributes two parts to the junction critical current. One has the same spatial dependence as that of the current distribution itself, while the other is independent of the position of the laser beam. This latter contribution depends upon the applied magnetic field and is a signature of the nonlocal effect. For a long junction, the nonlocal phase contribution to the change in the junction critical current can give rise to dramatic effects when the applied magnetic field is less than a critical value $H_c = \hbar c/(2ed\lambda_J)$. In this case the spatial dependence of the laser-induced change in the critical current can be qualitatively different from the unperturbed current distribution.

I. INTRODUCTION

Recently, various scanning techniques have been developed to probe the properties and structures of superconducting thin films and Josephson tunnel junctions.[1–4] Using a focused low-energy electron beam, Huebener and his co-workers[1,2] studied the hot spots in superconducting thin films and the distribution of the quasiparticle current in superconducting tunnel junctions. Using a patterned laser beam, Chi *et al.*[3] measured the spatial variations of the parameters of a thin film, such as the critical-current density J and the transition temperature T_c, as functions of positions. This technique was recently applied to Josephson tunnel junctions, and the spatial dependence of the tunneling probability was obtained. Of particular interest to us are the experiments by Chen and his co-workers[4] in which a focused laser beam was applied to Josephson tunnel junctions, and the change of the critical current was recorded as a function of the beam position. For low beam powers, the beam-position dependence of the laser-induced modification of the critical junction current was found to be in qualitative agreement with the calculated[5] (unperturbed) current distribution. Therefore, it appeared that the focused-laser scanning technique provided a direct probe of the current distribution in a Josephson junction. This agreement led to the idea that the laser beam changes only the critical-current density J in the irradiated region leaving the phase-difference function ϕ unperturbed.

We believe, however, that the physics involved is more complicated and reflects the macroscopic quantum properties of Josephson junctions. Specifically, the Josephson tunnel junction behaves as a macroscopic quantum system with the phase-difference function ϕ governed by the sine-Gordon-type Josephson equation.[6] Disturbing the parameter J in a local region can also produce a nonlocal modification of the function ϕ. The effective range of a phase disturbance is governed by the Josephson screening length λ_J which is typically of order of a millimeter, and in actual experimental situations is much larger than the beam size (typically about 10 μm). Hence, it is important to study the nonlocal phase response and its effects on the critical-current measurement. From the mathematical point of view, a junction is characterized by a set of parameters: J, the critical-current density; λ_L, the London penetration depth; and λ_J, which also depends on J and λ_L. Assuming that only J is modified locally by the laser beam, one ignores effects associated with the modifications of λ_L and λ_J which can be important, especially for long junctions where λ_J plays an important role.

Indeed, it is not difficult to demonstrate that the simple assumption that the laser beam changes only the amplitude J but not the phase difference function ϕ can be inconsistent. Consider a semi-infinite one-dimensional junction occupying the space $y > 0$, which is uniformly irradiated by a laser beam. The current distribution of such a junction is well known[7] to be given by

$$J(y)=J\sin\phi(y)$$
$$=\begin{cases}0 & \text{for } y<0\,,\\ 2J\,\mathrm{sech}\left[\dfrac{y}{\lambda_J}\right]\tanh\left[\dfrac{y}{\lambda_J}\right] & \text{for } y>0\,,\end{cases} \tag{1.1}$$

when the current is biased at its critical value. Therefore, the total current flowing through the junction is

$$I_c=\int_0^\infty dy\,J\sin\phi(y)=2\lambda_J J\,, \tag{1.2}$$

and the change of the junction critical current due to the laser irradiation that modified λ_J and J is

$$\Delta I_c=2\Delta(\lambda_J J)\simeq\lambda_J J\left[\frac{\Delta J}{J}-\frac{\Delta\lambda_L}{\lambda_L}\right]. \tag{1.3}$$

To obtain the last equation, we have used the fact that $\lambda_J\sim(\lambda_L J)^{-1/2}$ and kept the lowest-order terms. On the other hand, the simple assumption of the current amplitude J alone being modified implies that the junction critical current be changed by an amount,

$$\Delta I_c=\Delta J\int_0^\infty dy\,\sin\phi(y)=2\lambda_J\Delta J\,. \tag{1.4}$$

Comparing this with Eq. (1.3), we see that even if the effect due to the change of the London penetration depth can be neglected, there is a difference of a factor of 2.

This example clearly demonstrates the importance of the nonlocal response of the phase-difference function. The purpose of this article is to review the results of our theoretical study of the nonlocal phase modification in one-dimensional junctions, the effect of this modification on the critical current and its dependence on the external magnetic field.

The outline of the rest of this article is as follows. In Sec. II, we set up the framework for our calculation by describing the model and deriving a modified Josephson equation that includes the effect of the laser beam. Although we explicitly consider junctions irradiated by a focused laser beam, our approach can also be applied to cases where electron beams are used. A few illustrative cases are worked out in Sec. III, along with discussions, and Sec. IV contains a brief conclusion.

II. MODEL

We consider junctions with geometry shown in Fig. 1. The irradiated regions are indicated as the shaded areas. In Fig. 1(a), a strip of the junction is irradiated while in Fig. 1(b), a spot of the junction is irradiated. We assume that the focused laser beam produces local changes in the junction parameters. That is, in the irradiated regions, the critical Josephson current density J is reduced by a constant amount, while the London penetration depth λ_L and the Josephson screening length λ_J are increased by a constant amount from their respective unperturbed values. This approximation is appropriate for beams of dimensions larger than the thermal-diffusion length which is typically of the order of tens of micrometers. For beams of smaller size, we believe that the above approximation still works provided that an effective perturbation of appropriate strength is used.

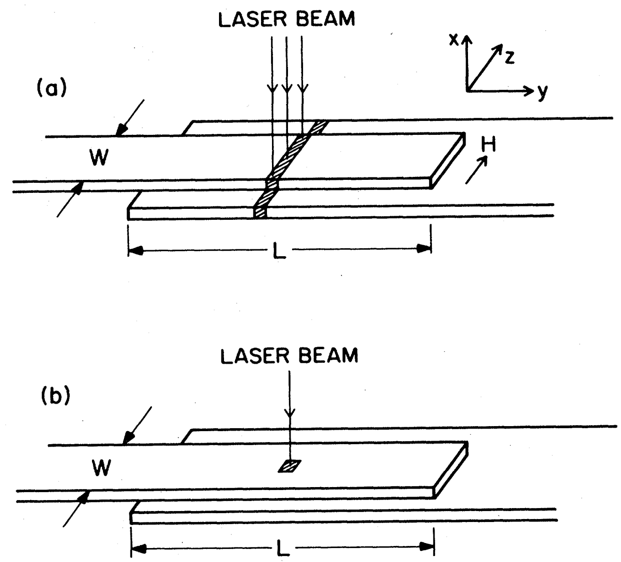

FIG. 1. Junction geometries considered. (a) A strip of the junction is irradiated by the laser beam. (b) A spot of the junction is irradiated.

With these in mind, we first derive a modified Josephson equation that governs the behavior of the phase-difference function $\phi(y,z)$. We begin with the Josephson relations:[6]

$$\frac{\partial\phi}{\partial y}=4e\lambda_L H_z/\hbar c \tag{2.1a}$$

and

$$\frac{\partial\phi}{\partial z}=-4e\lambda_L H_y/\hbar c\,, \tag{2.1b}$$

and the Maxwell equation,

$$\frac{\partial H_z}{\partial y}-\frac{\partial H_y}{\partial z}=\frac{4\pi}{c}J(y,z)\,. \tag{2.2}$$

Keeping in mind that λ_L and λ_J are now functions of space, combining Eqs. (2.1) and (2.2) we obtain

$$\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2}=\frac{1}{\lambda_J^2}\sin\phi+\frac{4e}{\hbar c}\left[H_z\frac{\partial\lambda_L}{\partial y}-H_y\frac{\partial\lambda_L}{\partial z}\right]. \tag{2.3}$$

For a given set of the boundary conditions, the phase-difference function can be calculated from Eq. (2.3), and the zero-voltage Josephson tunneling current is obtained from

$$I=\int dy\,dz\,J\sin\phi(y,z)\,. \tag{2.4}$$

Here the integration is on the junction area. Since the focused laser beam modifies λ_J and λ_L as well as J, it is evident from Eq. (2.3) that the phase difference function ϕ will also be modified by the laser beam. As we will see, this modification is nonlocal and can have a significant effect on the change of the junction critical current.

In the limit of weak laser intensities, we can separate the change of the junction critical current, induced by the focused laser beam, into two parts: one is due to the modification of the amplitude J while the other is due to the change of the phase difference function ϕ;

$$\Delta I_c = \Delta I_J + \Delta I_\phi \ , \tag{2.5a}$$

with

$$\Delta I_J = \int dy\, dz \Delta J \sin\phi_c(y,z) \ , \tag{2.5b}$$

and

$$\Delta I_\phi = \int dy\, dz\, J \cos\phi_c(y,z) \Delta\phi(y,z) \ . \tag{2.5c}$$

Here J is the unperturbed critical-current density, and ϕ_c is the phase difference function when the unperturbed junction is biased at the critical-current value. We notice that because of the local nature of ΔJ, ΔI_J has the same spatial dependence as the unperturbed-current density $J \sin\phi_c(y,z)$, while ΔI_ϕ can be quite complicated. A few cases are discussed in the next section.

III. CALCULATION AND DISCUSSION

In this section we present results for several illustrative cases in which short, medium-length, and long junctions are irradiated in different ways. Strong as well as weak laser intensities are considered so that nonlinear effects are taken into account. However, for simplicity, we only investigate one-dimensional junctions in which the width $W \ll \lambda_J$. The applied magnetic field is assumed to be in the z direction so that

$$\mathbf{H} = H\hat{z} \ . \tag{3.1}$$

A. Short junctions with a strip irradiated

The junction geometry is shown in Fig. 1(a). For a short junction, $L \ll \lambda_J$, so that the $\sin\phi$ term on the right-hand side can be neglected. Let $\Delta\lambda_L$ be the change of the London penetration depth in the irradiated region $(y_0 < y < y_0 + \epsilon)$. Then Eq. (2.3) reduces to

$$\frac{d^2\phi}{dy^2} = \frac{4eH\,\Delta\lambda_L}{\hbar c}[\delta(y - y_0) - \delta(y - y_0 - \epsilon)] \ . \tag{3.2}$$

This equation should be solved with the boundary conditions

$$\left.\frac{d\phi}{dy}\right|_{0,L} = \frac{4eH\lambda_L}{\hbar c} \ . \tag{3.3}$$

Therefore, $\phi(y)$ is equivalent to the electrostatic potential associated with two infinite uniformly charged plates. One with a charge density of $-(1/4\pi)(4eH\Delta\lambda_L/\hbar c)$, is located at $y = y_0$. The other, with the same amount of the opposite charge, is located at $y = y_0 + \epsilon$. The solution of $\phi(y)$ is well known:

$$\phi(y) = \begin{cases} k_B y + \phi_0, & 0 < y < y_0 \\ k_B' y + (k_B - k_B')y_0 + \phi_0, & y_0 < y < y_0 + \epsilon \\ k_B y + (k_B' - k_B)\epsilon + \phi_0, & y_0 + \epsilon < y < L \end{cases} \tag{3.4}$$

with $k_B \equiv 4eH\lambda_L/\hbar c$ and $k_B' \equiv 4eH(\lambda_H + \Delta\lambda_L)/\hbar c$.

When this is substituted into Eq. (2.4), the critical current can be obtained by adjusting the integration constant ϕ_0. The phase-difference functions for the perturbed and unperturbed cases are shown schematically in Fig. 2. We note that as long as $\Delta\lambda_L \neq 0$, the function $\phi(y)$ can be modified in a significant portion of the junction area.

We consider first laser beams with low intensities and small width so that $\Delta J/J$, $\Delta\lambda_L/\lambda_L$, and $\epsilon/L \ll 1$. It can be easily shown that in this case ϕ_0 is unchanged from its unperturbed value if the junction current is biased at its critical value. Let $\phi(y) = \phi_c(y) + \Delta\phi(y)$ with $\phi_c(y)$ being the unperturbed solution. Then $\Delta\phi(y)$ is given by

$$\Delta\phi(y) = \begin{cases} 0, & 0 < y < y_0 \\ \dfrac{4eH\,\Delta\lambda_L}{\hbar c}(y - y_0), & y_0 < y < y_0 + \epsilon \\ \dfrac{4eH\,\Delta\lambda_L}{\hbar c}\epsilon, & y_0 + \epsilon < y < L \ . \end{cases} \tag{3.5}$$

One may think that since the modification of the phase is $\Delta\phi(y) \sim (\Delta\lambda_L/\lambda_L)(\epsilon/L)$, its effects on the current density will be of the same order and is therefore much smaller than that due to the change of the local amplitude J which is of order $\Delta J/J$. However, one should also keep in mind that J is modified only in a small region of length ϵ. Therefore, its contribution to the modification of the (total) critical current of the junction would be proportional to $(\Delta J/J)(\epsilon/L)$, which may be comparable to that due to the phase modification.

Substituting Eq. (3.5) into Eq. (2.5) and recalling that ΔJ vanishes except in the irradiated region, the change of

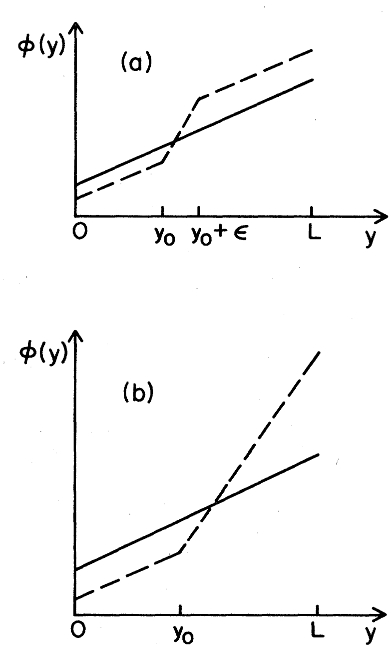

FIG. 2. Schematic drawing of the spatial dependences of the phase difference function ϕ in a short junction. Solid curves, ϕ when there is no laser irradiation. Dashed curves, ϕ when the laser irradiation is turned on. (a) A strip of the junction between $y > y_0$ and $y < y_0 + \epsilon$ is irradiated. (b) The area of $y > y_0$ is irradiated.

the critical current as a function of the beam position y_0 can be put into the form

$$\Delta I_c(y_0)=\Delta I_J(y_0)+\Delta I_\phi(y_0)\ , \tag{3.6a}$$

with

$$\frac{\Delta I_J(y_0)}{I_c}=\frac{\Delta J}{J}\frac{\epsilon}{L}\sin\phi_c(y_0) \tag{3.6b}$$

and

$$\frac{\Delta I_\phi(y_0)}{I_c}=\frac{\Delta\lambda_L}{\lambda_L}\frac{\epsilon}{L}\left\{(-1)^n\cos\left[\pi\left[\frac{\Phi}{\Phi_0}\right]\right]-\sin\phi_c(y_0)\right\}. \tag{3.6c}$$

Here Φ is the total flux in the junction in units of the flux quantum $\Phi_0=\hbar c/2e$ and n is the smallest positive integer greater than Φ/Φ_0. ΔI_J is due to the local modification of the amplitude J (the critical-current density), while ΔI_ϕ is due to the nonlocal phase modification. It reflects the increase of the magnetic flux that penetrates the junction due to the weakening of the superconductivity by the laser beam. The former has the same spatial dependence as the current distribution. The latter consists of two terms. One is independent of the beam position and depends upon the applied magnetic field as shown in Fig. 3. The other has the same spatial dependence as the current distribution and adds to ΔI_J. Therefore, in this case, the only unambiguous evidence for the existence of the nonlocal phase modification is the existence of a spatially independent change of ΔI_c.

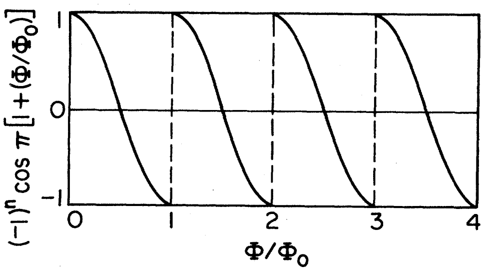

FIG. 3. Total flux in units of the flux quantum, $\Phi/\Phi_0(\Phi_0=\hbar c/2e)$ versus $(-1)^n\cos\{\pi[1+(\Phi/\Phi_0)]\}$.

For strong laser intensity, the local and nonlocal effects cannot be separated. To calculate $\Delta I_c(y_0)$, we substitute Eq. (3.4) into Eq. (2.4) and set $\partial I/\partial\phi_0=0$. This gives the equation that governs ϕ_0 at critical biases:

$$\begin{aligned}\tan\left[\phi_0+\frac{k_BL}{2}\right]=&\left\{\sin\left[k_B\frac{L}{2}\right]+\cos\left[k_B\frac{L}{2}+\Delta k_B\frac{\epsilon}{2}\right]\sin\left[\Delta k_B\frac{\epsilon}{2}\right]\right.\\&\left.+\left[\frac{1+\Delta J/J}{1+\Delta\lambda_L/\lambda_L}-1\right]\cos\left[k_B\frac{L}{2}-k_B\left[y_0+\frac{\epsilon}{2}\right]-\Delta k_B\frac{\epsilon}{2}\right]\sin\left[k_B'\frac{\epsilon}{2}\right]\right\}\\&\times\left\{\sin\left[k_B\frac{L}{2}+\Delta k_B\frac{\epsilon}{2}\right]\sin\left[\Delta k_B\frac{\epsilon}{2}\right]\right.\\&\left.-\left[\frac{1+\Delta J/J}{1+\Delta\lambda_L/\lambda_L}-1\right]\sin\left[k_B\frac{L}{2}-k_B\left[y_0+\frac{\epsilon}{2}\right]-\Delta k_B\frac{\epsilon}{2}\right]\sin\left[k_B\frac{\epsilon}{2}\right]\right\}^{-1},\end{aligned} \tag{3.7}$$

with $\Delta k_B\equiv k_B'-k_B$. The calculated ϕ_0 can then be used, along with Eq. (3.4), to obtain the critical current and hence $\Delta I_c(y_0)$ from Eq. (2.4). It was found that, as shown in Ref. 8, the spatial dependences of the laser-induced change in the junction critical current $\Delta I_c(y_0)$ and the unperturbed current distribution $J\sin\phi_0(y)$ are in good qualitative agreement. However, again there exists a spatially-independent constant shift of I_c which depends on the applied magnetic field and can make the maximum enhancement of the critical-current different from the maximum reduction.

In the above examples we have discussed linear and nonlinear effects produced by changing the laser intensity. Nonlinear effects can also be important if a laser beam with a wide width is used. In this case, it is possible to arrange conditions in which only linear effects associated with the local amplitude modification occur, and yet nonlinear effects of the nonlocal phase modification become important. For example, consider a weak laser beam with a wide width so that all of the junction where $y>y_0$ is irradiated. The solution for $\phi(y)$ is shown schematically in Fig. 2(b). In this case, the junction critical current will exhibit a y_0 dependence which depends in a nonlinear way on the nonlocal phase modifications.

B. Short junctions with a spot irradiated

The junction configuration for this case is shown in Fig. 1(b) with $L\ll\lambda_J$. Let the irradiated area be a square bounded by $y=y_0$, $y_0+\epsilon$ and $z=z_0$, $z_0+\epsilon'$. Because of the loss of the symmetry in the z direction, we have a two-dimensional problem. Setting $\phi(y,z)=\phi_0(y,z)$

$+\Delta\phi(y,z)$, the modified Josephson equation becomes

$$\frac{\partial^2\Delta\phi}{\partial y^2}+\frac{\partial^2\Delta\phi}{\partial z^2}=\left[k_B+\frac{\partial\Delta\phi}{\partial y}\right]\frac{1}{\lambda_L}\frac{\partial\lambda_L}{\partial y}+\frac{\partial\Delta\phi}{\partial z}\frac{1}{\lambda_L}\frac{\partial\lambda_L}{\partial z}$$
$$\simeq k_B\frac{1}{\lambda_L}\frac{\partial\lambda_L}{\partial y}$$
$$=k_B\frac{\Delta\lambda_L}{\lambda_L}[\delta(y-y_0)-\delta(y_0-y_0-\epsilon)]$$
$$\times[\Theta(z-z_0)-\Theta(z-z_0-\epsilon')]\ . \tag{3.8}$$

Here δ and Θ are, respectively, the Dirac delta function and the Heaviside step function. The boundary conditions are

$$\frac{\partial\phi}{\partial y}=0 \quad \text{for } y=0 \text{ and } L_y \tag{3.9a}$$

and

$$\frac{\partial\phi}{\partial z}=0 \quad \text{for } z=0 \text{ and } L_z\ . \tag{3.9b}$$

To obtain the second equality in Eq. (3.8), we have neglected the magnetic field generated by the inhomogeneity induced by the laser beam. Again, $\Delta\phi(y,z)$ is equivalent to the electrostatic potential of two strips of uniformly charged plates. One has density $-(1/4\pi)k_B(\Delta\lambda_L/\lambda_L)$ and is located at $(y=y_0, z_0<z<z_0+\epsilon')$. The other has the same amount of the opposite charge and is located at $(y=y_0+\epsilon, z_0<z<z_0+\epsilon')$. For a strong laser beam, the laser-induced change of the magnetic field cannot be neglected, and the "charge distribution" on the capacitor must be calculated self-consistently.

The electrostatic potential of an isolated dipole strip located at a position characterized by (y_0,z_0) can be easily calculated. One obtains

$$\tilde{\phi}(y,z;y_0,z_0)=-\frac{1}{4\pi}\left[\frac{\Delta\lambda_L}{\lambda_L}\right]k_B\left[\epsilon'\ln\left|\frac{y-y_0}{y-y_0-\epsilon}\right|\right.$$
$$-\tfrac{1}{2}|y-y_0|[Z_2\ln(1+Z_2^2)-Z_1\ln(1+Z_1^2)+2(\tan^{-1}Z_2-\tan^{-1}Z_1)]$$
$$\left.+\tfrac{1}{2}|y-y_0-\epsilon|[Z_4\ln(1+Z_4^2)-Z_3\ln(1+Z_3^2)+2(\tan^{-1}Z_4-\tan^{-1}Z_3)]\right]\ . \tag{3.10a}$$

Here

$$Z_1=\frac{z-z_0}{|y-y_0|},\quad Z_2=\frac{z-z_0-\epsilon'}{|y-y_0|},$$
$$Z_3=\frac{z-z_0}{|y-y_0-\epsilon|},\quad Z_4=\frac{z-z_0-\epsilon'}{|y-y_0-\epsilon|}\ .$$

The boundary conditions [Eqs. (3.9a) and (3.9b)] can be taken into account by using the method of images. In this way we obtain

$$\Delta\phi(y,z)=\sum_{m,n}[\tilde{\phi}(y,z;nL_y+y_0,mL_z+z_0)$$
$$+\tilde{\phi}(y,z;nL_y+y_0,mL_z-z_0-\epsilon')$$
$$-\tilde{\phi}(y,z;nL_y-y_0-\epsilon,mL_z+z_0)$$
$$-\tilde{\phi}(y,z;nL_y-y_0-\epsilon,mL_z-z_0-\epsilon')]\ . \tag{3.10b}$$

Here m and n are even integers which run from $-\infty$ to $+\infty$.

Various scanning situations of either fixed y or fixed z have been considered for junctions in different applied magnetic fields. The change of the junction critical current induced by this modification of the phase is calculated from Eq. (2.5c). The results[8] are found to be similar to those discussed in subsection A. Again, aside from a spatially-independent component in ΔI_c, the spatial dependences of $\Delta I_c(y,z)$ and the current distribution are qualitatively the same. However, this does not mean that the modified phase-difference function is the same as in the subsection A. They are different, but only within a distance of order ϵ from the spot irradiated by the laser beam. Therefore, it is the "long-range" response of the phase modification that is responsible for ΔI_ϕ, and of importance here are the strength and the location of the "dipole." If the beam size is smaller than the thermal diffusion length so that the parameters λ_J, λ_L, and J are not constants in the irradiated region, our model is still applicable, provided we use an effective perturbation of appropriate strength.

C. Long junctions with a strip irradiated

In this case, L is either larger or comparable to λ_J and the $\sin\phi$ term in Eq. (2.3) cannot be neglected. Thus we must solve the equation

$$\frac{d^2\phi}{dy^2}=\frac{1}{\lambda_J^2}\sin\phi+\left[\frac{4e}{\hbar c}\right]H(y)\frac{d\lambda_L}{dy}\ , \tag{3.11}$$

with the boundary conditions[5]

$$H(L)-H(0)=\frac{4\pi}{c}I \tag{3.12a}$$

and

$$H(L)+H(0)=2H\ . \tag{3.12b}$$

Here $H(y)$ is the local magnetic field at y including the applied field, self-field, and field change induced by the laser beam. The laser beam separates the junction into three regions. Within each region λ_J and λ_L are constants, although their values may be different for each region. Therefore, the solution for ϕ can be written in terms of the Jacobian elliptical functions.[5] We first consider the case of $k<1$. In the presence of the laser beam, the solution for ϕ can be written as

$$\phi(y)=\begin{cases} 2\sin^{-1}\mathrm{cn}\left[\dfrac{y-C_1}{k_1\lambda_J}\middle| k_1^2\right], & 0<y<y_0 \\ 2\sin^{-1}\mathrm{cn}\left[\dfrac{y-C_2}{k_2\lambda_J'}\middle| k_2^2\right], & y_0<y<y_0+\epsilon \\ 2\sin^{-1}\mathrm{cn}\left[\dfrac{y-C_3}{k_3\lambda_J}\middle| k_3^2\right], & y_0+\epsilon<y<L\ . \end{cases} \tag{3.13}$$

The case of $k>1$ can be solved in a similar manner and will be discussed next. The continuity conditions for ϕ and $H=(\hbar c/4e\lambda_L)/(\partial\phi/\partial y)$ at the boundaries $y=y_0$ and $y=y_0+\epsilon$ require that

$$\mathrm{cn}\left[\frac{y_0-C_2}{k_2\lambda_J'}\middle| k_2^2\right]=\mathrm{cn}\left[\frac{y_0-C_1}{k_1\lambda_J}\middle| k_1^2\right], \tag{3.14a}$$

$$\frac{1}{\lambda_L' k_2\lambda_J'}\,\mathrm{dn}\left[\frac{y_0-C_2}{k_2\lambda_J'}\middle| k_2^2\right]=\frac{1}{\lambda_L k_1\lambda_J}\,\mathrm{dn}\left[\frac{y_0-C_1}{k_1\lambda_J}\middle| k_1^2\right], \tag{3.14b}$$

$$\mathrm{cn}\left[\frac{y_0+\epsilon-C_2}{k_2\lambda_J'}\middle| k_2^2\right]=\mathrm{cn}\left[\frac{y_0+\epsilon-C_3}{k_3\lambda_J}\middle| k_3^2\right], \tag{3.14c}$$

and

$$\frac{1}{\lambda_L' k_2\lambda_J'}\,\mathrm{dn}\left[\frac{y+\epsilon-C_2}{k_2\lambda_J'}\middle| k_2^2\right] =\frac{1}{\lambda_L k_3\lambda_J}\,\mathrm{dn}\left[\frac{y+\epsilon-C_3}{k_3\lambda_J}\middle| k_3^2\right]. \tag{3.14d}$$

Using Eq. (3.13), one can easily show that the two boundary conditions [Eqs. (3.12a) and (3.12b)] become

$$\frac{1}{k_3}\,\mathrm{dn}\left[\frac{L-C_3}{k_3\lambda_J}\middle| k_3^2\right]+\frac{1}{k_1}\,\mathrm{dn}\left[\frac{-C_1}{k_1\lambda_J}\middle| k_1^2\right]=\frac{4e\lambda_L\lambda_J}{\hbar c}H \tag{3.15a}$$

and

$$\frac{1}{k_3}\,\mathrm{dn}\left[\frac{L-C_3}{k_3\lambda_J}\middle| k_3^2\right]-\frac{1}{k_1}\,\mathrm{dn}\left[\frac{-C_1}{k_1\lambda_J}\middle| k_1^2\right] =\frac{4\pi}{c}\left[\frac{2e\lambda_L\lambda_J}{\hbar c}\right]I\ . \tag{3.15b}$$

Here λ_L and λ_J are, respectively, the unperturbed values for the London penetration depth and the Josephson screening length, and λ_L' and λ_J' are the corresponding values in the irradiated region.

To obtain the critical current we first eliminate five of the six parameters (k_1, k_2, k_3, C_1, C_2, and C_3) using Eqs. (2.14) and (2.15a). I_c is then calculated by varying the remaining parameter to find the maximum value for the junction current I. This procedure has been carried out numerically for the case of strong laser powers. For weak powers, an analytical expression for the modified critical current can be obtained perturbatively as shown below.

In the weak-power limit, both $\Delta\lambda_L/\lambda_L$ and $\Delta\lambda_J/\lambda_J$ are small, and one can expand in these parameters. Ignoring higher-order contributions, we obtain from Eqs. (2.14a) and (2.14b)

$$\Delta k_2=\Delta k_1=f_k(y_0) \tag{3.16a}$$

and

$$\Delta C_2=\Delta C_1+f_C(y_0)\ , \tag{3.16b}$$

with

$$f_k(y_0)=-\frac{\left[\dfrac{\Delta\lambda_L}{\lambda_L}+\dfrac{\Delta\lambda_J}{\lambda_J}\right]\mathrm{dn}\dfrac{\partial\,\mathrm{cn}}{\partial C_0}}{\dfrac{\partial\,\mathrm{cn}}{\partial k_0}\dfrac{\partial\,\mathrm{dn}}{\partial C_0}-\left[\dfrac{\partial\,\mathrm{dn}}{\partial k_0}-\dfrac{\mathrm{dn}}{k_0}\right]\dfrac{\partial\,\mathrm{cn}}{\partial C_0}} \tag{3.17a}$$

and

$$f_c(y_0)=-\frac{\Delta\lambda_L}{\lambda_L}(y_0-C_0) +\frac{\left[\dfrac{\Delta\lambda_L}{\lambda_L}+\dfrac{\Delta\lambda_J}{\lambda_J}\right]\mathrm{dn}\dfrac{\partial\,\mathrm{cn}}{\partial k_0}}{\dfrac{\partial\,\mathrm{cn}}{\partial k_0}\dfrac{\partial\,\mathrm{dn}}{\partial C_0}-\left[\dfrac{\partial\,\mathrm{dn}}{\partial k_0}-\dfrac{\mathrm{dn}}{k_0}\right]\dfrac{\partial\,\mathrm{cn}}{\partial C_0}}\ . \tag{3.17b}$$

Here k_0 and C_0 are the parameters corresponding to the unperturbed case. We have defined $\Delta k_i\equiv k_i-k_0$ and $\Delta C_i\equiv C_i-C_0$. The functions dn and cn are shorthand notations for $\mathrm{dn}[(y_0-C_0)/k_0\lambda_J\,|\,k_0^2]$ and $\mathrm{cn}[(y_0-C_0)/k_0\lambda_J\,|\,k_0^2]$. Similarly, Eqs. (2.14c) and (2.14d) give

$$\Delta k_2=\Delta k_3+f_k(y_0+\epsilon) \tag{3.16c}$$

and

$$\Delta C_2=\Delta C_3+f_c(y_0+\epsilon)\ . \tag{3.16d}$$

In the same spirit, Eqs. (3.15a) and (3.15b) give

$$\frac{\partial\,\mathrm{dn}^L}{\partial C_0}\Delta C_3+\frac{\partial\,\mathrm{dn}^0}{\partial C_0}\Delta C_1+\left[\frac{\partial\,\mathrm{dn}^L}{\partial k_0}-\frac{\mathrm{dn}^L}{k_0}\right]\Delta k_3+\left[\frac{\partial\,\mathrm{dn}^0}{\partial k_0}-\frac{\mathrm{dn}^0}{k_0}\right]\Delta k_1=0 \tag{3.18a}$$

and

$$\Delta I=\frac{\hbar c^2}{8\pi e\lambda_I k_0\lambda_J}\left[\frac{\partial\,\mathrm{dn}^L}{\partial C_0}\Delta C_3-\frac{\partial\,\mathrm{dn}^0}{\partial C_0}\Delta C_1+\left[\frac{\partial\,\mathrm{dn}^L}{\partial k_0}-\frac{\mathrm{dn}^L}{k_0}\right]\Delta k_3-\left[\frac{\partial\,\mathrm{dn}^0}{\partial k_0}-\frac{\mathrm{dn}^0}{k_0}\right]\Delta k_1\right]. \tag{3.18b}$$

Here the superscripts L and 0 indicate that the Jacobian elliptical functions are evaluated at $y=L$ and $y=0$, respectively.

Eliminating all the parameters except for Δk_1 we obtain

$$\begin{aligned}\Delta I=&-\left[\frac{\hbar c^2}{8\pi e\lambda_L k_0\lambda_J}\right]\left\{[f_k(y_0+\epsilon)-f_k(y_0)]\left[2\frac{\partial\,\mathrm{dn}^0}{\partial C_0}\left[\frac{\partial\,\mathrm{dn}^L}{\partial k_0}-\frac{\mathrm{dn}^L}{k_0}\right]\Big/\left[\frac{\partial\,\mathrm{dn}^L}{\partial C_0}+\frac{\partial\,\mathrm{dn}^0}{\partial C_0}\right]\right]\right.\\&\left.+[f_c(y_0+\epsilon)-f_c(y_0)]\left[\left[2\frac{\partial\,\mathrm{dn}^L}{\partial C_0}\frac{\partial\,\mathrm{dn}^0}{\partial C_0}\Big/\left[\frac{\partial\,\mathrm{dn}^L}{\partial C_0}+\frac{\partial\,\mathrm{dn}^0}{\partial C_0}\right]\right]\right]\right\}\\&-\left[\frac{\hbar c^2}{8\pi e\lambda_L k_0\lambda_J}\right]\left\{\left[2\frac{\partial\,\mathrm{dn}^L}{\partial C_0}\left[\frac{\partial\,\mathrm{dn}^L}{\partial k_0}-\frac{\mathrm{dn}^L}{k_0}\right]\Big/\left[\frac{\partial\,\mathrm{dn}^L}{\partial C_0}+\frac{\partial\,\mathrm{dn}^0}{\partial C_0}\right]\right]\right.\\&\left.-\left[2\frac{\partial\,\mathrm{dn}^0}{\partial C_0}\left[\frac{\partial\,\mathrm{dn}^0}{2k_0}-\frac{\mathrm{dn}^0}{k_0}\right]\Big/\left[\frac{\partial\,\mathrm{dn}^L}{\partial C_0}+\frac{\partial\,\mathrm{dn}^0}{\partial C_0}\right]\right]\right\}\Delta k_1\,.\end{aligned} \tag{3.19}$$

If we choose (k_0,C_0) to be the set of parameters which gives the critical current in the unperturbed case, the coefficient of Δk_1 vanishes. Therefore, the change of the critical current is given by Eq. (3.19) with the term involving Δk ignored.

For the case of $k>1$, a similar calculation gives

$$\begin{aligned}\Delta I_c=&\left[\frac{\hbar c^2}{8\pi e\lambda_L k_0\lambda_J}\right]\left\{[f_k'(y_0+\epsilon)-f_k'(y_0)]\left[2\frac{\partial\,\mathrm{cn}^0}{\partial C_0}\left[\frac{\partial\,\mathrm{cn}^L}{\partial k_0}-\frac{\mathrm{cn}^L}{k_0}\right]\Big/\left[\frac{\partial\,\mathrm{cn}^L}{\partial C_0}+\frac{\partial\,\mathrm{cn}^0}{\partial C_0}\right]\right]\right.\\&\left.+[f_c'(y_0+\epsilon)-f_c'(y_0)]\left[2\frac{\partial\,\mathrm{cn}^L}{\partial C_0}\frac{\partial\,\mathrm{cn}^0}{\partial C_0}\Big/\left[\frac{\partial\,\mathrm{cn}^L}{\partial C_0}+\frac{\partial\,\mathrm{cn}^0}{\partial C_0}\right]\right]\right\},\end{aligned} \tag{3.20}$$

with

$$f_k'(y_0)=\frac{\left[\frac{\Delta\lambda_L}{\lambda_L}+\frac{\Delta\lambda_J}{\lambda_J}\right]\mathrm{cn}\frac{\partial\,\mathrm{dn}}{\partial C_0}}{\frac{\partial\,\mathrm{cn}}{\partial C_0}\frac{\partial\,\mathrm{dn}}{\partial k_0}-\frac{\partial\,\mathrm{dn}}{\partial C_0}\left[\frac{\partial\,\mathrm{cn}}{\partial k_0}-\frac{\mathrm{cn}}{k_0}\right]} \tag{3.21a}$$

and

$$f_c'(y_0)=-\frac{\Delta\lambda_J}{\lambda_J}(y_0-C_0)-\frac{\left[\frac{\Delta\lambda_L}{\lambda_L}+\frac{\Delta\lambda_J}{\lambda_J}\right]\mathrm{cn}\frac{\partial\,\mathrm{dn}}{\partial k_0}}{\frac{\partial\,\mathrm{cn}}{\partial C_0}\frac{\partial\,\mathrm{dn}}{\partial k_0}-\left[\frac{\partial\,\mathrm{cn}}{\partial k_0}-\frac{\mathrm{cn}}{k_0}\right]\frac{\partial\,\mathrm{dn}}{\partial C_0}}\,. \tag{3.21b}$$

Here

$$\mathrm{dn}\equiv\mathrm{dn}\left[\frac{y_0-C_0}{\lambda_J}\middle|\frac{1}{k_0^2}\right],\quad \mathrm{cn}\equiv\mathrm{cn}\left[\frac{y_0-C_0}{\lambda_J}\middle|\frac{1}{k_0^2}\right].$$

Therefore, the recipe for calculating the laser-induced change of the total critical current of the junction $\Delta I_c(y_0)$ in an applied field H is first to calculate the set of parameters (k_0,C_0) corresponding to the unperturbed junction biased at its critical-current value. This can be achieved by the method used in Ref. 5. This set of parameters can then be substituted into either Eq. (3.19) or Eq. (3.20), depending upon whether $k<1$ or $k>1$, to calculate $\Delta I_c(y_0)$. To obtain numerical values for $\Delta I_c(y_0)$, we still need two parameters, $(\Delta J/J)(\epsilon/L)$ and $(\Delta\lambda_L/\lambda_L)(\epsilon/L)$, which characterize the strength of the laser disturbance. The situation can be simplified to involve only one parameter by assuming that the laser beam elevates the local temperature[9] from T to T^*. Using the well-known relationships[10]

$$J(T)=\frac{\pi\Delta(T)}{2eR_N}\tanh\left[\frac{\Delta(T)}{2k_BT}\right] \tag{3.22a}$$

and

$$\lambda_L(T)=\lambda_L(0)/[1-(T/T_c)^4]^{1/2}\,, \tag{3.22b}$$

the parameters ΔJ and $\Delta\lambda_L$ can be calculated in terms of $\Delta T=T^*-T$. Here R_N is the junction resistance in the normal state.

It is found that $\Delta\lambda_L/\lambda_L$, although comparable for most temperatures, is smaller than $\Delta J/J$ except for temperature $T\gtrsim 0.9T_c$. Therefore, for low ambient temperature and

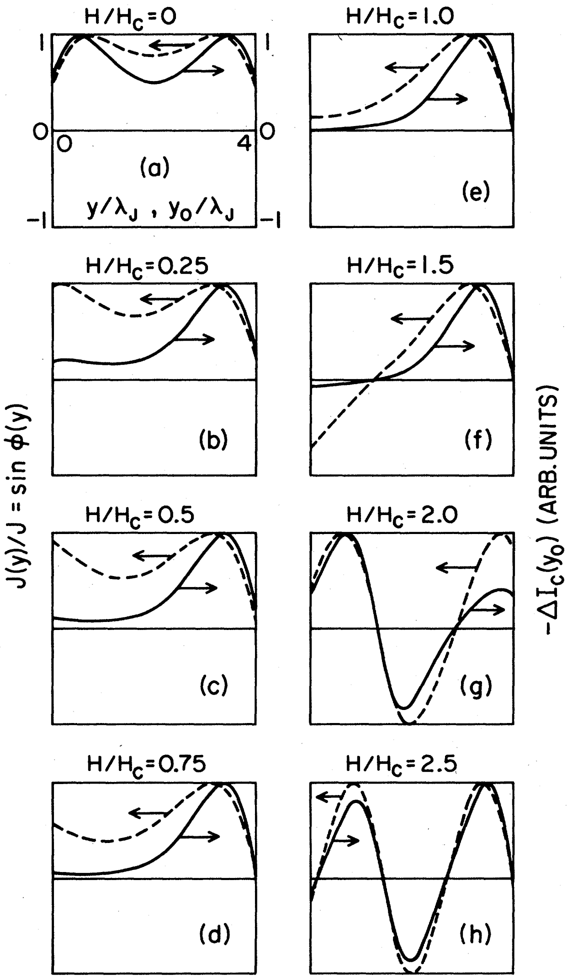

FIG. 4. Negative of the laser-induced modification of the junction critical current, $-\Delta I_c$, as a function of the beam position y_0 for various applied magnetic field H are shown as solid curves. The spatial dependence of the normalized current density, $\sin\phi(y)$, are plotted in dashed curves. $L=4\lambda_J$.

weak laser intensity, we can neglect $\Delta\lambda_L$.

In Fig. 4 we show results for a junction with length $L=4\lambda_J$ in various applied magnetic fields. The solid curves represent the normalized critical-current change $-\Delta I_c(y_0)$ (induced by weak laser irradiation between $y=y_0$ and $y=y_0+\epsilon$) calculated from Eqs. (3.19) and (3.20). We have set $\Delta\lambda_L=0$. For comparison, the normalized-current densities, $J\sin\phi(y)$, for the unperturbed junction biased at the critical-current values are plotted as the dashed curves. If the effect due to the phase modification was neglected, the solid curves and the dashed curves would overlap.

We see that although the solid curves do not overlap exactly with the dashed ones, for most applied magnetic fields the spatial dependences of $J\sin\phi(y)$ and $-\Delta I_c(y_0)$ are in good qualitative agreement. However, discrepancies do exist, and two points are worth noting. First, for $H/H_c=0.25$, 0.5, and 0.75, the large current densities on the left-hand edge of the junctions are not significantly reflected in $-\Delta I_c(y_0)$. Second, for large applied magnetic fields such as $H>2H_c$, the amplitudes of oscillation in $-\Delta I_c(y_0)$ are not uniform in contrast to those of $J\sin\phi(y)$, which are. In the case of $H=2H_c$, we see that the local maximum of the solid curve on the right-hand edge is significantly smaller than that on the left-hand edge. The amplitude is a decreasing function of y_0. In general, we found that the amplitude can be either an increasing function or a decreasing function of y depending upon the applied magnetic field. For a field such that $\partial I_c/\partial H>0$ (<0), the amplitude of oscillation in $-\Delta I_c(y_0)$ is a decreasing (increasing) function of y_0. Only when H has values which give $\partial I_c/\partial H=0$ is the amplitude of oscillation uniform.

These discrepancies are more significant in a longer junction. In Fig. 5 we plotted the results for a junction of length $L/\lambda_J=10$, in various magnetic fields. Again, the solid curves are the laser-induced changes of the junction critical current $-\Delta I_c(y_0)$ calculated from Eqs. (3.19) and (3.20). The dashed curves are the current distributions of the junction unperturbed by the laser beam. The discrepancies described above are evident.

In Fig. 6, the normalized critical current is shown as a function of the applied magnetic field H in units of H_c. For magnetic fields $H/H_c=1.25$, 1.75, and 2.25, we have $\partial I_c/\partial H<0$. In the corresponding curves for $-\Delta I_c(y_0)$ shown in Fig. 5, the amplitudes of oscillation increase with y_0. On the other hand, for $H/H_c=1.5$ and 2.0, $\partial I_c/\partial H>0$, and the solid curves shown in Figs. 5(g) and 5(i) have oscillation amplitudes which decrease with y_0.

The fact that for applied magnetic fields smaller than H_c, the large junction-current density near the left edge of the junction does not show up in the $-\Delta I_c(y_0)$ curves can be understood as follows. Consider a long junction such that $L\gg\lambda_J$. When the applied magnetic field satisfied $H_c>H>0$, the magnetic field and the tunneling current deep inside the junction vanish due to the Meissner effect. The current is concentrated on the edges of the junction. For $H=0$, the total currents carried by the left-hand edge equals that by the right-hand edge and is given by $2\lambda_J J$. For $H\neq 0$, the total current carried by the left-hand edge is reduced from its maximum value to $2\lambda_J J-(c/4\pi)H$ to assure the Meissner effect. The total current carried by the right-hand edge is unchanged to maximize the total current flowing through the junction. When a strip of the left-hand edge is irradiated, the maximum current that can flow through the left-hand edge of the junction is reduced. However, as long as it is still larger than $2\lambda_J J-(c/4\pi)H$, the value required to maintain the Meissner effect, the total current carried by the left-hand junction, will be $2\lambda_J J-(c/4\pi)H$, and the total junction critical current will be unchanged. Therefore, the large current density on the left-hand edge will not be reflected in $-\Delta I_c(y_0)$. On the other hand, if the laser beam is irradiated on the right-hand edge of the junction, the maximum total current which can flow through the right-hand junction is reduced. This in turn will further reduce the total current flowing through the left-hand of the junction to maintain the Meissner effect. Hence the junction critical current is reduced.

When the laser intensity is increased, we expect non-

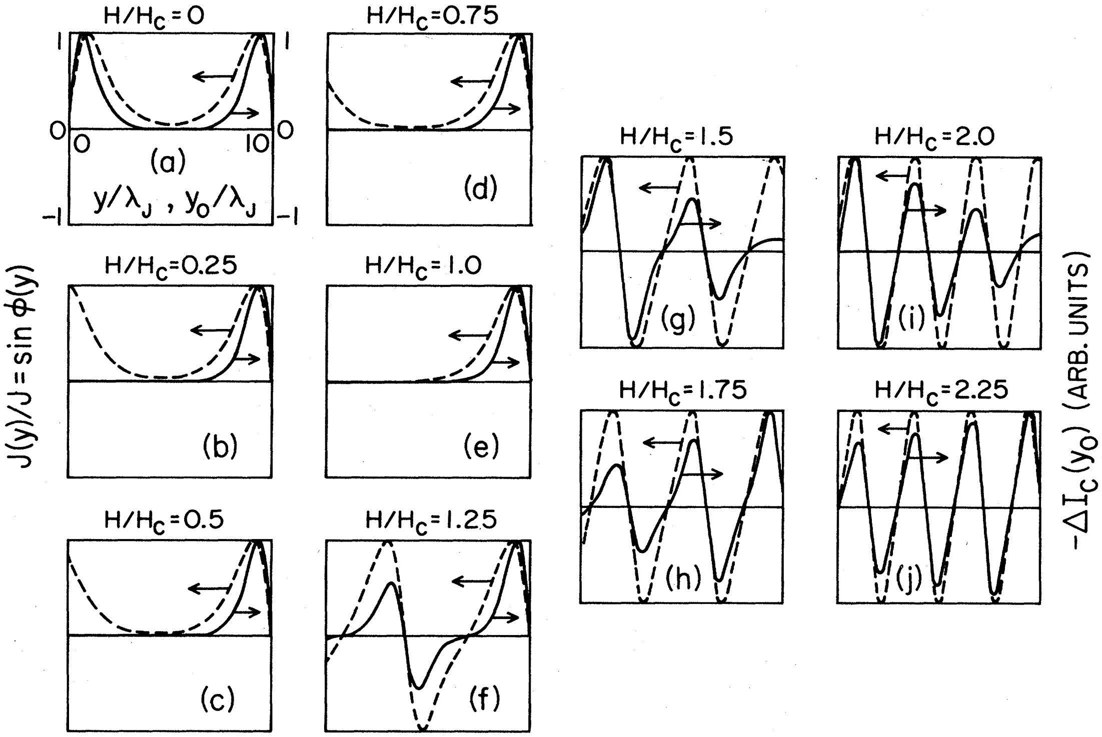

FIG. 5. Spatial dependences of the normalized $J(y)$ and $-\Delta I_c(y_0)$ for a junction with $L/\lambda_J=10$.

linear effects to appear. It is interesting to see if these can modify the result significantly. The laser-induced modification of the total junction current when a strip of the junction at $y_0<y<y_0+\epsilon$ is irradiated by a laser is shown in Figs. 7 and 8. The ambient temperature is taken to be zero and $\epsilon/L=\frac{1}{20}$. The solid curves correspond to the case of weak laser intensity. They are reproduced from those shown in Figs. 4 and 5. The dotted curves correspond to the case of strong laser intensity when the junction films are driven into a final state of temperature

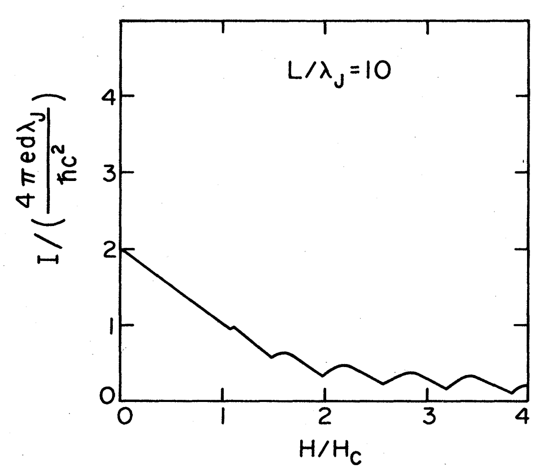

FIG. 6. Normalized critical current as a function of the applied magnetic field in units of $H_c=\hbar c/2ed\lambda_J$ for a junction of $L/\lambda_J=10$.

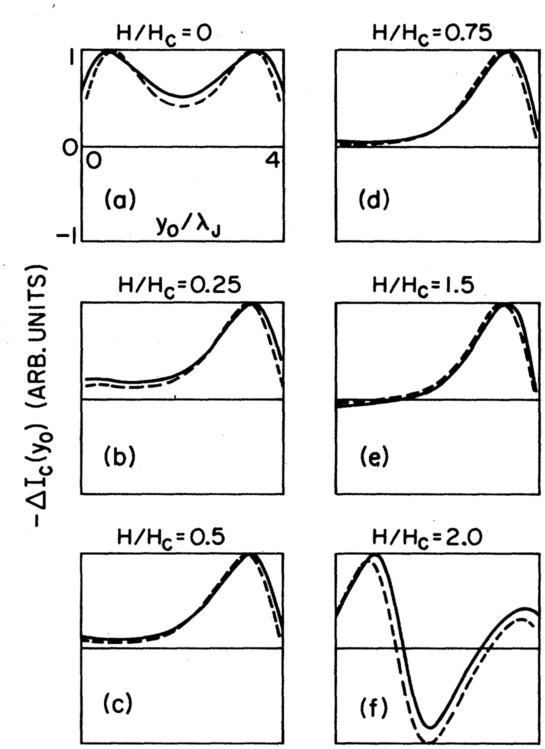

FIG. 7. Comparison of the spatial dependences of $-\Delta I_c(y_0)$ for cases of low laser power (solid curves) and high laser power (dotted curves). The parameters used are $L/\lambda_J=4$, $\epsilon/L=\frac{1}{20}$ and the elevated temperature $T_f=0.76T_c$. For details, see text.

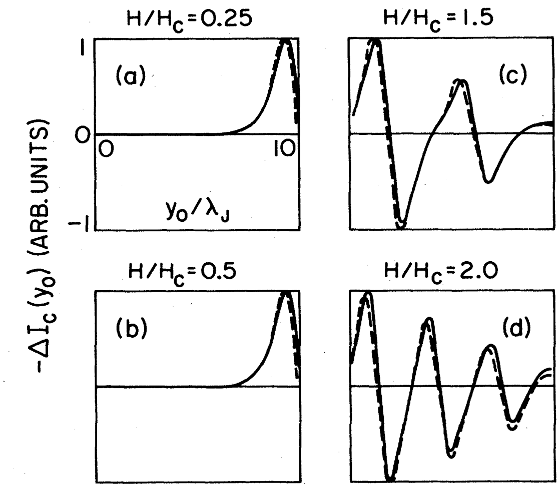

FIG. 8. Comparison of the spatial dependences of $-\Delta I_c(y_0)$ for cases of low laser power (solid curves) and high laser power (dotted curves). The parameters used are $L/\lambda_J=10$, $\epsilon/L=\frac{1}{20}$, and the elevated temperature $T_f=0.76T_c$. For details, see text.

$T_f=0.76T_c$. Hence $\Delta J/J=-0.46$ and $\Delta\lambda_L/\lambda_L=0.23$. The dotted curves have the same qualitative behavior as the dashed curves.

D. Semi-infinite junction

As a last example, we consider a semi-infinite junction located at $y\geq 0$. A laser beam irradiates a strip of the junction between y_0 and $y_0+\epsilon$. In this case, the boundary condition Eq. (3.12b) has to be replaced by $\phi(y\rightarrow\infty)=0$. Therefore, Eqs. (3.19) and (3.20) cannot be used to calculate the change of the critical junction current induced by the laser beam. However, the same method used in subsection C can be adopted here.

Before we consider the effect of the laser beam, let us first briefly review the results for an unperturbed junction. The general solution for $\phi(y)$, as mentioned before, can be expressed in terms of the Jacobian elliptical function

$$\phi(y)=2\sin^{-1}\text{cn}\left[\frac{y-C_0}{k_0\lambda_J}\middle| k_0^2\right] . \tag{3.23}$$

The boundary condition $\phi(y\rightarrow\infty)=0$ also implies $d\phi/dy\,|_{y\rightarrow\infty}=0$. This requires $k_0=1$ and hence

$$\begin{aligned} I&=\int_0^\infty dy\, J\sin\phi(y)\\ &=-2\int_0^\infty dy\, J\,\text{sn}\left[\frac{y-C_0}{\lambda_J}\middle| 1\right]\text{cn}\left[\frac{y-C_0}{\lambda_J}\middle| 1\right]\\ &=-2\int_0^\infty dy\, J\,\text{sech}\left[\frac{y-C_0}{\lambda_J}\right]\tanh\left[\frac{y-C_0}{\lambda_J}\right]\\ &=-2J\lambda_J\,\text{sech}\left[\frac{C_0}{\lambda_J}\right] . \end{aligned} \tag{3.24}$$

When $C_0=0$ and $k_0=1$, the tunneling current has its critical value,

$$|I_c|=2J\lambda_J . \tag{3.25}$$

Now we turn on the weak laser beam to irradiate a region between y_0 and $y_0+\epsilon$, and calculate the modification of the critical current.

The Meissner effect requires

$$H(y\rightarrow\infty)\sim(d\phi/dy)\,|_{y\rightarrow\infty}=0 ,$$

and hence $k_3=1$. The total junction current is therefore given by [from Eq. (3.12a)]

$$\begin{aligned} I&=-\frac{c}{4\pi}H(y=0)\\ &=-\frac{\hbar c^2}{8\pi e\lambda_L\lambda_J}\frac{1}{k_1}\text{dn}\left[\frac{-C_1}{k_1\lambda_J}\middle| k_1^2\right] . \end{aligned} \tag{3.26}$$

For weak laser power, we adopt a perturbative method. Expanding the parameters (k_1,C_1) with respect to their equilibrium values (k_0,C_0) and keeping the leading terms, we obtain

$$\Delta I_c(y_0)=\frac{\hbar c^2}{8\pi e\lambda_L\lambda_J}\Delta k_1=2J\lambda_J\Delta k_1 . \tag{3.27}$$

Here we have used the facts that

$$\text{dn}\left[-\frac{C_1}{k_1\lambda_J}\middle| k_1^2\right]=1 ,$$

and its derivatives with respect to both C_1 and k_1 vanish at $(C_1=0,\ k_1=1)$. The dependence on y_0 is implicitly in Δk_1 which must be calculated from Eqs. (3.16a), (3.16c), and (3.17a). After the details are worked out, one finds

$$\begin{aligned} \frac{\Delta I_c(y_0)}{-2J\lambda_J}&=f_k(y_0)-f_k(y_0+\epsilon)\\ &\simeq-\left[\frac{\Delta\lambda_L}{\lambda_L}-\frac{\Delta J}{J}\right]\frac{\epsilon}{\lambda_J}\text{sech}^2\left[\frac{y_0}{\lambda_J}\right]\tanh\left[\frac{y_0}{\lambda_J}\right],\\ &\qquad y_0>0 . \end{aligned} \tag{3.28}$$

To obtain the last equality, we have assumed $\epsilon\ll\lambda_J$.

The spatial dependence of $-\Delta I_c(y_0)$ is therefore not identical to that of the unperturbed-current distribution $J\sin\phi(y)$ given by Eq. (3.24), although qualitatively they have the same behavior.

As a check, one can show that Eq. (3.27) gives the correct result in the limit when the width of the beam is large so that the whole junction is under laser irradiation. Replacing ϵ by dy_0 and integrating from $y_0=0$ to $y_0=\infty$, one obtains the total ΔI_c to be $I_c(\Delta\lambda_L/\lambda_L+\Delta\lambda_J/\lambda_J)$. This, to the lowest order, is just $-\Delta(2J\lambda_J)$ as expected when the relationship of

$$\Delta\lambda_L/\lambda_L\simeq-2\Delta\lambda_J/\lambda_J-\Delta J/J$$

is used. The functional form is also found in excellent agreement with the $\Delta I_c(y_0)$ curve for the junction of $L/\lambda_J=10$ and $H=0$.

VII. CONCLUSION

We have studied the nonlocal response of one-dimensional Josephson tunnel junctions to a focused laser beam. Its effect on the critical junction current was calculated as a function of the laser-beam position and the applied magnetic field. For short junctions ($L \ll \lambda_J$), the nonlocal effect is proportional to $\Delta\lambda_L/\lambda_L$. When the ambient temperature $T \ll T_c$ and the laser intensity is low enough so that $\Delta\lambda_L/\lambda_L \ll \Delta J/J \ll 1$, the nonlocal effect is negligible. In this case, the measured laser-induced change of the junction critical current as a function of the beam position y_0, $-\Delta I_c(y_0)$, is proportional to the current distribution of the junction when it is unperturbed by the laser and biased at its critical-current value. When the ambient temperature is high or the laser intensity is strong so that the effective temperature of the irradiated region $T_f \gtrsim 0.7T_c$, nonlocal effect becomes important. Under these conditions, $-\Delta I_c(y_0)$ has a term which has the same spatial dependence as the current distribution plus a spatially-independent term. Since, for a short junction the amplitude of oscillation in $\Delta T_c(y_0)$ contributed by the local modification of J by the laser beam is uniform, the spatially-independent term contributed to $-\Delta I_c$ by the nonlocal phase modulation will make the peak height different from the dip size, and hence the presence of this feature is a signature of the existence of the nonlocal phase modulation.

For long junctions, the nonlocal phase response can have dramatic effect when the applied magnetic field does not exceed the critical value $H_c = \hbar c/(2ed\lambda_J)$. Specifically, for H between zero and H_c, the calculated spatial dependence of the laser-induced critical-current change $-\Delta I_c(y_0)$ is relatively insensitive to the applied magnetic field and is qualitatively similar to the current distribution for $H = H_c$. Therefore, $-\Delta I_c(y_0)$ does not reflect the current distribution near the left-hand edge of the junction, which depends upon the applied field strongly. Furthermore, the amplitude of oscillation in $-\Delta I_c(y_0)$ for $H > H_c$ is found, in general, not to be uniform in space. It can be either an increasing function of y_0 or a decreasing function of y_0, depending upon the applied magnetic field. Therefore, the detection of the nonlocal phase modulation induced by a focused laser beam should be easy in a long junction.

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