Heat capacity of free electrons at the degenerate-nondegenerate transition

G. Nimtz and J. P. Stadler
II. Physikalisches Institut, Universität zu Köln, Zülpicher Strasse 77, 5000 Köln 41, Federal Republic of Germany
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In this Brief Report the heat capacity of an electron gas at the degenerate-nondegenerate transition is presented. The values are deduced from hot-carrier data of InSb with $\approx 10^{14}$ electrons/cm^3 determined by Maneval, Zylberstejn, and Budd.

The heat capacity of an electron gas is found by differentiating the energy U of a system of electrons with respect to temperature T,

$$C_v = \frac{dU}{dT} = \int_0^\infty d\epsilon(\epsilon-\epsilon_F)\frac{df}{dT}D(\epsilon) \ , \tag{1}$$

where ϵ and ϵ_F are the particle energy and the Fermi energy, respectively, f the Fermi-Dirac function, and $D(\epsilon)$ the density of orbitals as a function of energy.[1] This integral is plotted as a function of temperature in Fig. 1. In the degenerate temperature region the limiting form

$$C_v = \gamma T \tag{2a}$$

holds, where γ is a constant. At high temperatures ($T \gtrsim T_F$; T_F is the Fermi temperature) the gas becomes nondegenerate, and the heat capacity is independent of temperature,

$$C_v = \tfrac{3}{2}nk \ , \tag{2b}$$

where n is the number of particles and k is the Boltzmann constant.

In ordinary metals the values of the Fermi temperature are of the order of 5×10^4 K. Therefore not any experimental data of the heat capacity of a nondegenerate electron gas are available. However, the experimental situation may be quite different in a semiconductor, where the carrier density can be small so that the value of the Fermi temperature is very low if compared with those of metals.

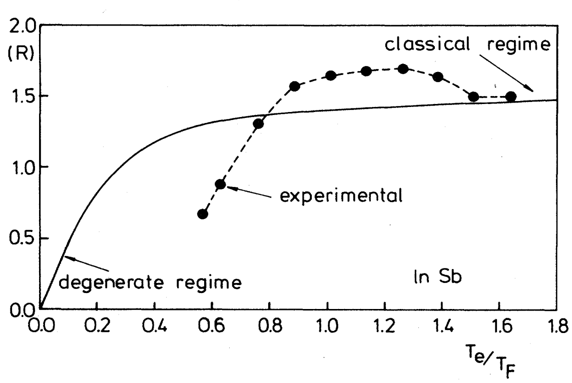

FIG. 1. Theoretical values of the normalized heat capacity of the electron gas. Circles are experimental values for 1.2×10^{14} electrons/cm^3 corresponding to a Fermi temperature of 8 K. R is the gas constant.

Recently, we came across two papers on hot-electron properties in InSb which were already published 15 years ago.[2,3] We shall show in this Brief Report that the experimental data presented there allow calculation of the heat capacity in a temperature region where the electron gas goes from the degenerate to the nondegenerate state.

Maneval, Zylberstejn, and Budd[2] presented experimental values of the hot-electron energy relaxation time τ_ϵ of the electron gas as a function of temperature. In their experiment the electron gas is heated in electric fields from 4 to about 15 K. The temperature of the electron system was determined in a static method by comparing Ohmic and hot-carrier conductivity. Using the conductivity as a thermometer of the electron temperature, they obtained a relation between E and T_e. In an independent dynamic determination they calculated the average energy by integrating the explicit electric field dependence of current density and energy relaxation time. Since both determinations of T_e were in very good agreement, an analysis based on an average energy is a justified approximation. Therefore, the energy relaxation time, according to the simplified energy balance equation, is given by the ratio of thermal energy to electric power,[4]

$$\tau_\epsilon(T_e) = \frac{C_v \Delta T_e}{jE} \ , \tag{3}$$

where ΔT_e is the temperature increase of the electrons due to the electric field E and j is the current density.

In the second work, a paper by Szymanska and Maneval,[3] experimental values of the electric power supplied to the electrons and normalized to the resulting temperature increase of the electron gas

$$X(T_e) = \frac{jE}{nk\Delta T_e} \tag{4}$$

are given.

In both studies n-type InSb samples with carrier densities of 1.2×10^{14} electrons/cm^3 were investigated, corresponding to a Fermi temperature of about 8 K. Thus we can use the experimental data presented in the two papers in order to calculate the heat capacity as a function of the electron-gas temperature from the relation

$$C_v(T_e) = \tau_\epsilon(T_e)X(T_e)nk \ . \tag{5}$$

The heat capacity is plotted as a function of T_e/T_F in Fig. 1. The agreement with the theoretical data is surprisingly good if we take into consideration that both $\tau_\epsilon(T_e)$ and $X(T_e)$ are expected to have some experimental errors and that Eq. (3) represents an approximation, as τ_ϵ is a function of T_e. The transition from the degenerate region of temperatures to the classical region is well demonstrated. As far as we know, this calculation yields the first published experimental values of the heat capacity of a nondegenerate electron gas.

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