Localization effects in the scattering of light from a randomly rough grating

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(Received 4 September 1984)

We calculate the resonant scattering of p-polarized light incident on a randomly rough grating ruled on a medium characterized by a dielectric constant $\epsilon(\omega)=\epsilon_1(\omega)+i\epsilon_2(\omega)$, where $\epsilon_1(\omega)<-1$ and $|\epsilon_2(\omega)| \ll |\epsilon_1(\omega)|$. Particular emphasis is placed on determining the contribution to the scattering that arises from the localization of surface polaritons due to the surface roughness. This localization is found to contribute a maximum to the angular dependence of the intensity of the non-specularly reflected light in the antispecular direction.

Recent work on disordered systems of $d \leq 2$ dimensions has identified the terms in a diagrammatic expansion in the spatial disorder that are responsible for the localization of elementary excitations of the system.[1-3] These maximally crossed, or fan, diagrams were originally associated by Langer and Neal[4] with nonanalytic terms in the expansion of the electrical resistivity of an electron gas in the impurity concentration. Only more recently has a fuller understanding of the relationship of maximally crossed diagrams to localization phenomena been obtained, culminating in the self-consistent calculation by Vollhardt and Wölfle of the electrical conductivity and localization length of the two-dimensional electron gas.[3]

In this paper we apply the above-mentioned diagrammatic techniques to calculate the resonant scattering of light from surface polaritons on a randomly rough dielectric grating. The surface polaritons of the grating exhibit localization due to the surface roughness, i.e., they cease being propagating surface excitations. The localization in this case is an example of one-dimensional Anderson localization, because the electromagnetic field of the surface polariton decays exponentially with increasing distance from the dielectric-vacuum interface into each medium with a decay length that is smaller than its wavelength along the interface. The effects of this localization on the resonant scattering of light are obtained by including maximally crossed diagrams as intermediate states between the incident and reflected rays. The expansion in the surface roughness that we make is based on the unitarity- and reciprocity-preserving formulation of the problem given by Brown *et al.*[5,6]

We consider a grating whose profile is given by $x_3=\zeta(x_1)$. The region $x_3>\zeta(x_1)$ is vacuum, while the region $x_3<\zeta(x_1)$ is filled by an isotropic dielectric medium characterized by the complex dielectric constant $\epsilon(\omega)=\epsilon_1(\omega)+i\epsilon_2(\omega)$, with $\epsilon_1(\omega)<-1$ and $|\epsilon_2(\omega)| \ll |\epsilon_1(\omega)|$ at the frequency ω of the incident light.

The surface profile function $\zeta(x_1)$ is assumed to be a Gaussianly distributed random variable, with the following properties:

$$\langle \zeta(x_1)\rangle = 0 \ , \tag{1a}$$

$$\langle \zeta(x_1)\zeta(x_1')\rangle = \sigma^2 \exp\left[-\frac{|x_1-x_1'|^2}{a^2}\right] . \tag{1b}$$

The angular brackets denote an average over the ensemble of the realizations of $\zeta(x_1)$, and $\sigma^2=\langle \zeta^2(x_1)\rangle$ is the mean-square deviation of the surface from flatness. In evaluating higher-order moments of $\zeta(x_1)$ we assume that it is a Gaussianly distributed random variable.[7]

The light in our system is taken to be p polarized and the plane of incidence is the x_1x_3 plane. The magnetic field vector is then in the x_2 direction, $\mathbf{H}(\mathbf{x},t) = (0,H_2(x_1x_3|\omega),0)\exp(-i\omega t)$, and its single, nonzero component satisfies the equations

$$\left[\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_3^2}+\frac{\omega^2}{c^2}\right] H_2^{>}(x_1x_3|\omega)=0 \ , \quad x_3>\zeta(x_1) \tag{2a}$$

$$\left[\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_3^2}+\epsilon(\omega)\frac{\omega^2}{c^2}\right] H_2^{<}(x_1x_3|\omega)=0 \ , \quad x_3<\zeta(x_1) \ . \tag{2b}$$

The solution of Eq. (2a) in the region $x_3>\zeta(x_1)_{\max}$ that satisfies the boundary condition at infinity can be written in the form

$$H_2^{>}(x_1x_3|\omega)=e^{ikx_1-i\alpha_0(k\omega)x_3} + \int \frac{dq}{2\pi} R(q|k) e^{iqx_1+i\alpha_0(q\omega)x_3} \ , \tag{3}$$

where

$$\alpha_0(q\omega)=\left[\frac{\omega^2}{c^2}-q^2\right]^{1/2} , \quad q^2<\frac{\omega^2}{c^2} \tag{4a}$$

$$= i\left[q^2-\frac{\omega^2}{c^2}\right]^{1/2} , \quad q^2>\frac{\omega^2}{c^2} \tag{4b}$$

while $\alpha_0(k\omega)$ is real $(k^2<\omega^2/c^2)$. A similar expansion can be written for the refracted field in the dielectric medium. However, in a diffraction problem we are interested in only the electromagnetic field in the vacuum above the dielectric. It is therefore convenient to eliminate the field in the dielectric medium and to work with only the field in the vacuum. This can be done by an application of Green's theorem, the extinction theorem, and the Rayleigh hypothesis, in a manner described in Refs. 5 and 6. It is found in this way that the field in the dielectric medium enters the problem only indirectly, viz., through the boundary condition that $H_2^>(x_1,x_3\,|\,\omega)$, given by Eq. (3), satisfies on the surface $x_3=\zeta(x_1)$

$$\int dx_1 \int dp \frac{i}{\alpha(p\omega)} e^{ip(x_1-x_1')+i\alpha(p\omega)(x_3'-x_3)} \Bigg[H_2^>(x_1x_3\,|\,\omega)[-\zeta'(x_1)ip-i\alpha(p\omega)] - \epsilon(\omega)\left[-\zeta'(x_1)\frac{\partial}{\partial x_1}+\frac{\partial}{\partial x_3}\right] H_2^>(x_1x_3\,|\,\omega)\Bigg]_{x_3=\zeta(x_1)} =0 , \quad x_3'>\zeta(x_1') \tag{5}$$

where $\alpha(p\omega)=[\epsilon(\omega)(\omega^2/c^2)-p^2]^{1/2}$, with $\mathrm{Re}[\alpha(p\omega)]>0$, $\mathrm{Im}[\alpha(p\omega)]>0$. When the expansion given by Eq. (3) is substituted into Eq. (5), we obtain the integral equation satisfied by the reflection amplitude $R(q\,|\,k)$,

$$\int \frac{dq}{2\pi} \frac{I(\alpha(p\omega)-\alpha_0(q\omega)\,|\,p-q)}{\alpha(p\omega)-\alpha_0(q\omega)}[\alpha(p\omega)\alpha_0(q\omega)+pq]R(q/k)=\frac{I(\alpha(p\omega)+\alpha_0(k\omega)\,|\,p-k)}{\alpha(p\omega)+\alpha_0(k\omega)}[\alpha(p\omega)\alpha_0(k\omega)-pk] , \tag{6}$$

where

$$I(\alpha\,|\,Q)=\int dx_1 e^{-iQx_1}e^{-i\alpha\zeta(x_1)} . \tag{7}$$

Equation (6) can be recast as a standard scattering problem by writing[5,6]

$$R(p\,|\,k)=2\pi\delta(p-k)R_0(k)-2iG_0(p)T(p\,|\,k)G_0(k)\alpha_0(k) , \tag{8}$$

where

$$R_0(p)=\frac{\epsilon(\omega)\alpha_0(p\omega)-\alpha(p\omega)}{\epsilon(\omega)\alpha_0(p\omega)+\alpha(p\omega)} \tag{9}$$

is the Fresnel coefficient for reflection of p-polarized light from a flat dielectric surface, and

$$G_0(p)=\frac{i\epsilon(\omega)}{\epsilon(\omega)\alpha_0(p\omega)+\alpha(p\omega)} \tag{10}$$

is the surface-polariton Green's function for $\zeta(x_1)=0$. The scattering matrix $T(p\,|\,k)$ is postulated to satisfy the equation

$$T(p\,|\,k)=V(p\,|\,k)+\int \frac{dq}{2\pi} V(p\,|\,q)G_0(q)T(q\,|\,k) . \tag{11}$$

Equations (6), (8), and (11) serve to define the effective scattering potential $V(p\,|\,k)$. To leading order in $\zeta(x_1)$ we find that

$$V(p\,|\,k)=\frac{\epsilon(\omega)-1}{\epsilon(\omega)^2}\hat{\zeta}(p-k)[\epsilon(\omega)pk-\alpha(p\omega)\alpha(k\omega)] +O(\zeta^2) , \tag{12}$$

with $\hat{\zeta}(p)=\int dx\, \zeta(x)\exp(-ipx)$. It is also convenient to introduce the surface-polariton Green's function $G(p\,|\,k)$ for the rough surface as the solution of the Dyson equation

$$G(p\,|\,k)=2\pi\delta(p-k)G_0(k)+G_0(p)\int \frac{dq}{2\pi} V(p\,|\,q)G(q\,|\,k) \tag{13a}$$

$$=2\pi\delta(p-k)G_0(k)+G_0(p)T(p\,|\,k)G_0(k) . \tag{13b}$$

The scattering efficiency (the total scattered flux in the x_3 direction normalized by the total incident flux in the x_3 direction), averaged over the ensemble of realizations of $\zeta(x_1)$, is

$$I(q\,|\,k)=\frac{1}{L_1}\frac{\alpha_0(q\omega)}{\alpha_0(k\omega)}\langle\,|R(q\,|\,k)|^2\rangle , \quad q^2<\frac{\omega^2}{c^2} \tag{14}$$

where L_1 is the length of the surface $x_3=0$ in the x_1 direction. From Eqs. (8) and (14) we see that the efficiency for diffuse (i.e., nonspecular) scattering can be written as

$$I(q\,|\,k)_{\mathrm{diff}}=\frac{4\alpha_0(q\omega)\alpha_0(k\omega)}{L_1}|G_0(q)|^2 \times\langle\,|T(q\,|\,k)|^2\rangle_{\mathrm{diff}}|G_0(k)|^2 , \tag{15}$$

where $\langle\,|T(q\,|\,k)|^2\rangle_{\mathrm{diff}}$ is the contribution to $\langle\,|T(q\,|\,k)|^2\rangle$ that contains no factor of $2\pi\delta(q-k)$. From Eq. (13b) we see that this result can be reexpressed in the form

$$I(q\,|\,k)_{\mathrm{diff}}=\frac{4\alpha_0(q\omega)\alpha_0(k\omega)}{L_1}\langle\,|G(q\,|\,k)|^2\rangle_{\mathrm{diff}} , \tag{16}$$

so that we need only to obtain $\langle\,|G(q\,|\,k)|^2\rangle$ from Eq. (13a) to determine $I(q\,|\,k)$.

We note that $\langle\,|G(q\,|\,k)|^2\rangle$ is similar in form to the $k\to 0$ and $\omega\to 0$ limit of the configuration-averaged product of two one-electron Green's functions

$$\langle G^R(p+k/2,p'+k/2;E_F+\omega) \times G^A(p'-k/2,p-k/2;E_F)\rangle$$

considered by Vollhardt and Wölfle.[3] The calculation of $\langle |G(q|k)|^2\rangle$, as we shall see below, can be carried out by summing the same ladder and maximally crossed diagrams that were found to be relevant to the determination of the two-electron Green's functions. Unlike the electron calculations, however, the sums in the calculation of $\langle |G(q|k)|^2\rangle$ are not divergent due to the nonzero value of $\epsilon_2(\omega)$. The latter gives the surface polaritons a finite lifetime independent of any processes associated with the surface roughness. This situation is analogous to the elimination of the divergence in the $k\to 0$, $\omega\to 0$ limit of the density response function of a two-dimensional gas of noninteracting electrons in a random distribution of impurity potentials, when time-reversal symmetry is broken by the application of an external dc magnetic field.[3,8]

In order to calculate $\langle |G(q|k)|^2\rangle$ we first need to know the average of the single-particle Green's function $\langle G(q|k)\rangle$. This can be obtained by the procedure of Ref. 6. Since averaging restores infinitesimal translational invariance to our system we find that $\langle G(q|k)\rangle = 2\pi\delta(q-k)G(k)$, where $G(k)$ satisfies the Dyson equation

$$G(k)=G_0(k)+G_0(k)M(k)G(k)\ . \tag{17}$$

The self-energy $M(k)$ in the lowest-order self-consistent approximation is given by the pair of equations

$$\langle M(q|k)\rangle=\int\frac{dp}{2\pi}\langle V(q|p)G(p)V(p|k)\rangle\ , \tag{18a}$$

$$\langle M(q|k)\rangle=2\pi\delta(q-k)M(k)\ . \tag{18b}$$

The average appearing on the right-hand side of Eq. (18a) is found from Eqs. (16) and (12) to be

$$\begin{aligned}\langle V(q|p)V(p|k)\rangle &= 2\pi\delta(q-k)\pi^{1/2}a\sigma^2\left[\frac{\epsilon(\omega)-1}{\epsilon^2(\omega)}\right]^2\\ &\quad\times[\epsilon(\omega)kp-\alpha(k\omega)\alpha(p\omega)]^2\exp\left[-\frac{a^2}{4}(k-p)^2\right]\ .\end{aligned} \tag{19}$$

To solve the coupled Eqs. (17) and (18) we first note the result that follows from Eq. (10) in the limit $\epsilon_2\to 0$,

$$G_0(k)\cong\frac{C_1}{k-K_{sp}-i\Delta_\epsilon}-\frac{C_1}{k+K_{sp}+i\Delta_\epsilon}\ , \tag{20}$$

where for $K_{sp}\gg\Delta_\epsilon$

$$C_1=\frac{\epsilon_1(\omega)\sqrt{-\epsilon_1(\omega)}}{1-\epsilon_1^2(\omega)}\ , \tag{21a}$$

$$\Delta_\epsilon=\frac{\epsilon_2(\omega)K_{sp}}{2\epsilon_1(\omega)[\epsilon_1(\omega)+1]}\ , \tag{21b}$$

$$K_{sp}=\frac{\omega}{c}\left[\frac{\epsilon_1(\omega)}{\epsilon_1(\omega)+1}\right]^{1/2}\ . \tag{21c}$$

In Eq. (20) Δ_ϵ describes the damping of the surface polariton on a flat surface due to the loss mechanisms in the dielectric medium, i.e., to $\epsilon_2(\omega)$.

The result given by Eq. (20) combined with Eq. (17) suggests the following approximation for $G(k)$:

$$G(k)\cong\frac{C_1}{k-K_{sp}-i\Delta_{tot}}-\frac{C_1}{k+K_{sp}+i\Delta_{tot}}\ , \tag{22}$$

where $\Delta_{tot}=\Delta_\epsilon+\Delta_{sp}$ and $\Delta_{sp}=C_1\,\mathrm{Im}[M(K_{sp})]$. From the solution of Eqs. (18) we find that

$$\Delta_{sp}\cong 2\pi^{1/2}a\sigma^2C_1^2\left[\frac{\epsilon_1(\omega)-1}{\epsilon_1(\omega)}\right]^2K_{sp}^4\exp(-a^2K_{sp}^2)\ . \tag{23}$$

In Eqs. (22) and (23) Δ_{sp} describes the damping of the surface polariton by its roughness-induced conversion into radiative modes.

Using the averaging procedure outlined in Appendix A of Ref. 6 we can now write a Bethe-Salpeter equation for $\langle G(q|k)G^*(p|k)\rangle$,

$$\begin{aligned}\langle G(q|k)G^*(p|k)\rangle &= 2\pi\delta(q-k)2\pi\delta(p-k)|G(k)|^2\\ &\quad+G(q)G^*(p)\int\frac{dr}{2\pi}\int\frac{ds}{2\pi}\langle\Gamma(q,r|p,s)\rangle\\ &\qquad\times\langle G(r|k)G^*(s|k)\rangle\ ,\end{aligned} \tag{24}$$

where the irreducible vertex function is given by the solution of

$$\begin{aligned}\Gamma(q,r|p,s) &\cong v_0(q|r)v_0^*(p|s)\\ &\quad+\int\frac{dy}{2\pi}\int\frac{dz}{2\pi}v_0(q|y)v_0^*(p|z)G(y)G^*(z)\\ &\qquad\times[\Gamma(y,r|z,s)-\langle\Gamma(y,r|z,s)\rangle]\end{aligned} \tag{25}$$

with

$$v_0(q|r)=V(q|r)-\langle M(q|r)\rangle\ . \tag{26}$$

If we write

$$\langle G(q|k)G^*(p|k)\rangle=(2\pi)^2\delta(q-p)F(p|k) \tag{27}$$

and

$$\langle\Gamma(q,r|p,s)\rangle=2\pi\delta(q-r-p+s)\Gamma_0(q,r|p,s)\ , \tag{28}$$

we have from Eq. (24) that

$$\begin{aligned}F(p|k) &= \delta(p-k)|G(k)|^2\\ &\quad+|G(p)|^2\int\frac{ds}{2\pi}\Gamma_0(p,s|p,s)F(s|k)\\ &= \delta(p-k)|G(k)|^2+|G(p)|^2\tau(p|k)|G(k)|^2\ ,\end{aligned} \tag{29}$$

where

$$\tau(p|k)=\Gamma_0(p,k|p,k)+\int\frac{ds}{2\pi}\Gamma_0(p,s|p,s)|G(s)|^2\tau(s|k)\ . \tag{30}$$

From Eq. (22) we see that as $\Delta_{tot}\to 0$

$$|G(k)|^2 \to \frac{\pi C_1^2}{\Delta_{tot}}[\delta(k-K_{sp})+\delta(k+K_{sp})] \; . \tag{31}$$

When we use this approximation in Eq. (30) we find that

$$\begin{aligned}\tau(p\,|\,k) &= \Gamma_0(p,k\,|\,p,k) \\ &\quad + \frac{C_1^2}{2\Delta_{tot}}[\Gamma_0(p,K_{sp}\,|\,p,K_{sp})\tau(K_{sp}\,|\,k) \\ &\qquad + \Gamma_0(p,-K_{sp}\,|\,p,-K_{sp})\tau(-K_{sp}\,|\,k)] \; ,\end{aligned} \tag{32}$$

where

$$\tau(\pm K_{sp},k) = \frac{\Gamma_0(\pm K_{sp},k\,|\,\pm K_{sp},k)+A\,\Gamma_0(\mp K_{sp},k\,|\,\mp K_{sp},k)}{1-A^2} \tag{33}$$

with $A=(C_1^2/2\Delta_{tot})\Gamma_0(-K_{sp},K_{sp}\,|\,-K_{sp},K_{sp})$.

To lowest order in $\zeta(x_1)$ we find from Eq. (25)

$$\begin{aligned}\langle\Gamma(q,r\,|\,p,s)\rangle &= \langle V(q,r)V^*(p,s)\rangle = 2\pi\delta(q-r-p+s)\pi^{1/2}a\sigma^2\left|\frac{\epsilon(\omega)-1}{\epsilon^2(\omega)}\right|^2[\epsilon(\omega)qr-\alpha(q\omega)\alpha(r\omega)] \\ &\quad \times[\epsilon(\omega)ps-\alpha(p\omega)\alpha(s\omega)]^*\exp\left[-\frac{a^2}{4}(q-r)^2\right] .\end{aligned} \tag{34}$$

This interaction can be represented diagramatically as in Fig. 1(a). Using Eq. (34) in Eqs. (32), (29), (27), and (16) we find the corresponding contribution to $I(q\,|\,k)_{diff}$,

$$\begin{aligned}I^{(L)}(q\,|\,k)_{diff} &\cong 8\pi\alpha_0(q)\alpha_0(k)\,|G(q)|^2\,|G(k)|^2 \\ &\quad \times\left[K(q,k)+\frac{C_1^2}{2\Delta_{tot}}\frac{1}{1-\left[\frac{\Delta_{sp}}{\Delta_{tot}}\right]^2}\left[K(q,K_{sp})K(K_{sp},k)+K(q,-K_{sp})K(-K_{sp},k)\right.\right. \\ &\qquad \left.\left.+\frac{\Delta_{sp}}{\Delta_{tot}}[K(q,K_{sp})K(-K_{sp},k)+K(q,-K_{sp})K(K_{sp},k)]\right]\right] ,\end{aligned} \tag{35}$$

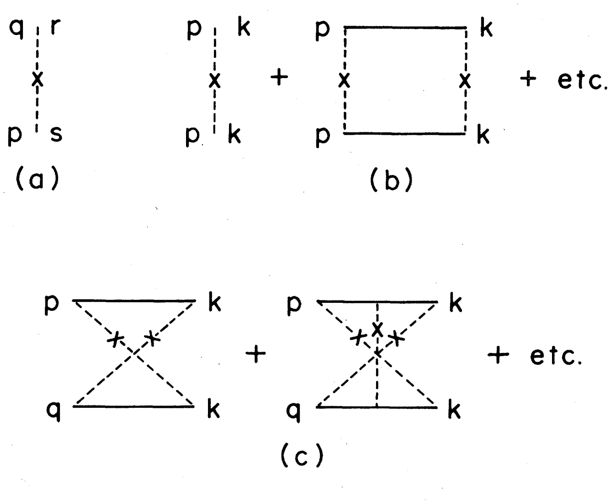

FIG. 1. (a) Diagrammatic representation of $\langle\Gamma(q,r\,|\,p,s)\rangle$ in the lowest order of $\zeta(x)$ [see Eq. (34)]. (b) Ladder-diagram sum [see Eq. (35)] for the reducible vertex function in terms of the $\langle\Gamma\rangle$ shown in (a). Upper and lower solid lines are G and G^*, respectively. (c) Contribution from the maximally crossed diagrams to the irreducible vertex $\Lambda(p,k\,|\,q,k)$ expressed in terms of the interaction $\langle\Gamma\rangle$ represented in (a).

where

$$\begin{aligned}K(q,k) &= \pi^{1/2}a\sigma^2\left|\frac{\epsilon(\omega)-1}{\epsilon^2(\omega)}\right|^2|\epsilon(\omega)qk-\alpha(q\omega)\alpha(k\omega)|^2 \\ &\quad \times\exp\left[\frac{a^2}{4}(k-q)^2\right] ,\end{aligned} \tag{36}$$

and $(\Delta_{sp}/\Delta_{tot})$ is the perturbation-theory expansion parameter. This result represents the sum of the so-called ladder diagrams[3,6] depicted in Fig. 1(b). The irreducible vertex part entering this sum together with the approximation given by Eq. (18) satisfies the Ward identity (for $\epsilon_2 \to 0$) relating the self-energy and the irreducible vertex function[6]

$$\text{Im}[M(k)] = \int\frac{dk_1}{2\pi}\text{Im}[G(k_1)]\Gamma_0(k_1,k\,|\,k_1,k) \; . \tag{37}$$

The ladder diagrams summed to obtain Eq. (35) correspond to the $q\to 0$, $\omega\to 0$ limit of Eq. (16) in Vollhardt and Wölfle.[3] For our case, however, the result is finite because $\epsilon_2(\omega)\neq 0$. However, if we let $\epsilon_2(\omega)=0$ in Eq. (35), the expression becomes divergent as expected, because then $(\Delta_{sp}/\Delta_{tot})=1$.

We now turn to the diagrams in $\langle\Gamma\rangle$ that contain intermediate states contributing to localization of the surface polaritons. These are the maximally crossed diagrams represented in Fig. 1(c).[3] If we represent the irreducible vertex function corresponding to the sum of these diagrams by

$$\langle \Gamma(q,r\mid p,r)\rangle = \Lambda(q,r\mid p,r)\ , \tag{38}$$

then $\Lambda(q,p\mid q_0,p)$ is given by

$$\Lambda(q,p\mid q_0,p)=\int\frac{ds}{2\pi}\int\frac{dt}{2\pi}\langle V(q\mid s)V^*(t\mid p)\rangle \times G(s)G^*(t)\underset{\sim}{\Lambda}(s,p\mid q_0,t)\ , \tag{39}$$

where

$$\underset{\sim}{\Lambda}(s,p\mid q,t)=\langle V(s\mid p)V^*(q\mid t)\rangle + \int\frac{dr}{2\pi}\int\frac{dy}{2\pi}\langle V(s\mid r)V^*(y\mid t)\rangle \times G(r)G^*(y)\underset{\sim}{\Lambda}(r,p\mid q,y)\ . \tag{40}$$

Upon solving Eqs. (39) and (40) we find (when $\Delta_{sp}/\Delta_{tot}\ll 1$)

$$\Lambda(q,p\mid q_0,p)=2\pi\delta(q-q_0)\frac{4\Delta_{tot}C_1^2}{(q+p)^2+4\Delta_{tot}^2}\frac{1}{1-\left[\dfrac{\Delta_{sp}}{\Delta_{tot}}\right]^2} \times\left\{K(p,K_{sp})K(p,-K_{sp})+\frac{1}{2}\frac{\Delta_{sp}}{\Delta_{tot}}\{[K(p,K_{sp})]^2+[K(p,-K_{sp})]^2\}\right\}\equiv 2\pi\delta(q-q_0)\Lambda_0(q,p\mid q,p)\ , \tag{41}$$

where we have used the result that

$$\lim_{\Delta_{tot}\to 0}\frac{2\Delta_{tot}}{(q+p)^2+4\Delta_{tot}^2}\to\pi\delta(q+p)\ . \tag{42}$$

The perturbation-theory expansion parameter for both ladder and maximally crossed diagrams is $(\Delta_{sp}/\Delta_{tot})$. Setting

$$\Gamma_0(p,s\mid p,s)\cong\Lambda_0(p,s\mid p,s) \tag{43}$$

in Eq. (30), we find

$$\tau(p,k)=\Lambda_0(p,k\mid p,k)\ . \tag{44}$$

Then the contribution to $I(q\mid k)_{diff}$ from the maximally crossed diagrams is

$$I^{(c)}(q\mid k)_{diff} = 8\pi\alpha_0(q\omega)\alpha_0(k\omega)\,|G(q)|^2\,|G(k)|^2\Lambda_0(q,k\mid q,k)\ . \tag{45}$$

From Eq. (41) it is easy to see that the contribution to the scattering efficiency from $I^{(c)}(q\mid k)_{diff}$ is greatest for $q=-k$, i.e., for reflected light in the direction opposite to that of the incident beam.

The differential reflection coefficient, $\partial R/\partial\theta$, is obtained from $I(q\mid k)$ and is found to be given by

$$\frac{\partial R}{\partial\theta}=\frac{\omega}{2\pi c}\cos\theta\, I(q\mid k)\ ,$$

where θ is the angle between the scattered wave vector and the outward normal to the mean surface. In what follows we will ignore the specular contribution to $\partial R/\partial\theta$ and concentrate only on the diffuse scattering obtained as the sum of $I^{(L)}(q\mid k)$ and $I^{(c)}(q\mid k)$ given in Eqs. (35) and (45).

We present some results for $\partial R/\partial\theta$ for the case of light

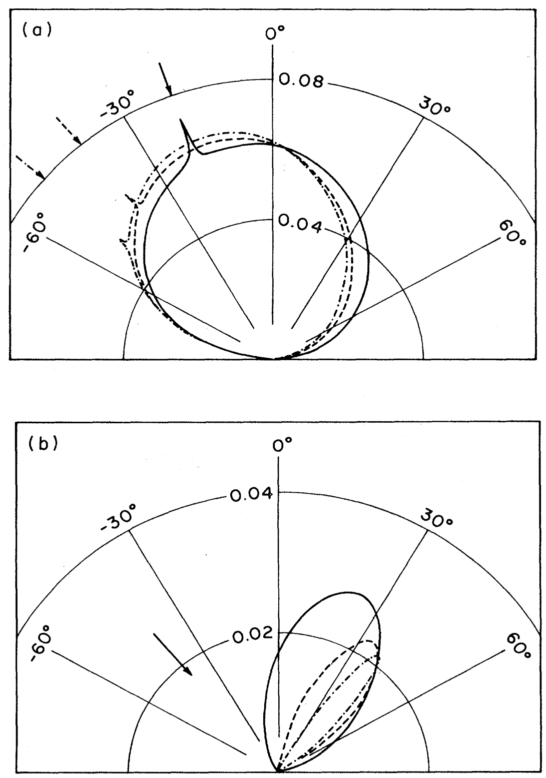

FIG. 2. (a) Plot of $\partial R/\partial\theta$ in rad^{-1} vs θ for 4579-Å light incident on an Ag grating [$\epsilon(\omega)=-7.5+0.24i$] with $a=1000$ Å and $\sigma=50$ Å. Curves for angles of incidence of 20° (solid line), 40° (dashed line), and 50° (dot-dashed line) are shown. Direction of the incoming light for each curve is indicated on the figure by an arrow. (b) Plot of $\partial R/\partial\theta$ in rad^{-1} vs θ for 4579-Å light incident on an Ag grating [$\epsilon(\omega)=-7.5+0.24i$]. The angle of incidence of 40° is shown by an arrow. Curves for $a=2500$ Å, $\sigma=31.62$ Å (solid line); $a=5000$ Å, $\sigma=22.36$ Å (dashed line); and $a=10\,000$ Å, $\sigma=15.18$ Å (dot-dashed line) are shown.

of wavelength $\lambda=4579$ Å incident on a rough silver grating. The dielectric constant of silver at this wavelength is $\epsilon(\omega)=-7.5+0.24i$.[9] We find in general that $\partial R/\partial\theta$ exhibits two qualitatively different types of behavior depending on the ratio a/λ. For $a/\lambda \ll 1$ localization effects are significant, but for $a/\lambda \gtrsim 1$ the localization contributions are negligible. We illustrate the $a/\lambda \ll 1$ limit in Fig. 2(a) where $\partial R/\partial\theta$ versus θ is plotted for $a=1000$ Å, $\sigma=50$ Å, and angles of incidence of 20°, 40°, and 50°. Note that the peaks in $\partial R/\partial\theta$ in the antispecular direction that arise from polariton localization are found to increase as the angle of incidence is decreased. The general form of the antispecular peaks is Lorentzian. Their width increases with increasing $\epsilon_2(\omega)$. In the opposite limit $\Delta_{tot}\rightarrow 0$ they become singularities.

The $a/\lambda \gtrsim 1$ region is illustrated in Fig. 2(b) for an angle of incidence of 40° and for several values of a and σ. From Eqs. (35), (36), (41), and (45) we see that $a\sigma^2$ sets the scale of $\partial R/\partial\theta$. Hence, to facilitate comparison of the results obtained, a and σ were chosen such that $a\sigma^2=2.5\times10^6$ Å for all curves in Fig. 2(b). We see that as a increases relative to λ the diffuse scattering concentrates around the specular direction. In addition, the antispecular contribution to $\partial R/\partial\theta$ from localization effects is so small that it cannot be seen on the scale of our figure.

Thus, we conclude that under suitable conditions, indicated above, effects of surface polariton localization should be observable in the angular dependence of the light scattered nonspecularly from a randomly rough grating.

ACKNOWLEDGMENT

This work was supported in part by National Science Foundation Grant No. INT81-15141.

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