Natural damping of nonlocal surface Bernstein modes in the high-magnetic-field quantum limit

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The rate at which nonlocal surface Bernstein modes decay into electron-hole pairs is calculated in the high-magnetic-field quantum limit. The infinite-barrier model is used for the planar surface confining the electrons of a degenerate semi-infinite plasma. In this model for a metal, semimetal, or semiconductor, a uniform magnetic field is applied in the direction perpendicular to the surface. The dynamical properties of the Landau quantized plasma are described in the random-phase approximation.

In 1958, Bernstein[1] showed that a classical gaseous bulk plasma could have undamped nonlocal modes near each integer multiple $n\omega_c$ ($n \geq 2$) of the cyclotron frequency for propagation perpendicular to an applied magnetic field. In 1965, Horing[2] demonstrated that the spectrum of a bulk quantum plasma has branches analogous to Bernstein modes near multiples of the cyclotron frequency for propagation nearly perpendicular to the magnetic field in a Landau quantized description, and for propagation perpendicular to the magnetic field these modes are also undamped. This was exhibited in the random-phase approximation (RPA) by examining the frequency- and wave-number-dependent bulk dielectric function $\epsilon(\mathbf{p},\omega)$ with real and imaginary parts ϵ_R and ϵ_I, respectively. In the long-wavelength limit, one may study the $n=2$ Bernstein-type mode using the approximation[2,3]

$$\epsilon_R(\mathbf{p},\omega) \approx 1-\frac{p_z^2}{p^2}\frac{\omega_p^2}{\omega^2}-\frac{p_\parallel^2}{p^2}\frac{\omega_p^2}{\omega^2-\omega_c^2} -\frac{\omega_p^2}{m^* S_1\omega_c^2}\frac{p_\parallel^4}{p^2}\frac{1}{\omega^2-(2\omega_c)^2} \ . \tag{1}$$

The magnetic field is applied in the z direction and the wave vector $\mathbf{p}$ has components p_z and $p_\parallel$, parallel and perpendicular to the magnetic field, respectively. ω_p and ω_c are the plasma frequency and the cyclotron frequency, respectively, of an electron with effective mass m^*. $S_1 \equiv n_B/\sigma_B$ where n_B and σ_B are the electron number and energy density, respectively, in the bulk plasma. For a degenerate bulk plasma in the high-field quantum limit (HFQL) with all electrons in the lowest Landau state, we have[4]

$$\epsilon_I(\mathbf{p},\omega)=\frac{4\pi e^2}{p^2}\frac{\pi n_B}{2\hbar}\left[\frac{m^*}{2p_z^2\zeta}\right]^{1/2} e^{-\Lambda} \times \sum_{n=0}^{\infty}\frac{\Lambda^n}{n!}\left[\eta_+\left[\zeta-\left[\omega-n\omega_c-\frac{\hbar p_z^2}{2m^*}\right]^2\frac{m^*}{2p_z^2}\right] -\eta_+\left[\zeta-\left[\omega+n\omega_c+\frac{\hbar p_z^2}{2m^*}\right]^2\frac{m^*}{2p_z^2}\right]\right] , \tag{2}$$

where $\Lambda \equiv \hbar p_\parallel^2/2m^*\omega_c$, η_+ is the Heaviside unit step function, and ζ the magnetic-field-dependent chemical potential. Also, $S_1 \approx 2/\hbar\omega_c$ in the HFQL. For propagation perpendicular to the magnetic field, it is readily verified that there is no damping. For propagation off the perpendicular direction to the magnetic field, the bulk Bernstein-like mode near $2\omega_c$ [given by a zero of ϵ_R in Eq. (1)] suffers natural damping by its decay into electron-hole pairs. This decay process is governed by the cutoff factors η_+ in Eq. (2).

In 1973, Horing and Yildiz[3] (to be referred to as HY) predicted that in the presence of a uniform magnetic field in the z direction perpendicular to the planar surface, a semi-infinite plasma has *surface Bernstein modes.* Special attention was given to the $n=2$ nonlocal surface Bernstein mode and HY calculated the frequency of this mode in the long-wavelength limit. It is the purpose of this paper to calculate the decay rate of the $n=2$ nonlocal surface Bernstein mode into electron-hole pairs when a strong magnetic field is applied. Unlike the bulk Bernstein mode which is devoid of natural damping for perpendicular propagation, the surface Bernstein mode does in fact suffer decay into electron-hole pairs for propagation along the surface and perpendicular to the magnetic field.

For arbitrary magnetic field strengths, the decay rate is given by[5,6]

$$\gamma=-\left[\frac{D_I(p_\parallel,\omega)}{\partial D_R(p_\parallel,\omega)/\partial\omega}\right]_{\omega=\Omega} , \tag{3}$$

where the right-hand side of Eq. (3) is to be evaluated at the surface Bernstein mode frequency Ω. D_R and D_I are the real and imaginary parts of the surface mode "dispersion formula" in the RPA:

$$D(p_\parallel,\omega)=1+\frac{2p_\parallel}{\pi}\int_0^\infty \frac{dp_z}{p_z^2+p_\parallel^2}\frac{1}{\epsilon(\mathbf{p},\omega)} \ . \tag{4}$$

Substituting Eq. (1) and Eq. (2) into Eq. (4) and assuming that $|\epsilon_I| << |\epsilon_R|$, we obtain

$$D_I(p_\parallel,\omega)\approx -\frac{2p_\parallel}{\pi}\int_0^\infty \frac{dp_z}{p_z^2+p_\parallel^2}\frac{\epsilon_I(\mathbf{p},\omega)}{\epsilon_R^2(\mathbf{p},\omega)}$$

$$\approx -p_\parallel\omega_p^2\left[\frac{m^{*3}}{2\hbar^2\zeta}\right]^{1/2}e^{-\Lambda}\sum_{n=0}^{\infty}\frac{\Lambda^n}{n!}\int_0^\infty\frac{dp_z}{p_z}\frac{1}{(Ap_z^2+B)^2}\left[\eta+\left[\zeta-\left[\omega-n\omega_c-\frac{\hbar p_z^2}{2m^*}\right]^2\frac{m^*}{2p_z^2}\right]-(\omega\to-\omega)\right], \tag{5}$$

where $\Lambda=\hbar p_\parallel^2/2m^*\omega_c$ and

$$A\equiv 1-\frac{\omega_p^2}{\omega^2}, \tag{6a}$$

$$B\equiv\left[1-\frac{\omega_p^2}{\omega^2-\omega_c^2}\right]p_\parallel^2-\frac{\omega_p^2}{m^*S_1\omega_c^2}\frac{p_\parallel^4}{\omega^2-(2\omega_c)^2}, \tag{6b}$$

and $(\omega\to-\omega)$ signifies a term similar to the preceding one with ω replaced by $-\omega$. The range of integration in Eq. (5) as delimited by the Heaviside unit step function is $p_n^{(-)}(\omega)<p_z<p_n^{(+)}(\omega)$ where for $\omega>n\omega_c$

$$p_n^{(\pm)}(\omega)\equiv\left[p_F^2+\frac{2m^*}{\hbar}(\omega-n\omega_c)\right]^{1/2}\pm p_F \tag{7}$$

and for $\omega<n\omega_c$

$$p_n^{(\pm)}(\omega)=p_F\pm\left[p_F^2-\frac{2m^*}{\hbar}(n\omega_c-\omega)\right]^{1/2}. \tag{8}$$

Here the Fermi wave number is $p_F\equiv(2\zeta m^*/\hbar^2)^{1/2}$.

In the long-wavelength limit, the $n=2$ nonlocal surface Bernstein mode of HY is given by

$$\Omega^2=(2\omega_c)^2+\frac{p_\parallel^2\omega_p^2}{m^*S_1\omega_c^2C}, \tag{9}$$

where

$$C\equiv\frac{\omega_p^2}{3\omega_c^2}\left[\frac{\omega_p^2-7\omega_c^2}{4\omega_c^2-\omega_p^2}\right], \tag{10}$$

so that $C\geq 0$ for $4\leq\omega_p^2/\omega_c^2\leq 7$; otherwise, C is negative. In the HFQL when $\hbar\omega_c>\zeta$ and only the lowest Landau eigenstate is populated, we find that $p_n^{(\pm)}(-\Omega)$ is complex for $n=0,1,2,\ldots$ so that the range of integration vanishes for all the terms with $\omega\to-\omega$ in Eq. (5) at the low-wave-number surface Bernstein mode frequency Ω. The range of integration also vanishes similarly in the HFQL for $n=3,4,5,\ldots$: To see this one may set $\omega=\Omega$ in Eq. (7) and determine that $p_n^{(\pm)}(\Omega)$ is complex for $n=3,4,5,\ldots$ for the HFQL. Therefore, in our calculation of $D_I(p_\parallel,\omega)$ in the HFQL we only need to calculate the contributions due to the first *three* terms $n=0,1,$ and 2 in Eq. (5) for positive Ω.

For $\omega=\Omega$, A and B in Eq. (6) are given approximately by $A\approx 1-\omega_p^2/4\omega_c^2$ and $B\approx p_\parallel^2/A$. In both cases $C>0$ and $C<0$, we obtain the $n=2$ integration limits $p_2^{(\pm)}(\Omega)$ from Eq. (7) in the HFQL as $p_2^{(+)}(\Omega)\approx 2p_F$ and

$$p_2^{(-)}(\Omega)\approx p_\parallel^2\omega_p^2(4\hbar p_F S_1\omega_c^3|C|)^{-1}\ll p_F.$$

However, when $n=0,1$ in the HFQL we have $p_n^{(\pm)}(\Omega)=O((m^*\omega_c/\hbar)^{1/2})\approx O(p_F)$. In view of these results, we obtain from Eq. (5)[7]

$$D_I(p_\parallel,\Omega)\approx -p_\parallel\omega_p^2\left[\frac{m^{*3}}{2\hbar^2\zeta}\right]^{1/2}\frac{e^{-\Lambda}}{A^2}\left[\sum_{n=0,1}\frac{\Lambda^n}{n!}\frac{1}{4}\left[\frac{1}{(p_n^{(-)})^4}-\frac{1}{(p_n^{(+)})^4}\right]\right.$$
$$\left.+\frac{\Lambda^2}{2!}\left\{\frac{1}{2}\left[\frac{A}{p_\parallel}\right]^4\left[\frac{(p_\parallel/A)^2}{(p_2^{(-)})^2+(p_\parallel/A)^2}+\ln\left[\frac{(p_2^{(-)})^2}{(p_2^{(-)})^2+(p_\parallel/A)^2}\right]\right]-\frac{1}{4}\frac{1}{(p_2^{(+)})^4}\right\}\right]. \tag{11}$$

In the long-wavelength limit we have $\Lambda\ll 1$, and since the logarithmic term makes the dominant contribution in Eq. (11), we obtain approximately

$$D_I(p_\parallel,\Omega)\approx -p_\parallel\omega_p^2\left[\frac{m^{*3}}{2\hbar^2\zeta}\right]^{1/2}\frac{\hbar^2A^2}{8m^{*2}\omega_c^2}\ln\left|\frac{8p_F\omega_c^2C}{p_\parallel\omega_p^2A}\right|, \tag{12}$$

since $S_1\approx 2/\hbar\omega_c$ in the HFQL for a degenerate plasma.[3]

Assuming that $|\epsilon_I|\ll|\epsilon_R|$, the real part of the RPA surface "dispersion formula" is

$$D_R(p_\parallel,\omega)\approx 1+\frac{2p_\parallel}{\pi}\int_0^\infty\frac{dp_z}{p_z^2+p_\parallel^2}\frac{1}{\epsilon_R(\mathbf{p},\omega)}. \tag{13}$$

Substituting Eq. (1) into Eq. (13), the resulting integral is elementary. For long-wavelength surface Bernstein modes of a degenerate plasma in the HFQL, we obtain

$$\left[\frac{\partial D_R(p_\parallel,\omega)}{\partial\omega}\right]_{\omega=\Omega}\approx\frac{m^*C^2}{\hbar p_\parallel^2\omega_p^2}(4\omega_c^2-\omega_p^2). \tag{14}$$

Substituting Eq. (12) and Eq. (14) into Eq. (3), we obtain the decay rate of the long-wavelength $n=2$ surface Bernstein mode of a degenerate semi-infinite magnetoplasma in the HFQL:

$$\gamma=\left[\frac{\hbar p_\parallel^3}{8m^*p_F}\right]\left[\frac{A}{C}\right]^2\frac{\omega_p^4}{\omega_c^2(4\omega_c^2-\omega_p^2)}\ln\left|\frac{8p_F\omega_c^2C}{p_\parallel\omega_p^2A}\right|. \tag{15}$$

One could similarly obtain the damping of the higher modes belonging to the surface Bernstein mode magnetoplasmon spectrum.

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[6] G. L. Rafanelli and N. J. Horing, in *Proceedings of the IV International Conference on Solid Surfaces* [Suppl. Rev. "La Vide, les Couches Mince" **201**, 918 (1980)].
[7] In a straightforward way, we obtain

$$I \equiv \int \frac{dp_z}{p_z} \frac{1}{(p_z^2 + B/A)^2} = \frac{1}{2}\left(\frac{A}{B}\right)^2 \left[\frac{B}{Ap_z^2 + B} + \ln\left(\frac{Ap_z^2}{Ap_z^2 + B}\right)\right] .$$

When ω is equal to the long-wavelength surface Bernstein mode frequency Ω, we find that $B \approx p_{\parallel}^2/A$. Therefore, if $p_{\parallel}^2/A^2 p_z^2 << 1$, we find by expanding the logarithmic term above that

$$I \approx -1/4p_z^4 .$$

This approximation may be applied to the $n=0$ and $n=1$ terms in Eq. (5) as well as to $p_z = p_2^{(+)}(\Omega)$. However, it is not applicable when $p_z = p_2^{(-)}(\Omega)$ since $p_2^{(-)} = O(p_{\parallel}^2/p_F)$, and here $p_{\parallel}^2/A[p_2^{(-)}(\Omega)]^2$ is *not* small, so that in this case the logarithmic term may *not* be expanded as before, and it must be kept intact.