PHYSICAL REVIEW B VOLUME 31, NUMBER 6 15 MARCH 1985

Phonon-scattering-limited mobility in a quantum-well heterostructure

Vijay K. Arora and Athar Naeem
Department of Physics, King Saud University, Riyadh, Saudi Arabia
(Received 13 February 1984; revised manuscript received 11 June 1984)

The theory of electron-lattice scattering is examined for a quantum-well heterostructure in the size quantum limit when most of the electrons populate the lowest quantized subband. The acoustic-phonon scattering is found to be one of the important mechanisms of scattering at intermediate temperatures when donors supplying the electrons are removed in an adjacent nonconducting layer.

I. INTRODUCTION

Nowadays there is an intense interest in the quantum transport studies of semiconducting layered heterojunctions, thin films, and inversion layers.[1] In these microstructures, grown by molecular-beam epitaxy (MBE) and other sophisticated techniques, the de Broglie wavelength of an electron can easily become comparable to the device size in one or more of three Cartesian directions. The system then behaves essentially as a quasi-low-dimensional system.[2]

In a quantum-well heterostructure of the GaAs/$Ga_xAl_{1-x}As$ prototype, Coulomb scattering by ionized impurities can be reduced if the impurities are placed in the $Ga_xAl_{1-x}As$ layers with the higher energy gap, and if the carriers are confined to the GaAs layer with lower energy gap.[3] Because of this carrier confinement, the size quantization plays an important role in determining their electrical[4] and optical properties.[3,5] The scattering is considerably reduced[6] when donor impurities are present outside the well, and high values of the mobilities have been reported.[7,8] Still higher mobilities can be obtained by separating the carriers from the ionized impurities by introducing an undoped spacer layer.[7,9]

Drummond *et al.*[7] found an optimum space length, but Störmer *et al.*[9] did not identify any optimum spacer thickness in modulation-doped structure up to a spacer thickness of 150 Å. In carefully prepared samples, ionized-impurity scattering plays a secondary role in these modulation-doped heterostructures. Due to the reduction in ionized-impurity scattering, the mobility along the heterojunction is considerably improved over mobility in bulk materials. The mobilities exceeding 10^6 cm^2/V s at low temperatures are now not so difficult to obtain with modulation doping.[8] Störmer[8] has listed possible interactions which are likely to play a role in limiting mobility. The increased cleanliness of the growth process by using MBE reduces scattering by unwanted defects associated with device samples, which act as scattering centers. But the lattice-phonon scattering remains unavoidable, especially at intermediate and high temperatures.

II. ACOUSTIC-PHONON SCATTERING

The electron scattering by acoustic phonons is one of the important mechanisms of scattering. Price[10] has considered the piezoelectric coupling as well as the deformation-potential coupling. It is shown that the former may become important at temperatures below 100 K. On the other hand, the work of Basu and Nag[11] has indicated a suppressed piezoelectric scattering for GaAs, InSb, and InAs in the temperature range 4.2—100 K. It, therefore, seems reasonable to assume that the piezoelectric scattering by acoustic phonons does not play an active role at intermediate temperatures.

The deformation-potential scattering by acoustic phonons is considered by several workers;[4–6,10–16] some of these expressions differ by a numerical factor. The most agreed upon value of the mobility[4–6,10,12] limited by this scattering mechanism is given by

$$\mu_a = 2e\hbar^3 \rho_d u^2 d / (3m^{*2} E_1^2 k_B T) . \tag{2.1}$$

Here ρ_d is the crystal density, u is the longitudinal sound velocity, d is the width of the quantum well, m^* is the carrier's effective mass, E_1 is the deformation-potential constant, and T is the temperature. The mobility values of Refs. 11, 13, and 16 are lower than that of Eq. (2.1) by a factor of $\frac{2}{3}$. This is due to the neglect of the quantized motion perpendicular to the electron layer and agrees with the QTD-I (quasi-two-dimensional-I) model of Ref. 4. The mobility values of Refs. 14 and 15 are larger than those of Eq. (2.1), probably because of the neglect of phonon emission,[6] which is as important as phonon absorption in the elastic scattering limit. For GaAs, the deformation-potential constant E_1 is taken to be 7 eV,[10,15] 12 eV,[16] or 8.7 eV.[11] Since the mobility is inversely proportional to E_1^2, the conclusion drawn from these studies[11,15,16] can be drastically altered if different values of E_1 are used. With the values quoted by Price,[10] the acoustic-phonon—scattering-limited mobility μ_a is calculated as

$$\mu_a = 2.9\times 10^6 \frac{d\ (\text{nm})}{T\ (\text{K})} \frac{\text{cm}^2}{\text{V s}} . \tag{2.2}$$

At $T = 100$ K and $d = 10$ nm, the mobility value obtained is $\mu_a = 2.7\times 10^5$ cm^2/V s. Equations (2.1) and (2.2) show the mobility increasing linearly with the well width d and inversely proportional to the device temperature. The comparison with the bulk mobility[4] μ_{ab} gives

$$\mu_a / \mu_{ab} = d / (2\pi^{1/2} \lambda_D) \tag{2.3}$$

 ©1985 The American Physical Society

with

$$\lambda\!\!\!^{-}_D=\hbar/(2m^*k_BT)^{1/2} . \quad (2.4)$$

At 100 K and 10-nm well width, $\mu_a/\mu_{ab}=0.35$. Therefore, the conductivity decreases by a factor of 3 or so. The de Broglie wavelength $\lambda_D=51.2$ nm at 100 K. Thus the condition for onset of the size quantum limit (SQL) is quite well satisfied for GaAs under these conditions.

III. NONPOLAR-OPTICAL-PHONON SCATTERING

The similar calculations for nonpolar-optical-mode—scattering-limited[10,12] (also see Ref. 1 and 18 for other references) mobility μ_{no} yield the expression

$$\mu_{no}=4e\hbar^2 d\rho_d\omega_0/(3m^{*2}D_{no}^2N_0) , \quad (3.1)$$

with

$$D_{no}=E_{op}\omega_0/u , \quad (3.2)$$

where E_{op} is the deformation-potential-constant, $\hbar\omega_0$ is the optical-phonon energy, N_0 is the number of phonons ($N_0=[\exp(\hbar\omega_0/k_BT)-1]^{-1}$). In Eq. (3.1), implicit is the assumption that the energy ϵ_k of an electron in the plane of the quantum well is less than the phonon energy ($\epsilon_k<\hbar\omega_0$) at intermediate temperature so that phonon emission is not possible. The comparison with the bulk mobility[17] in this case shows

$$\mu_{no}/\mu_{nob}=(2/3\pi)(d/\lambda\!\!\!^{-}_D)(\hbar\omega_0/k_BT)^{1/2} . \quad (3.3)$$

As in the case for the acoustic phonons, the nonpolar-optic-phonon—scattering-limited mobility also increases linearly with the width of the well. This scattering mechanism is important in nonpolar semiconductors of silicon type and has been considered, for example, by Roychoudhury and Basu for silicon inversion layers[18] (also see Ref. 1). If both emission and absorption of nonpolar optical phonons are considered,[10,12] the resulting expression for mobility is given by

$$\mu=\mu_{no}\{1-\exp(-\hbar\omega_0/k_BT)[1+(\hbar\omega_0/k_BT)] \times[1-(N_0/2N_0+1)]\} , \quad (3.4)$$

where μ_{no} is as given by Eq. (3.1) for the case of optical-phonon absorption only.

A few comments on the neglect of phonon emission should prove useful at this stage. In considering the Ohmic mobility, the carriers contributing to the conductivity are in the neighborhood of the thermal energy k_BT. If $k_BT<\hbar\omega_0$, the carriers are deficient in energy to emit an optical phonon, which is indicated by a step function $\Theta(\epsilon-\hbar\omega_0)$ in the scattering rate, where ϵ is the carrier energy. However, in nondegenerate semiconductors, there are always a few carriers in the high-energy tail of the distribution function which have enough energy to emit optical phonons. The fractional concentration of these electrons in GaAs is proportional to $\exp(-\hbar\omega_0/k_BT)=\exp(-420/T)\sim10^{-2}$ at $T=100$ K, and 0.25 at $T=300$ K. Therefore, the contribution of electron scattering by the emission of an optical phonon can be neglected at low temperatures. But, at room temperature, these electrons could make an appreciable contribution. The ratio of scattering rates due to emission $\tau_k^{-1}(E)$ to those due to absorption $\tau_k^{-1}(A)$ of an optical phonon is given by[12]

$$\tau_k^{-1}(E)/\tau_k^{-1}(A)=(N_0+1)/N_0 , \quad (3.5)$$

which is ≈70 at $T=100$ K and ≈4 at $T=300$ K for $\hbar\omega_0=36$ meV. Thus, although the relative concentration of carriers able to emit an optical phonon is larger at room temperature than that at $T=100$ K, the relative probability of emission to that of absorption at room temperature is smaller than that at intermediate temperatures. Since mobility is proportional to τ_k, mobility change due to the carriers able to emit optical phonons is further reduced when both absorption and emission are considered, which is reflected in Eq. (3.4). At higher temperatures ($T>300$ K), relaxation rates for emission and absorption tend to be equal (because $N_0\gg1$). However, a careful analysis is desirable to investigate this point further especially if the spacing between the quantized levels is such that $3\epsilon_0\leq\hbar\omega_0$ (intersubband scattering) and hot-electron effects[19] are important in limiting the device speed. Here $\epsilon_0=\pi^2\hbar^2/2m^*d^2$ is the ground-state energy of an electron in the quantum well.

In non-Ohmic transport, it is an accepted thesis that the saturation velocity is obtained by hot electrons by emission of an optical phonon (see, for example, Ref. 17). It has recently been indicated[19] that the saturation velocity is due to the field broadening $\hbar\tau_F^{-1}\sim e\varepsilon\lambda\!\!\!^{-}_D$ in a strong electric field ε, when collision mechanisms are suppressed by the field broadening (quasiballistic transport).

IV. POLAR-OPTICAL-PHONON SCATTERING

In a quantum-well heterostructure made of polar materials, the nonpolar-optical-phonon scattering is found to be negligible, and polar-optical-phonon scattering is found to be quite important.[10] Although the formal expressions for polar-optic-phonon scattering are quite different from those of three-dimensional systems, the numerical values of the mobility and of its temperature dependence for a GaAs-based superlattices or heterojunction near 100 K are comparable to those in bulk GaAs for typical layer parameters if the carrier concentration is low enough that screening effects can be neglected.[10,20] Basu and Nag[16] found that the polar-mode-scattering-limited mobility contributes 80% of the lattice mobility. This conclusion will change if the correct expression [Eq. (2.1)] for acoustic-phonon scattering and $E_1=7$ eV (Refs. 8 and 13) are used. Their calculations are based on an iterative solution of the Boltzmann transport equation.

At intermediate temperatures, there is little difference between mobilities obtained from the relaxation-time approximation and by the iterative solution.[14,16] The differences can be significant only at higher temperature, but then the condition for size quantum limit under which these results are applicable may not apply.

The relaxation-rate calculation of Price[10] gives, for polar-optical-phonon-limited (po) mobility, the expression

$$\mu_{\rm po}^{P}=2\hbar\epsilon_p/(m^* k_0 e\pi N_0)\ , \tag{4.1}$$

with

$$\epsilon_p^{-1}=\epsilon_\infty^{-1}-\epsilon_s^{-1}\ , \tag{4.2}$$

$$k_0=(2m^*\omega_0/\hbar)^{1/2}\ , \tag{4.3}$$

where ϵ_∞ is the high-frequency and ϵ_s is the static dielectric constant. Compared with the bulk mobility $\mu_{\rm pob}$ (with $k_B T \ll \hbar\omega_0$), the $\mu_{\rm po}^{P}/\mu_{\rm pob}$ is obtained as

$$\mu_{\rm po}^{P}/\mu_{\rm pob}=2/\pi=0.64\ . \tag{4.4}$$

Therefore, the mobility decreases because of carrier confinement.

Ridley[2] has indicated an important distinction between scattering rate and momentum-relaxation rate which turns out to be of considerable importance. The principal difference from the bulk situation is the fuzzier momentum conservation perpendicular to the walls. For $qd=1$ (where q is the phonon momentum), the difference between two approximations could be as large as 60%. The mobility $\mu_{\rm po}^{R}$ obtained from the momentum-relaxation rate derived by Ridley[12] is found to be

$$\mu_{\rm po}^{R}=4\pi\epsilon_p\hbar^2/(e\omega_0 N_0 m^{*2} d)\ , \tag{4.5}$$

which when compared with the result of Price[10] gives

$$\mu_{\rm po}^{R}/\mu_{\rm po}^{P}=4\pi^2/(k_0 d)\ . \tag{4.6}$$

For a quantum well of width $d=10$ nm, $\mu_{\rm po}^{R}/\mu_{\rm po}^{P}=15.7$. Therefore, the mobility is considerably larger in Ridley's approach than in Price's approach and is also dependent on the well thickness. As well thickness decreased, the mobility will become larger. In contrast to the case of acoustic phonons, the momentum-relaxation rate associated with the absorption of optical phonons vanishes, though the scattering rate of Price[8] remains finite. In the regime where both acoustic-phonon scattering and polar-optic-phonon scattering limit the mobility, the mobility is maximum at $d=d^*$ given by

$$d^*=4\pi^{3/4}(\mu_{\rm pob}\lambdabar_D/\mu_{ab}k_0)^{1/2}\ , \tag{4.7}$$

with the maximum value of the mobility given by

$$\mu^*=\pi^{1/4}(k_0\lambdabar_D)^{-1/2}(\mu_{ab}\mu_{\rm pob})^{1/2}\ . \tag{4.8}$$

As in Sec. III the mobility expression of Eq. (4.5) is derived for phonon absorption only using the momentum-relaxation rate τ_k^{-1} [see Eq. (52) of Ref. 12]. It may be noted that the expression in Ref. 12 are in SI units, whereas those used here are in cgs units. Also, the relaxation rate used here is smaller than that of Ref. 12 by a factor of 2, which seems to be due to an error in the derivation in Ref. 12. Ridley[12] derives expressions for phonon absorption as well as phonon emission [Eqs. (52) and (53) of Ref. 12]. A casual look at these expressions gives the impression that $\tau_k^{-1}(E)\sim d^{-1}$ should dominate over $\tau_k^{-1}(A)\sim d$ at small thicknesses in quantum-well heterostructure. But a closer look at the derivation of these expressions indicates that $\tau_k^{-1}(E)$ vanishes for $\epsilon_k<\hbar\omega_0$ because of the presence of a step function $\Theta(\epsilon_k-\hbar\omega_0)$ [not explicitly included in Eq. (53) of Ref. 12]. Therefore, at low temperatures, consistent with what has been said in Sec. III, the contribution of scattering by phonon emission to the mobility is expected to be small.

One of the problems with the theory of the optical-phonon scattering is the difficulty in defining the momentum-relaxation rate in the Boltzmann transport formalism when scattering of the carriers is highly inelastic.[21] In the density-matrix formalism,[22] there is an anisotropic term which makes the momentum-relaxation rate distinct from the scattering rate, which is what Ridley[12] has considered.

The ratio of the electron-scattering rate by emission (for $\epsilon_k=\hbar\omega_0$ of an optical phonon to that by absorption is given by

$$\tau_k^{-1}(E)/\tau_k^{-1}(A)=[8(N_0+1)/N_0](\epsilon_0/\hbar\omega_0)\ . \tag{4.9}$$

If both emission and absorption of polar optical phonons[12] is considered, the resulting mobility expression is given by

$$\mu=\mu_{\rm po}^{R}[1-\delta\exp(-x_0)+\delta^2\exp(\delta)E_1(x_0+\delta)]\ , \tag{4.10}$$

with

$$x_0=\hbar\omega_0/k_BT\ ,\quad \delta=8(N_0+1)\epsilon_0/N_0k_BT\ .$$

Here $E_1(x)$ is the exponential integral, which for asymptotic values of x is approximated as $E_1(x)\approx\exp(-x)/x$. In the temperature range of 100—300 K, δ is large ($\delta\approx10^2$ at 300 K and even larger at lower temperatures), and $x_0=4.2$ at 100 K and 1.4 at 300 K. Under these conditions $\mu\approx\mu_{\rm po}^{R}$ in Eq. (4.10), and hence the contribution of emission of polar optical phonons to the mobility can be neglected. If $\epsilon_0>\hbar\omega_0$, the possibility of electron scattering by emission of an optical phonon is considerably enhanced.

V. DISCUSSION

All the above results are applicable under the conditions of size-quantum limit (SQL) when most of the electrons occupy the lowest quantized subband. The separation between the two lowest subbands is $3\epsilon_0$, where $\epsilon_0=\pi^2\hbar^2/2m^*d^2=56$ meV at $d=10$ nm. In terms of thermal energy $k_BT=26$ meV at room temperature $3\epsilon_0=6.5k_BT$ ($3\epsilon_0\gg k_BT$) and hence SQL applies in this situation. Actually, the minimum thickness d_m required for onset of the SQL is given by

$$d_m=\sqrt{3}\lambda_D/2\ , \tag{5.1}$$

with

$$\lambda_D=2\pi\lambdabar_D\ . \tag{5.2}$$

At room temperature, the onset of SQL takes place at $d_m=26$ nm ($\lambda_D=29$ nm). For thickness smaller than 26 nm, SQL holds pretty well.

Taking a typical value of $T=300$ K, and $d=10$ nm, the mobilities are calculated as follows:

$$\begin{aligned}\mu_a&=0.96\times 10^5\ \mathrm{cm}^2/\mathrm{V\,s}\ ,\\ \mu_{\mathrm{po}}^P&=0.07\times 10^5\ \mathrm{cm}^2/\mathrm{V\,s}\ ,\\ \mu_{po}^R&=1.04\times 10^5\ \mathrm{cm}^2/\mathrm{V\,s}\ .\end{aligned} \tag{5.3}$$

The mobilities at 77 K and $d=10$ nm are those given below,

$$\begin{aligned}\mu_a&=3.7\times 10^5\ \mathrm{cm}^2/\mathrm{V\,s}\ ,\\ \mu_{\mathrm{po}}^P&=5.0\times 10^5\ \mathrm{cm}^2/\mathrm{V\,s}\ ,\\ \mu_{\mathrm{po}}^R&=7.9\times 10^5\ \mathrm{cm}^2/\mathrm{V\,s}\ .\end{aligned} \tag{5.4}$$

These results indicate that using Price's scattering rates, the polar-optic-phonon scattering is the dominant mechanism at room temperature and is comparable to acoustic-phonon scattering at liquid-nitrogen temperatures. On the other hand, using Ridley's momentum-relaxation rates, the polar-optic-phonon scattering is ineffective in limiting the mobility at liquid-nitrogen temperatures and becomes comparable to acoustic-phonon scattering at room temperature.

With the use of momentum-relaxation rate of Ridley,[12] it appears that the acoustic-phonon scattering may continue to be a dominant mechanism up to room temperature if phonon scattering is considered. For device design, it will be quite appropriate to consider Eq. (14) for optimal thickness for maximum mobility. The detailed comparison with experimental results is sometimes difficult as the width of the quantum well is not always explicitly stated in the published literature. Kotera *et al.*[23] have obtained $\mu\sim T^{-n}$ with $n=1-1.5$, as compared to $\mu\sim T^{-1}$ predicted by Eq. (2.1).

Chiu *et al.*[24] performed an order-of-magnitude calculation for the qualitative comparison of the polar-optical-phonon limited mobility and the drift velocity in GaAs/AlGaAs high-electron mobility transistor. A shifted Maxwellian distribution is used to calculate the mobility relaxation time, which is thought by these authors to be a poor approximation. They found a two-dimensional electron-gas mobility lower than the bulk mobility, consistent with Eq. (4.4) or similar to Hess.[14] The simple approximate calculation of Ref. 24 indicates that as the electric field strength increases (or the effective thickness decreases), the mobility decreases, in disagreement with the measured mobility of two-dimensional electron gas which is considerably higher than that of bulk gas,[25] thereby calling for further investigation. One of the important reasons for the mobility enhancement in a quantum-well heterolayer, especially at low temperatures, is the removal of the donors supplying the conduction electrons outside the well.[8] Therefore, the ionized impurity scattering, which limits the mobility in bulk samples, is considerably reduced in a quantum-well heterolayer, resulting in a mobility enhancement. A useful comparison of Eq. (2.1) or (4.5) (which are obtained for mobility with no transverse gate field) with those in Ref. 24 is difficult because of the possible presence of hot-electron effects[19] and the qualitative nature of the calculations reported. It is quite possible that a dominant scattering mechanism (e.g., polar optic phonon scattering in GaAs) in bulk samples, may become a secondary mechanism in confined systems, as indicated via Eqs. (4.7) and (4.8). For thickness $d\ll d^*$, the acoustic-phonon scattering is the dominant mechanism of scattering, whereas for large thicknesses ($d>d^*$), the polar optic phonon may also contribute and may ultimately become dominant. In the former case ($d<d^*$), $\mu\sim T^{-1}$ is expected, whereas in the latter case ($d>d^*$), $n>1$ is expected if the mobility is expressed as a power law ($\mu\sim T^{-n}$). The experimental results of Hiyamizu *et al.*[25] ($\mu\sim T^{-2.08}$) may conform to the latter regime, not ignoring the fact that the hot-electron effects may still be important due to the change in the carrier distribution function.[19]

One of the most useful concepts in understanding what governs the relative position of electronic levels on either side of the interface is the concept of dipole layer, which in a simple form can be envisioned as a parallel plate capacitor.[26] In a field-effect transistor, where high field is present in transverse direction, an induced polarization similar to that predicted for the bulk semiconductor[19] may change the Ohmic transport parallel to the interface.

At low temperatures, two other scattering mechanisms may limit the mobility. These are alloy (al) scattering and point-defect (PD) scattering. Simplified models[27] of both of these scattering mechanisms give similar results as those for acoustic phonon scattering:

$$\mu_{\mathrm{PD}}/\mu_{\mathrm{PDb}}=\mu_{\mathrm{al}}/\mu_{\mathrm{alb}}=d/2\pi^{1/2}\lambda_D\ , \tag{5.5}$$

with

$$\mu_{\mathrm{PDb}}=4e\pi\hbar^4/3(2\pi m^*k_BT)^{1/2}m^{*2}n_dV_0^2\ , \tag{5.6}$$

$$\mu_{\mathrm{alb}}=4e\pi\hbar^4N_{\mathrm{al}}/3(2\pi m^*k_BT)^{1/2}\alpha(1-\alpha)(E_a-E_b)^2\ . \tag{5.7}$$

Here n_d is the volume density of point defects, V_0 is the potential parameter for point defect potential: $V=V_0\sum_{i=1}^{n_d}\delta(r-r_i)$, N_{al} is the number of atoms per unit volume in the crystal, α is the fraction of atoms of type a, and E_a-E_b is the difference between the band edge of atoms of type a and of type b constituting the alloy. The principal difference in behavior of the mobilities μ_{PD} and μ_{al} are in its temperature independence, in contrast to the acoustic-phonon scattering which depends inversely on temperature.

The scattering rates derived by Harrison and Hauser[28] differ from those used in Eq. (5.7) by a numerical factor. The mobility values of Eq. (5.5) are of the same order as acoustic phonon scattering and may limit the mobility at very low temperatures in alloy structures. Evidence of this scattering is available in the work of Ghosh and Layman,[29] who have indicated a decrease in transconductance with an increase of aluminum composition in the spacer layer.

In Ref. 7, a temperature-independent mobility was assumed to fit low-temperature experimental data which was attributed to interface scattering whose origin has not been explicitly stated. The expressions for ionized impurity scattering used in Ref. 7 are in error, as has been recently indicated,[6] and acoustic-phonon scattering is completely neglected. Ogale and Madhukar[30] have recently

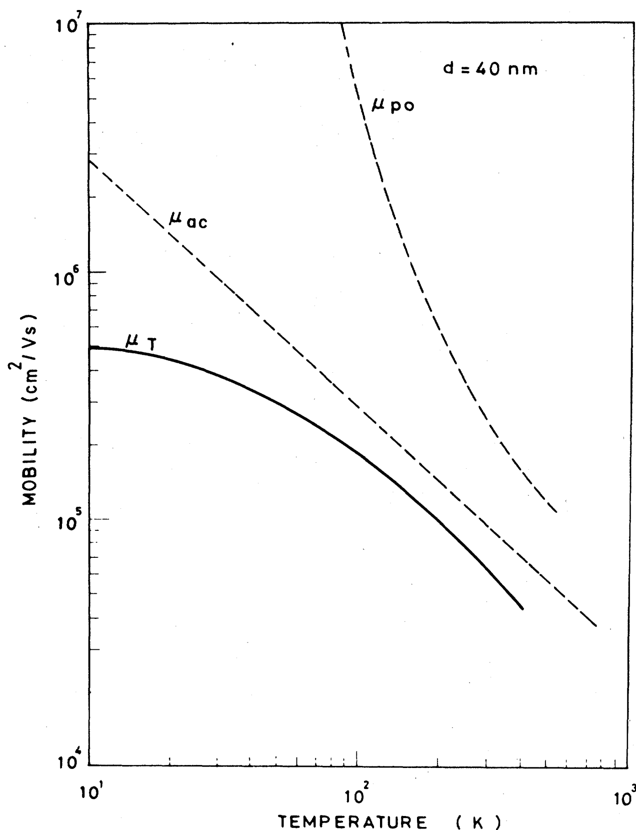

FIG. 1. Mobility in a quantum-well heterolayer, as a function of temperature, limited by acoustic-phonon scattering (μ_{ac}), polar-optical-phonon scattering (μ_{po}), and total mobility (μ_T), which includes the alloy scattering ($\mu_{al}=6.4\times10^5$ $cm^2/V\,s$).

shown that in a very narrow quantum-well structure one has to take account of the finite height of the confining potential barrier because of the large zero-point energy of the confined carriers. Dingle,[3] in a review article, discusses the change in energy due to the finite barrier height. For a symmetrical rectangular well, there is always at least one bound state for all values of the barrier height. The bound-state energy is smaller than that in an infinite potential well, indicating an increase in the effective thickness of the quantum well. The acoustic phonon scattering is then reduced (or mobility enhanced), and polar-optical-phonon absorption scattering is enhanced (or mobility reduced) due to this increase in the effective thickness. The conduction-band discontinuity in the GaAs/(Ga,Al)As interface is approximately 300 meV.[8] Price and Stern[31] have indicated that the penetration depth is small and hence the enhanced scattering due to the tunneling of electronic wave function can be neglected. At low temperatures, it can be argued, as concluded by Ogale and Madhukar,[30] that the tunneling of electrons into the classically forbidden region enhances the scattering from remote impurities because the carriers tunnel into a region when they are closer to impurities. But in that regime the temperature-independent alloy scattering may become more important than the ionized impurity scattering.

In Fig. 1 we show on a log-log plot the mobility as a function of temperature, limited by acoustic-phonon scattering [Eq. (2.1)], that limited by the absorption of polar optical phonons [Eq. (4.5)] and the combined mobility due to acoustic phonons, polar optical phonons, and the alloy structure [$\mu_{al}=6.4\times10^5$ $cm^2/V\,s$ from Eqs. (5.5) and (5.7) for parameters[10] appropriate to GaAs with $d=40$ nm]. It is clear from the plot that the alloy scattering is dominant at low temperature, acoustic-phonon scattering is dominant at intermediate temperature, and polar-optical-phonon scattering becomes important near room temperature.

One of the basic problems in identifying the dominant scattering mechanisms is the lack of knowledge of the growth history of the sample,[25,32–34] which may introduce additional centers of scattering. As Störmer[8] has pointed out, the MBE technique is now being perfected to the point where these unwanted scattering centers can be eliminated and mobility can be considerably enhanced.

To conclude, acoustic-phonon scattering is one of the important scattering mechanisms at intermediate temperatures. In a carefully grown sample, alloy scattering may become important at low temperatures. The case of phonon scattering by polar and nonpolar optical phonons needs a careful further investigation, perhaps starting with derivation of the transport equation from the fundamental principles, such as that using the density matrix, and by developing the appropriate sum rules for this scattering including multisubband effects.

Note added in proof: Since the time this article was submitted for publication, several new works have appeared which support the conclusions of this work. E. E. Mandez, P. J. Price, and M. Heilbum [Appl. Phys. Lett. **45**, 294 (1984)] have studied the temperature dependence of the mobility of two-dimensional electron gas formed at the interface of high-quality GaAs-GaAlAs heterostructures. The inverse mobility is shown to increase linearly with temperature at intermediate temperature with a slope which increases linearly with temperature, consistent with Eq. (2.1). A good agreement is found by using a value of 13.5 eV rather than 7.0 eV considered in the present paper. J. Lee and M. O. Vassell [Jpn. J. Appl. Phys. **23**, 1207 (1984)] have calculated the drift mobility of a quasi-two-dimensional electron gas by considering acoustic deformation and polar optical phonon scattering, using momentum relaxation rates. They find the scattering rates considerably larger than momentum relaxation rates as described by Eq. (4.6). The calculated mobility is found to be dominated by polar optical phonon scattering at thicknesses larger than critical thickness $d^*=135$ Å, and by acoustic phonon scattering at thicknesses smaller than d^*, in qualitative agreement with Eq. (4.7). G. Fishman and D. Calecki [Phys. Rev. B **29**, 5778 (1984)] consider the intersubband scattering in addition to intrasubband scattering considered here. W. Walukiewicz, H. E. Ruda, J. Lagouski, and H. C. Gatos [Phys. Rev. B **29**, 4818 (1984)] formulate a theoretical model for electron

scattering and calculate an inherent mobility limit imposed by phonon, alloy, and remote impurity scattering. The consideration of scattering rates in formulating a transport problem for polar-optical-phonon scattering tends to overestimate the contribution of this type of scattering. A theoretical framework which uses momentum relaxation rates in distinction from scattering rates is, therefore, needed for correct interpretation of experimen-

[1]T. Ando, A. B. Fowler, and F. Stern, Rev. Mod. Phys. **54**, 437 (1982).
[2]V. K. Arora, Phys. Rev. B **26**, 2247 (1982).
[3]R. Dingle, in *Festkörperprobleme XV (Advances in Solid State Physics)*, edited by H. J. Queisser (Pergamon, Braunschweig, Germany, 1975).
[4]V. K. Arora and F. G. Awad, Phys. Rev. B **23**, 5570 (1981).
[5]H. N. Spector, Phys. Rev. B **28**, 971 (1983).
[6]J. Lee, H. N. Spector, and V. K. Arora, Appl. Phys. Lett. **42**, 363 (1983); J. Appl. Phys. **54**, 6995 (1983).
[7]T. J. Drummond, H. Morkoç, K. Hess, and A. Y. Cho, J. Appl. Phys. **52**, 5231 (1981).
[8]H. L. Störmer, Surf. Sci. **132**, 519 (1983).
[9]H. L. Störmer, A. Pinczuk, A. C. Gossard, and Weigmann, Appl. Phys. Lett. **38**, 691 (1981).
[10]P. J. Price, Ann. Phys. **133**, 217 (1981).
[11]P. K. Basu and B. R. Nag, J. Phys. C **14**, 1519 (1981).
[12]B. K. Ridley, J. Phys. C **15**, 5899 (1982).
[13]S. Kawaji, J. Phys. Soc. Jpn. **27**, 906 (1969).
[14]K. Hess, Appl. Phys. Lett. **35**, 484 (1979).
[15]J. B. Roy, P. K. Basu, and B. R. Nag, Solid State Commun. **40**, 491 (1981).
[16]P. K. Basu and B. R. Nag, Phys. Rev. B **22**, 4849 (1980).
[17]E. Conwell, *High Field Transport in Semiconductors*, Suppl. 9 of *Solid State Physics*, edited by F. Seitz, D. Turnbull, and H. Ehrehreich (Academic, New York, 1967), p. 167.
[18]D. Roychoudhury and P. K. Basu, Phys. Rev. B **22**, 6325 (1980).
[19]V. K. Arora, Phys. Status Solidi B **115**, K107 (1983); Phys. Rev. B (to be published).
[20]P. J. Price, Surf. Sci. **113**, 199 (1982).
[21]B. R. Nag, *Electron Transport in Compound Semiconductors* (Springer, Berlin, 1980), p. 141.
[22]V. K. Arora, M. Prasad, and M. A. Al-Mass'ari, Physica **106B**, 311 (1981); V. K. Arora, Acta Cienc. Indica **3**, 334 (1978).
[23]N. Kotera, Y. Katayama, I. Yoshida, and K. F. Komatsubara, J. Vac. Sci. Technol. **9**, 754 (1972).
[24]L. C. Chiu, S. Margalit, and A. Yariv, Jpn, J. Appl. Phys. **22**, L82 (1983).
[25]S. Hiyamizu, T. Fujii, T. Mimura, K. Nanbu, J. Saito, and H. Hashimoto, Jpn. J. Appl. Phys. **20**, L455 (1981).
[26]A. Zur, T. C. McGill, and D. L. Smith, Surf. Sci. **132**, 456 (1983).
[27]V. K. Arora and H. N. Spector, J. Appl. Phys. **54**, 831 (1983).
[28]J. W. Harrison and J. R. Hauser, Phys. Rev. B **13**, 5347 (1976).
[29]C. L. Ghosh and R. L. Layman, IEEE Electron Devices Lett. **EDL-5**, 3 (1984).
[30]S. B. Ogale and A. Madhukar, J. Appl. Phys. **55**, 483 (1984).
[31]P. J. Price and F. Stern, Surf. Sci. **132**, 577 (1983).
[32]A. Madhukar, Surf. Sci. **132**, 344 (1983).
[33]R. Heckingbottom, G. J. Davies, and K. A. Prior, Surf. Sci. **132**, 375 (1983).
[34]G. Margaritondo, Surf. Sci. **132**, 469 (1983); Solid State Electron. **26**, 499 (1983).