Energy levels of two- and three-dimensional polarons in a magnetic field

F. M. Peeters
University of Antwerp (Universitaire Instelling Antwerpen), Department of Physics, Universiteitsplein 1, B-2610 Antwerpen-Wilrijk, Belgium

J. T. Devreese
University of Antwerp (Universitaire Instelling Antwerpen), Department of Physics, Universiteitsplein 1, B-2610 Antwerpen-Wilrijk, Belgium; University of Antwerp (Rijksuniversitair Centrum Antwerpen), B-2020 Antwerpen, Belgium; and Eindhoven University of Technology, Department of Physics, NL-5600 MB Eindhoven, The Netherlands

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The influence of the electron-phonon interaction on the Landau levels of two- (2D) and three-dimensional (3D) electrons are studied within second-order perturbation theory. It is found that polaron effects in ideal 2D systems are larger than in 3D systems which agrees with recent results of Das Sarma and Larsen. Analytical and numerical results are presented for the electron-phonon correction to the different Landau levels (n) below the LO-phonon continuum. Special attention is paid to the small and large magnetic-field limit and to the splitting of the Landau levels which occurs at $n\omega_c \approx \omega_0$ (ω_c is the cyclotron frequency and ω_0 is the LO-phonon frequency). Existing results are rederived and generalized.

I. INTRODUCTION

The energy levels of an electron in a magnetic field are quantized into Landau levels. If the electron moves in a polar semiconductor it also interacts with longitudinal-optical (LO) phonons; the Landau levels are then modified in the following way[1]: (i) They are shifted to lower energy. (ii) The slope of the Landau levels versus magnetic field is changed because of a mass renormalization of the electron. (iii) The Landau levels do not cross the energy level at $\hbar\omega_c/2+\hbar\omega_0$ because of Von Neumann's "noncrossing" theorem which leads to a *splitting* of degenerate energy levels. (iv) The Landau levels are pinned to the energy $\hbar\omega_0+\frac{1}{2}\hbar\omega_c$ in the limit of a large magnetic field.

In the present paper we calculate the energy levels of a two-dimensional (2D) and a three-dimensional (3D) polaron in a magnetic field at zero temperature and below the LO-phonon continuum. We will use the continuum model for the electron-phonon interaction, a parabolic conduction band for the electron will be adopted and only one electron is present in our system. The effect of the electron-phonon interaction on the energy levels will be studied within different types of second-order perturbation theory.

For the 3D polaron, considerable work has been devoted (see, e.g., Ref. 2 and references therein) to the study of the magnetic field dependence of the electron-phonon correction to the energy of the Landau levels. These studies were mainly concentrated on the weak magnetic field limit[2,3] or were restricted to the lowest Landau levels.[2,4] The influence of intermediate and strong electron-phonon coupling on the first two Landau levels was discussed by Larsen[2] and the present authors.[5] In the present paper the electron-phonon correction to all the Landau levels will be calculated within second-order perturbation theory for arbitrary magnetic-field strength but for energies below the LO-phonon continuum.

Recently there has been increasing interest in 2D quantum systems.[6] In, e.g., GaAs-$Al_xGa_{1-x}As$ heterostructures and p-Insb metal-oxide semiconductors (MOS) structures the electrons are confined to a 2D layer with a thickness of the order of 10—100 Å. In those weakly polar semiconductors polaron effects can be important.[7] In the present study we disregard the finite extent of the electron wave function in the direction perpendicular to the interface and consider the system as ideally two dimensional. We expect that this approximation will overestimate the polaron effects as was shown by Das Sarma[8] in the case of zero magnetic field. An advantage of considering the ideal 2D case is that analytical formulas for the energy levels can be obtained in closed form. Furthermore, we do not expect that finite subband effects will change the present results in a qualitative manner. Recently Das Sarma[9] and Larsen[10] studied the 2D weak coupling polaron in the presence of a magnetic field. They elaborated on the magnetic field dependence of the electron-phonon correction to the energy of the first two Landau levels and on the small magnetic field limit of the energy shift for arbitrary Landau level. We will generalize these results to arbitrary magnetic field strength and to arbitrary Landau level.

The present paper is organized as follows: In Sec. II explicit formulas for the electron-phonon correction to the different Landau levels are given within different types of second-order perturbation theory for all Landau levels both for the 2D and the 3D polaron. These formulas are valid for the whole magnetic field range and for energies below the continuum for emission of LO phonons. In Sec. III results are reported valid in the limit of small and large magnetic fields and for the resonant condition

$n\omega_c \approx \omega_0$. All these results are calculated here for arbitrary Landau level. We show that for the lowest Landau levels and for weak magnetic fields earlier results by Larsen[2] for the 3D polaron and by Das Sarma[9] and Larsen[10] for the 2D polaron are reobtained.

II. POLARON CORRECTIONS TO THE LANDAU LEVELS OF 2D AND 3D ELECTRONS

The energy levels of an electron in a magnetic field (at zero temperature) are shifted over ΔE_n by the interaction of the electron with the LO phonons,

$$E_n = (n+\tfrac{1}{2})\hbar\omega_c + \Delta E_n \; , \tag{1}$$

where $\omega_c = eB/m_b c$ with m_b the electron band mass. Within second-order perturbation theory the energy shift of the nth Landau level is given by[3,4]

$$\Delta E_n = -\sum_{m=0}^{\infty}\sum_{\mathbf{q}} \frac{|M_{nm}(\mathbf{q})|^2}{D_{nm}} \; , \tag{2}$$

where

$$M_{nm}(\mathbf{q}) = \langle m,k_z;\mathbf{q} \,|\, H_I \,|\, n,0;\mathbf{0}\rangle \tag{3}$$

is the matrix element of the electron-phonon interaction operator H_I between electron and phonon states. The ket $|m,k_z;\mathbf{q}\rangle = |m,k_z\rangle \otimes |\mathbf{q}\rangle$ describes a state composed of an electron in the Landau level m with momentum $\hbar k_z$ along the z direction (the z direction is chosen as the direction of the magnetic field) and an LO-phonon with momentum $\hbar\mathbf{q}$ and energy $\hbar\omega_0$. In the 2D case and with the magnetic field perpendicular to the 2D layer the electron cannot move along the z direction and as a result the electron state $|m\rangle$ is completely quantized (there is no electron momentum $\hbar k_z$ in the z direction). The electron-phonon interaction term in the Hamiltonian is

$$H_I = \sum_{\mathbf{q}} (V_{\mathbf{q}} a_{\mathbf{q}} e^{i\mathbf{q}\cdot\mathbf{r}} + V_{\mathbf{q}}^* a_{\mathbf{q}}^\dagger e^{-i\mathbf{q}\cdot\mathbf{r}}) \; , \tag{4}$$

with $a_{\mathbf{q}}$ ($a_{\mathbf{q}}^\dagger$) the annihilation (creation) operator of an LO phonon with momentum $\hbar\mathbf{q}$. The interaction coefficients are

$$|V_{\mathbf{q}}|^2 = \frac{1}{q^2}\frac{2\sqrt{2}\pi\alpha}{V} \quad \text{in 3D} \; , \tag{5a}$$

and

$$|V_{\mathbf{q}}|^2 = \frac{1}{q}\frac{\sqrt{2}\pi\alpha}{A} \quad \text{in 2D} \; , \tag{5b}$$

where we used units such that $\hbar = m_b = \omega_0 = 1$ and where $V(A)$ is the volume (surface area) of the system. The energy dominator in Eq. (2) is given by

$$D_{nm} = \hbar\omega_{\mathrm{LO}} - \Delta_n + \frac{\hbar^2 q_z^2}{2m} + \hbar\omega_c(m-n) \quad \text{in 3D} \tag{6a}$$

and

$$D_{nm} = \hbar\omega_{\mathrm{LO}} - \Delta_n + \hbar\omega_c(m-n) \quad \text{in 2D} \; . \tag{6b}$$

As was discussed by Lindemann *et al.*[4] the different types of perturbation theories are obtained by making the following choices for the energy shift Δ_n: (1) $\Delta_n = 0$ leads to the Rayleigh-Schrödinger perturbation theory (RSPT) which gives accurate results for ΔE_n if $\omega_c \ll \omega_0$; (2) $\Delta_n = \Delta E_n$ results in Wigner-Brillouin perturbation theory (WBPT) which can account for the splitting of the degenerate energy levels; and (3) $\Delta_n = \Delta E_n - \Delta E_0^{\mathrm{RSPT}}$ gives an "improved Wigner-Brillouin theory" (IWBPT) as developed in Ref. 4 and which gives the correct pinning behavior for α small. In Ref. 11 it was found that IWBPT gives good results for $\alpha \lesssim 0.1$. $\Delta E_0^{\mathrm{RSPT}}$ is the weak-coupling electron-phonon correction to the electron ground-state energy as calculated within RSPT. By inspection of Eqs. (2), (6a) and (6b) one has

$$\Delta E_0^{\mathrm{IWBPT}} = \Delta E_0^{\mathrm{RSPT}} \quad \text{for all } \omega_c \; .$$

Instead of inserting the explicit expression for the matrix element $M_{nm}(\mathbf{q})$ into Eq. (2) and then performing the $\mathbf{q}$ integral, as is traditionally done,[3,4] we will follow a different approach which enables us to get rid of the summation in Eq. (2) over the different Landau levels. In Eq. (2) insert

$$\frac{1}{D_{nm}} = \int_0^\infty du\, e^{-D_{nm}u} \; , \tag{7}$$

which is meaningful when $D_{nm} > 0$. Physically this means that we are limiting ourselves to the study of the Landau levels below the LO-phonon continuum. Inserting Eq. (7) into Eq. (2) and using the explicit form for H_I [see Eq. (4)] we obtain, after some rearrangements, the following result:

$$E_n = -\sum_{\mathbf{q}} |V_{\mathbf{q}}|^2 \int_0^\infty du\, e^{-(1-\Delta_n)u} \langle n,\mathbf{0} \,|\, e^{i\mathbf{q}\cdot\mathbf{r}(u)} \times e^{-i\mathbf{q}\cdot\mathbf{r}(0)} \,|\, n,\mathbf{0}\rangle \; , \tag{8}$$

where the average over the phonon state was taken [in Eq. (8) $\hbar = m_b = \omega_0 = 1$]. In Eq. (8) $\mathbf{r}(u)$ is the electron position operator at imaginary time $u = -it$ as described by the Hamiltonian without electron-phonon interaction. From Refs. 12 and 13 one has, for real time,

$$x(t) = -\frac{\pi_2}{\omega_c} - \frac{1}{\sqrt{2\omega_c}}(Ce^{-i\omega_c t} + C^\dagger e^{i\omega_c t}) \; , \tag{9a}$$

$$y(t) = \frac{\pi_1}{\omega_c} - \frac{i}{\sqrt{2\omega_c}}(Ce^{-i\omega_c t} - C^\dagger e^{i\omega_c t}) \; , \tag{9b}$$

$$z(t) = z(0) + \frac{p_z}{m}t \; , \tag{9c}$$

where in the 2D case no z direction has to be considered. π_1 and π_2 are the center of orbit coordinate operators which have the commutation relation $[\pi_1,\pi_2] = i\omega_c$. The creation ($C^\dagger$) annihilation ($C$) operators for a Landau state have the well-known commutation relation $[C,C^\dagger] = 1$. Using the techniques outlined in Sec. III D of Ref. 11, we obtain

$$e^{i\mathbf{q}\cdot\mathbf{r}(t)}e^{-i\mathbf{q}\cdot\mathbf{r}(0)}=e^{iq_zp_zt}e^{iq_z^2t}\exp\left[\frac{iq_+}{\sqrt{2\omega_c}}(1-e^{i\omega_c t})C^\dagger\right]\exp\left[\frac{iq_-}{\sqrt{2\omega_c}}(1-e^{-i\omega_c t})C\right]\exp\left[-\frac{q_\perp^2}{2\omega_c}(1-e^{-i\omega_c t})\right], \tag{10}$$

where $q_\pm=q_x\pm iq_y$ and $q_\perp^2=q_x^2+q_y^2$. In Eq. (10) we need the expectation value

$$\langle n\,|\,e^{a_+C^\dagger}e^{a_-C}\,|\,n\rangle=\sum_{m=0}^{n}\begin{bmatrix}n\\m\end{bmatrix}\frac{(a_+a_-)^n}{m!}\,,$$

which together with Eq. (10) gives, for imaginary time $t=iu$,

$$\langle n,0\,|\,e^{i\mathbf{q}\cdot\mathbf{r}(u)}e^{-i\mathbf{q}\cdot\mathbf{r}(0)}\,|\,n,0\rangle=e^{-q_z^2u/2}\exp\left[-\frac{q_\perp^2}{2\omega_c}(1-e^{-\omega_c u})\right]\sum_{m=0}^{n}\begin{bmatrix}n\\m\end{bmatrix}\frac{1}{m!}\left[\frac{2q_\perp^2}{\omega_c}\sinh^2\left[\frac{\omega_c u}{2}\right]\right]^m. \tag{11}$$

Inserting Eq. (11) into Eq. (8) and performing the $\mathbf{q}$ integral we obtain, for the energy shift in 3D,

$$\Delta E_n=\frac{\alpha}{\sqrt{\pi\omega_c}}\int_0^\infty\frac{du}{\sqrt{u}}e^{-u(1-\Delta_n)/\omega_c}G_n(u)\,, \tag{12}$$

where we defined the function

$$G_n(u)=\sum_{m=0}^{n}\begin{bmatrix}n\\m\end{bmatrix}\frac{(2m-1)!!}{m!}[A(u)]^m\times\sum_{k=0}^{m}\begin{bmatrix}m\\k\end{bmatrix}\frac{(-1)^{m-k}}{F(u)^k}B_k(u)\,, \tag{13}$$

with

$$A(u)=\frac{2}{u}\frac{\sinh^2(u/2)}{C(u)}\,, \tag{14a}$$

$$C(u)=1-F(u)\,, \tag{14b}$$

$$F(u)=\frac{1}{u}(1-e^{-u})\,, \tag{14c}$$

$$B_0(u)=\frac{1}{\sqrt{C(u)}}\ln\left[\frac{1+\sqrt{C(u)}}{\sqrt{F(u)}}\right], \tag{14d}$$

$$B_k(u)=\sum_{s=0}^{k-1}\frac{(-1)^s}{2s+1}\begin{bmatrix}k-1\\s\end{bmatrix}C(u)^s,\quad k\geq 1 \tag{14e}$$

and $(2m-1)!!=(2m-1)(2m-3)\times\cdots\times 3.1$, where $(2m-1)!!=1$, for $m=0$. Note that $F(0)=1$, $C(0)=0$, $B(0)=1$, $A(0)=1$, and thus $G_n(0)$ is finite and the integrand in Eq. (12) is defined at $u=0$. For $u\to\infty$ we obtain $G_n(u)\sim e^{nu}$ and Eq. (12) is meaningful when $n\omega_c<\omega_0-\Delta_n$. Equation (12) is a generalization to arbitrary Landau level n of the result given in the Appendix[14] of Ref. 4 for $n=0,1,2,3$.

In the 2D case we have to disregard the z direction in the above formulas. A calculation analogous to the above 3D case leads to the rather simple result

$$\Delta E_n=-\alpha\frac{\pi}{2}\frac{1}{(\omega_c)^{1/2}}\sum_{m=0}^{n}\begin{bmatrix}n\\m\end{bmatrix}\left[\frac{(2m-1)!!}{2^m}\right]^2\frac{1}{m!}\times\frac{\Gamma((1-\Delta_n)/\omega_c-m)}{\Gamma((1-\Delta_n)/\omega_c+\frac{1}{2})}\,, \tag{15}$$

with $\Gamma(x)$ the gamma function. For $\Delta_n=0$, Eq. (15) was recently obtained by Larsen[10] for $n=0,1,2$.

In Fig. 1 the first three Landau levels are plotted as function of the magnetic field for an electron-phonon coupling constant $\alpha=0.1$. The thin-dashed lines represent the unperturbed energy levels. The Landau levels for the 2D (thick, full curves) and the 3D (thick, dashed curves) polaron are obtained from Eqs. (15) and (12), respectively, within the IWBPT. From Fig. 1 it is apparent that the polaron energy levels are: (1) shifted to lower energies, (2) at small magnetic fields they are bended downward due to a mass renormalization, (3) at $n\omega_c\approx\omega_0$ the nth Landau levels do not cross the LO phonon + $n=0$ Landau level, and (4) for $\omega_c\to\infty$ all the Landau levels become pinned to $\hbar\omega_0+\Delta E_0^{\mathrm{RSPT}}$. When one compares the 2D result with the equivalent 3D result one notes that in 2D the energy shift, the mass renormalization, and the level splitting at $n\omega_c\approx\omega_0$ are more pronounced than for the 3D polaron.

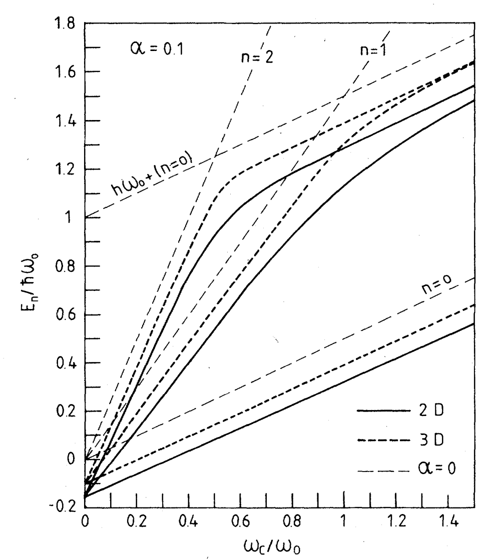


FIG. 1. The first three Landau levels as function of the magnetic field for the unperturbed energy levels (thin dashed curves), the 2D polaron (thick solid curves) and the 3D polaron (thick dashed curves). The electron-phonon coupling constant is $\alpha=0.1$.

This confirms a similar conclusion made recently by Das Sarma.[9]

III. ANALYTICAL RESULTS FOR LIMITING VALUES OF THE MAGNETIC FIELD

A. Weak magnetic field limit ($\omega_c/\omega_0 \ll 1$)

The electron-phonon correction, ΔE_n, to the Landau levels will be calculated in this section in the limit of small magnetic fields, i.e., $\omega_c \ll \omega_0$. For the 3D polaron, this is realized by expanding the function $G_n(u)$ in Eq. (12) in powers of u

$$G_n(u)=1+\frac{2n+1}{6}u+\frac{18n^2+18n-1}{180}u^2+\frac{90n^3+135n^2-37n-6}{3780}u^3+\cdots ,$$

which results in

$$\Delta E_n=-\frac{\alpha}{1-\Delta_n}\left[1+\frac{2n+1}{12}\left[\frac{\omega_c}{1-\Delta_n}\right]+\frac{18n^2+18n-1}{240}\left[\frac{\omega_c}{1-\Delta_n}\right]^2+\frac{90n^3+135n^2-37n-6}{2016}\left[\frac{\omega_c}{1-\Delta_n}\right]^3+\cdots\right]. \tag{16}$$

When $\omega_c/\omega_0 \ll 1$, nondegenerate perturbation theory (i.e., RSPT) has to be used and one has $\Delta_n=0$. The magnetic field correction to the electron-phonon self-energy for weak magnetic fields is then given by

$$\Delta E_n=-\alpha\left[1+\frac{2n+1}{12}\omega_c+\frac{18n^2+18n-1}{240}\omega_c^2+\frac{90n^3+135n^2-37n-6}{2016}\omega_c^3+\cdots\right], \tag{17}$$

where ω_c and $\Delta E_n/\hbar$ are in units of ω_0. Note that the polaron corrections are larger for higher Landau levels. The expression for the ground-state energy ($n=0$) and for the first excited state ($n=1$) was already obtained by Larsen.[2] Equation (17) is a generalization of Larsen's result to arbitrary Landau energy level.

The electron-phonon contribution to the energy splitting between two successive Landau levels can now easily be obtained from Eq. (17):

$$\Delta E_{n+1}-\Delta E_n=-\alpha\frac{\omega_c}{6}\left[1+\frac{9(n+1)}{10}\omega_c+\frac{135n(n+2)+94}{168}\omega_c^2+\cdots\right]. \tag{18}$$

The cyclotron resonance frequency,

$$\hbar\omega_c^*=E_1-E_0 , \tag{19}$$

defines a cyclotron mass $m^*/m_b=\omega_c/\omega_c^*$ (m_b is the electron band mass), which for small magnetic fields is given by

$$\frac{m^*}{m_b}=\frac{1}{1-\frac{\alpha}{6}(1+\frac{9}{10}\omega_c+\frac{47}{84}\omega_c^2+\cdots)} . \tag{20}$$

For the 2D polaron problem we insert the asymptotic expansion of the Γ function for large argument into Eq. (15) and obtain

$$\Delta E_n=-\alpha\frac{\pi/2}{\sqrt{1-\Delta_n}}\left[1+\frac{2n+1}{8}\frac{\omega_c}{1-\Delta_n}+\frac{18n(n+1)+1}{128}\left[\frac{\omega_c}{1-\Delta_n}\right]^2+\frac{5}{1024}(2n+1)(10n^2+10n-1)\left[\frac{\omega_c}{1-\Delta_n}\right]^3+\cdots\right], \tag{21}$$

which to first order in α, (i.e., $\Delta_n=0$) becomes

$$\Delta E_n=-\alpha\frac{\pi}{2}\left[1+\frac{2n+1}{8}\omega_c+\frac{18n(n+1)+1}{128}\omega_c^2+\frac{5}{1024}(2n+1)(10n^2+10n-1)\omega_c^3+\cdots\right]. \tag{22}$$

The same expression up to ω_c^2 and for all n was recently found by Larsen in Ref. 10. For $n=0$ and $n=1$ the result (22) agrees with the corrected result of Das Sarma.[9]

The electron-phonon contribution to the energy splitting between two successive Landau levels in 2D in the limit $\omega_c/\omega_0 \ll 1$ is given by

$$\Delta E_{n+1}-\Delta E_n=\alpha\frac{\pi}{8}\omega_c\left[1+\frac{9(n+1)}{8}\omega_c+\frac{5}{128}(30n^2+60n+29)\omega_c^2+\cdots\right], \tag{23}$$

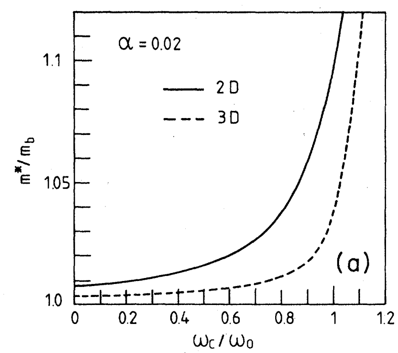
which leads to the cyclotron mass

$$\frac{m^*}{m_b}=\frac{1}{1-\alpha\frac{\pi}{8}(1+\frac{9}{8}\omega_c+\frac{145}{128}\omega_c^2+\cdots)} . \tag{24}$$

In Fig. 2 we plotted the cyclotron mass (calculated within the IWBPT) for the 2D and the 3D polaron in function of the magnetic field for two values of the electron-phonon coupling constant: (i) $\alpha=0.02$ [Fig. 2(a)] which corresponds to the case of InSb and (ii) $\alpha=0.07$ [Fig. 2(b)] which is the electron-phonon coupling constant of GaAs. Figs. 2(a) and 2(b) show clearly the enhancement of the mass renormalization due to polaron effects in systems with reduced dimensionality. The strong enhancement of the cyclotron mass around $\omega_c \approx \omega_0$ is a consequence of the pinning of the Landau levels to the energy $\hbar\omega_c/2+\hbar\omega_0$.

B. Splitting and pinning of the Landau levels

To study the splitting and pinning of the Landau level E_n we are interested, respectively, in the region where $n\omega_c \approx \omega_0$ and $\omega_c/\omega_0 \rightarrow \infty$. For the Landau level E_n at a magnetic field such that $n\omega_c=\omega_0$, two energy levels cross[15] (see Fig. 1): namely the energy level with an electron in the Landau level n becomes degenerate with the energy level in which the electron is in the lowest Landau level and where one real LO phonon is present. For $n\omega_c \gtrsim \omega_0$, Rayleigh-Schrödinger perturbation theory is no longer applicable to calculate E_n. One has to use degenerate perturbation theory, i.e., WBPT or its improved version IWBPT, to calculate E_n self-consistently.



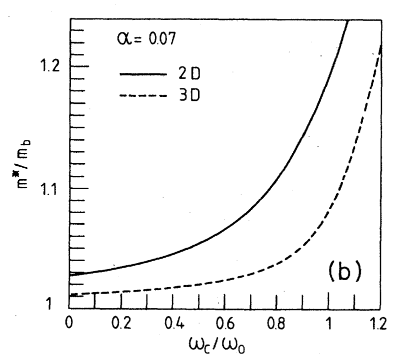


FIG. 2. Cyclotron resonance mass as function of the magnetic field for the 2D (solid curve) and the 3D (dashed curve) polaron and for (a) $\alpha=0.02$ and (b) $\alpha=0.07$.

In the magnetic field range under study one has $1-\Delta_n \simeq \omega_c$ and as a result the dominant contribution to the integral in Eq. (12) is obtained from the behavior of $G_n(u)$ for $u \rightarrow \infty$. In the limit $u \rightarrow \infty$, $G_n(u) \simeq A_n e^{nu}$ with

$$A_n=\frac{(2n-1)!!}{2^n n!}\sum_{s=0}^{n-1}\frac{(-1)^s}{2s+1}\begin{bmatrix} n-1 \\ s \end{bmatrix} ,$$

which can be simplified to $A_n=1/2n$ for $n \geq 1$. After performing the u integral in Eq. (12) we obtain for the electron-phonon contribution to the Landau level $n \geq 1$,

$$\Delta E_n=\frac{\alpha/2n}{\sqrt{1-\Delta_n-n\omega_c}} . \tag{25}$$

For $n=1$ and $\Delta_1=E_1-\frac{3}{2}\omega_c=\Delta E_1$, Eq. (25) reduces to the WBPT result of Larsen[2] (see also Ref. 15).

When $n\omega_c=\omega_0$, the electron-phonon interaction lifts the degeneracy of the energy levels E_n and $E_0+\hbar\omega_0$. The splitting between E_n and $E_0+\hbar\omega_0$ within WBPT and IWBPT is twice

$$|\Delta E_n|=(\alpha/2n)^{2/3} \tag{26}$$

in the limit $\alpha \ll 1$. For $n=1$ the $\alpha^{2/3}$ dependence of the splitting was already found by Levinson and Rashba.[16] Note also that the splitting is smaller for higher Landau levels!

In the limit $\omega_c \rightarrow \infty$, it is still possible to find a solution for Eq. (25). Using WBPT, i.e., $\Delta_n=\Delta E_n$, we found to lowest order in α

$$E_n=1+\tfrac{1}{2}\omega_c-\frac{1}{4n^4}\frac{\alpha^2}{\omega_c^2} . \tag{27}$$

It then follows that the Landau level E_n is *pinned* to $\hbar\omega_0+\hbar\omega_c/2$ at large magnetic fields. When one applies IWBPT the following result is found:

$$E_n=1+\Delta E_0^{\mathrm{RSPT}}+\tfrac{1}{2}\omega_c-\frac{1}{4n^4}\frac{\alpha^2}{\omega_c^2} , \tag{28}$$

and the Landau level n is pinned to $\hbar\omega_0+\hbar\omega_c/2$ shifted with the electron-phonon contribution to the electron ground-state energy calculated within RSPT. The result given by Eq. (28) for $\omega_c \rightarrow \infty$ is the correct one. The α^2 dependence in Eqs. (28) and (29) was also obtained in Ref. 16.

For the 2D polaron in the magnetic field region, $1-\Delta_n \simeq n\omega_c$, the term $m=n$ gives the dominant contribution to the sum of Eq. (15) and we obtain

$$\Delta E_n=-\frac{\sqrt{\pi}(2n-1)!!}{2^{n+1}n!}\frac{\alpha\sqrt{\omega_c}}{1-\Delta_n-n\omega_c} . \tag{29}$$

The splitting of the Landau level n at $n\omega_c=\omega_0$ is twice

$$|\Delta E_n|=\alpha^{1/2}\left[\left[\frac{\pi}{n}\right]^{1/2}\frac{(2n-1)!!}{2^{n+1}n!}\right]^{1/2} . \tag{30}$$

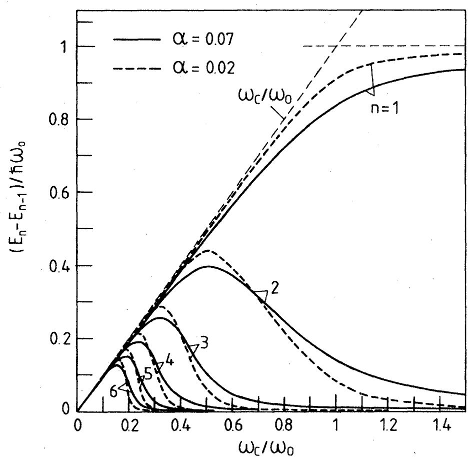


FIG. 3. Energy difference between two successive Landau levels for the 2D polaron as function of the magnetic field for $\alpha=0.02$ (dashed curves) and for $\alpha=0.07$ (solid curves).

For $n=1$, this $\sqrt{\alpha}$ dependence of the splitting was recently obtained by Das Sarma.[9,17] Note the difference between 2D [Eq. (30)] and 3D [Eq. (26)] in the α dependence of ΔE_n at $n\omega_c=\omega_0$.

In the asymptotic limit of large magnetic fields the Landau n becomes, within WBPT,

$$E_n=1+\tfrac{1}{2}\omega_c-\frac{\alpha}{\sqrt{\omega_c}}\frac{\sqrt{\pi}(2n-1)!!}{2^{n+1}n!}\,, \tag{31}$$

while for the IWBPT we get the following result:

$$E_n=1+E_0^{\mathrm{RSPT}}+\tfrac{1}{2}\omega_c-\frac{\alpha}{\sqrt{\omega_c}}\frac{\sqrt{\pi}(2n-1)!!}{2^{n+1}n!}\,. \tag{32}$$

The energy difference between two successive Landau levels,

$$E_n-E_{n-1}=\hbar\omega_c+(\Delta E_n-\Delta E_{n-1})\,,$$

is plotted in Fig. 3 as a function of the magnetic field for $\alpha=0.02$ and $\alpha=0.07$ in the case of a 2D polaron. For $n=1$, the energy difference $\hbar\omega_c=E_1-E_0$ becomes equal to the LO-phonon energy ω_0 in the limit $\omega_c\to\infty$. For transitions between higher Landau levels, the transition frequency $(E_n-E_{n-1})/\hbar$ decreases to zero when $\omega_c\to\infty$. Because of the pinning of the energy levels E_n $(n>0)$ to $\hbar\omega_0+\Delta E_0^{\mathrm{RSPT}}+\hbar\omega_c/2$.

IV. CONCLUSIONS

It is interesting to compare our result for the spectrum of a polaron in a magnetic field in the limit of a weak magnetic field with the semiclassical result of Bajaj.[18] His calculation was based on the Bohr-Sommerfeld quantization rule and resulted in 3D in

$$\Delta E_n=-\alpha\left[1+\frac{n+\frac{1}{2}}{6}\omega_c+\frac{3(n+\frac{1}{2})^2}{40}\omega_c^2+\cdots\right], \tag{33}$$

and in 2D one obtains[9]

$$\Delta E_n=-\alpha\frac{\pi}{2}\left[1+\frac{n+\frac{1}{2}}{4}\omega_c+\frac{9(n+\frac{1}{2})^2}{64}\omega_c^2+\cdots\right]. \tag{34}$$

Comparing Eqs. (33) and (34) with Eqs. (17) and (22) we note that Bajaj's result is correct to order ω_c but that the coefficient of the ω_c^2 term is incorrect. It is the latter coefficient which determines the magnetic field correction to the cyclotron mass to order ω_c. Note that the term ω_c^2 term in Eqs. (33) and (34) is asymptotically correct for $n\to\infty$, i.e., in the classical limit.

The influence of the electron-phonon interaction on the cyclotron resonance line of electrons in inversion layers on InSb was recently observed by Horst *et al.*[7] An enhancement of polaron effects was observed in this confined electron system in comparison with the 3D polaron in InSb which is in qualitative agreement with the present calculation and the results of Das Sarma[9,17] and Larsen.[10] Because of the finite electron layer width and the importance of nonparabolic band effects in InSb, no direct comparison is possible between our results and the experimental results of Horst *et al.*[7]

In order to observe transitions between higher Landau levels one has to populate these levels first. This can be done by heating up the electrons by, e.g., an electric field. The feasibility of this method experimentally was demonstrated in Refs. 4 and 19 in the case of 3D polarons in GaAs. It would be of interest to do a similar experiment for inversion layers and to investigate the influence of the electron-phonon interaction on the different Landau levels. The splitting of the energy levels can be compared with the present results. One has to keep in mind that in our investigation some simplifying assumptions (parabolic conduction band, ideal 2D layer, one-electron picture, influence of impurities is neglected, etc.) have been made which can make a possible direct quantitative comparison with experiment less straightforward.

In conclusion, we have calculated the energy spectrum of a 2D and 3D polaron in a magnetic field below the LO-phonon continuum to second order in the electron-phonon coupling. Our result, which was presented in a closed analytical form, is valid for zero temperature, arbitrary magnetic field strength and arbitrary Landau level number n. The analytical analysis for $\omega_c/\omega_0\ll 1$, $n\omega_c\approx\omega_0$, $\omega_c\to\infty$, and our numerical results for arbitrary ω_c for the polaron energy levels, indicate that in ideal 2D systems the polaron effects are considerably enhanced with respect to 3D systems. This conclusion generalizes the results by Das Sarma[9,17] and Larsen[10] to arbitrary Landau level n.

The polaron energy levels above the LO-phonon continuum have a finite lifetime. To calculate the polaron spectrum in this energy range one can apply similar techniques as were used in Ref. 20 for the 3D polaron.

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