Domain-wall renormalization-group study of the random Heisenberg model

W. L. McMillan*
Department of Physics and Materials Research Laboratory, University of Illinois at Urbana-Champaign, 1110 W. Green St., Urbana-Illinois 61801
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The random Heisenberg model with a Gaussian distribution of nearest-neighbor interactions is studied for the pure spin-glass case where the average interaction vanishes. The distribution of domain-wall energies at zero temperature is calculated using a spin-quench algorithm to find the ground-state energy for finite lattices. A renormalization-group transformation is set up which preserves the domain-wall energy distribution when the lattice parameter is changed. In the strong-coupling regime (zero temperature) the model iterates toward weak coupling and therefore exhibits a "phase transition at zero temperature" in both two and three dimensions. The correlation-length exponent is $\nu=0.714\pm0.015$ in two dimensions and $\nu=1.54\pm0.19$ in three dimensions. The lower critical dimension is four.

I. INTRODUCTION

It has recently been shown[1–3] that the three-dimensional random Ising model exhibits a spin-glass phase transition at finite temperature, whereas in two dimensions there is only a "phase transition at zero temperature".[4] Here, we study the random Heisenberg model using the domain-wall renormalization-group (DWRG) method[4] and find a phase transition at zero temperature in both two and three dimensions and a lower critical dimension of four.

II. DOMAIN-WALL RENORMALIZATION GROUP

We consider a hypercubic lattice of dimensionality d with n^d sites, lattice spacing a, and lattice size $L=na$. The Hamiltonian is

$$H=-\sum_{ij} J_{ij}\vec{S}_i\cdot\vec{S}_j \tag{1}$$

with nearest-neighbor interactions J_{ij} chosen from a Gaussian distribution with zero mean and variance $\tilde{J}$. We choose either periodic boundary conditions in all d directions or antiperiodic boundary conditions in one direction and periodic boundary conditions in the others; antiperiodic boundary conditions introduce a domain wall. For a given configuration of interactions let E^p and E^a be the ground-state energy for periodic and antiperiodic boundary conditions, respectively. The domain-wall energy at zero temperature is then $W=E^a-E^p$. For each configuration of interactions, one obtains a different domain-wall energy and one is interested in the distribution function of the domain-wall energies. We characterize that distribution by its mean $\overline{W}$ and variance $\tilde{W}$; $\overline{W}$ is zero for the model considered here.

We set up a renormalization-group transformation by considering two systems with the same physical size but different spacings. The first system has lattice spacing a, n^d lattice sites with $L=na$, and Hamiltonian parameter $\tilde{J}$. The second system has lattice spacing $a'>a$, n'^d lattice sites with $L=n'a'=na$, and Hamiltonian parameter $\tilde{J}'$. We require that the two systems have the same macroscopic properties and that the variances of the distribution of the domain-wall energies be equal:

$$\tilde{W}_{n'}(\tilde{J}')=\tilde{W}_n(\tilde{J}) . \tag{2}$$

We choose $\tilde{J}'$ so that (2) is satisfied and (2) is the implicit recursion relation for $\tilde{J}$.

III. SPIN-QUENCH ALGORITHM

In order to implement the DWRG method we need a numerical algorithm to generate the ground-state energy for a particular configuration of interactions. We begin with a randomly chosen configuration of spins and use a spin-quench algorithm to find the ground state or possibly a metastable state. We examine each spin in turn, find the local field acting on that spin due to interactions with its neighbors, and rotate the spin to its minimum energy configuration in the local field. This procedure is iterated until the total energy converges to the desired accuracy. We find metastable states for the larger lattices in all three dimensionalities and we use several starting spin configurations in the search for the ground state. In this way we find the ground-state energy for periodic and antiperiodic boundary conditions; the energy difference is the domain-wall energy for that particular configuration of interactions. The calculation is repeated for N configurations of interactions, where N is between 5000 and 20 000, to provide N samples of the domain-wall energy from which we estimate the variance of the domain-wall energy distribution. For a model with a constant density of metastable states we expect the variance of the domain-wall energy distribution to converge as $1/M$ for large M, where M is the number of starting spin configurations used in the search for the ground state. We observe this $1/M$ behavior and use it to extrapolate the data to large M; the

TABLE I. Spin-quench results for the random Heisenberg model. d is the dimensionality, n^d is the number of lattice sites, N is the number of interaction configurations, $\tilde{J}$ is the variance of the interaction distribution, and $\tilde{W}$ is the variance of the domain-wall energy distribution. The errors quoted are three estimated standard deviations.

d	n	N	$\tilde{W}/\tilde{J}$
2	3	10000	1.259±0.030
2	4	10000	0.904±0.021
2	5	10000	0.689±0.017
2	6	10000	0.530±0.013
2	8	10000	0.353±0.008
2	12	5000	0.202±0.007
3	3	10000	1.870±0.042
3	4	10000	1.449±0.032
3	5	9000	1.231±0.030
3	6	5000	1.122±0.038
4	3	20000	2.849±0.045
4	4	10000	2.718±0.063

corrections are small and we use an M of three or four starting spin configurations.

IV. RESULTS

The spin-quench results for the variance of the domain-wall energy distribution are given in Table I. The errors quoted are three estimated standard deviations. The scaling form for $\tilde{W}$ is

$$\tilde{W}_n = B n^{\lambda} \tilde{J} . \quad (3)$$

We fit (3) to the data in Table I using a least-squares procedure with the χ^2 test; the fitting parameters are given in Table II for several ranges of n. The value of χ^2 normalized to the largest acceptable value of χ^2 at the 98% confidence level, is given in the table; if this ratio is less than 1, the fit is acceptable. In two dimensions we find an unacceptable fit when lattice sizes from 3^2 to 12^2 are included, a marginal fit when 4^2 to 12^2 are included, and a good fit when only 5^2 to 12^2 are included. Corrections to scaling are important for the smallest lattice and the exponent is biased when the smallest lattice is included in the fit (for the unacceptable fits we estimate the uncertainty in the eigenvalue from the value which causes χ^2 to double). The bias in the eigenvalue is no larger than the statistical uncertainty in the eigenvalue, even when the

TABLE II. Fitting parameters for the least-squares fits of the data in Table I to the scaling form of Eq. (3). d is the dimensionality, "range" is the range if n values are used in the fit, "χ^2" is the ratio of the value of χ^2 found in the fit to the largest acceptable value at the 98% confidence level, and "eigenvalue" is the λ parameter in Eq. (3). The errors quoted are from the χ^2 test at the 98% confidence level.

d	Range	χ^2	eigenvalue
2	$3 \le n \le 12$	4.82	−1.32±0.07
2	$4 \le n \le 12$	1.08	−1.37±0.04
2	$5 \le n \le 12$	0.17	−1.40±0.03
3	$3 \le n \le 6$	2.92	−0.77±0.08
3	$4 \le n \le 6$	0.80	−0.65±0.08
4	$3 \le n \le 4$		−0.16±0.07

corrections to scaling are sufficiently large to be detected with the χ^2 test. The errors quoted herein are the statistical errors and one should keep in mind that the systematic errors may be of comparable magnitude.

In two dimensions, the eigenvalue is $\lambda = -1.40 \pm 0.03$. The interaction iterates toward weak coupling and there is a phase transition at zero temperature with correlation-length exponent $\nu = -1/\lambda = 0.174 \pm 0.015$. In three dimensions the eigenvalue is $\lambda = -0.65 \pm 0.08$. The interaction iterates toward weak coupling and there is a phase transition at zero temperature with correlation-length exponent $\nu = -1/\lambda = 1.54 \pm 0.19$. In four dimensions we find $\lambda = -0.16 \pm 0.07$; given the systematic errors found for small lattices in two and three dimensions we believe that this value is not significantly different from zero and that the lower critical dimension (at which the eigenvalue vanishes) for the random Heisenberg model is four. Several theoretical papers[5–7] have predicted that the lower critical dimension of the random Ising model is four; that prediction is now known to be incorrect.[1,2]

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*Deceased.

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