Comparison of models with competing interactions: Axial next-nearest-neighbor Ising versus two-plus-four spin interactions

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The phase diagrams of the one-dimensional quantum versions of the two-dimensional axial next-nearest-neighbor Ising (ANNNI) and the competing two-plus-four spin-interaction models are compared. The analysis of the zero-temperature wave-vector-dependent spectrum indicates a qualitative difference between the two models: while our finite-chain results support the existence of an incommensurate phase for the ANNNI case, they suggest that there is no such phase in the two-plus-four spin case.

Models of systems with competing interactions have received considerable attention because of the rich phase structure of such systems. The complexity of the phases includes features such as incommensurate phases,[1] multiphase points with infinite-ground-state degeneracy,[2] Lifshitz points,[3] and disorder lines.[4]

The axial next-nearest-neighbor Ising (ANNNI) model and its quantum analog is one of the simplest realizations of such complex structures and has been studied extensively.[5] But other models describing competition of the interactions have been proposed. In particular, multispin interactions are being investigated and applied to fields as diverse as surface,[6] plasma,[7] and nuclear physics.[8] It is now natural to ask about the features of such multispin systems in the case when the interactions compete. This has been done by one of the authors[9] for the $2+4$ spin model concluding that the phase diagram is similar to the one of the ANNNI model. A qualitative argument[10] indicates, however, that the (difficult to study) incommensurate phase is absent in this case. In this note we wish to present evidence that in this respect, the multispin model is indeed different from the ANNNI model. The criteria to identify modulated phases are quite limited for the various approximations used for these models. Our conclusions are based on the wave-vector-dependent spectrum obtained from finite-chain calculations. The two models have different spectra in the region where the competition of the interactions is important. We first define the models, then sketch their phase diagrams, and finally compare the numerical data of the finite-chain calculations.

The quantum Hamiltonians of the two models used in this study are H_A for (ANNNI) and H_M (for multispin). They are defined in terms of Pauli matrices $\vec{\sigma}_i$ at the L sites of a periodically bounded chain

$$H_A = -J_2 \sum_i \sigma_i^x \sigma_{i+1}^x - J_2' \sum_i \sigma_i^x \sigma_{i+2}^x - h \sum_i \sigma_i^z \ , \quad (1)$$

$$H_M = -J_2 \sum_i \sigma_i^x \sigma_{i+1}^x - J_4 \sum_i \sigma_i^x \sigma_{i+1}^x \sigma_{i+2}^x \sigma_{i+3}^x - h \sum_i \sigma_i^z \ . \quad (2)$$

The quantum versions studied here are the extreme anisotropic limits of the two-dimensional classical systems and are expected to describe the same phase structure. The regime of competing interactions ($J_2, h > 0; J_2', J_4 < 0$) can be divided in both models into paramagnetic [$h \to \infty$; $\kappa_A \equiv -(J_2'/J_2)$, $\kappa_M \equiv -(J_4/J_2) = \text{const}$], ferromagnetic ($h$, κ_A, κ_M small), and degenerate phases (when the ground state has a high degeneracy: fourfold for the ANNNI model at $h=0$, $\kappa_A > 0.5$ and eightfold for the $2+4$ spin model at $h=0$, $\kappa_M > 0.5$). The phase lines separating these regions have been determined for both models[5,9] and are reproduced in Fig. 1. In the ANNNI model, an additional feature that is important for our calculation is the existence of a disorder line,[4] which separates paramagnetic regions with low-lying excitations of zero and nonzero wave vectors, respectively.

The more delicate question, whether there exists an incommensurate phase with a characteristic wave vector that varies with the parameters h and κ between the paramagnetic and the degenerate phase, will now be discussed. For the

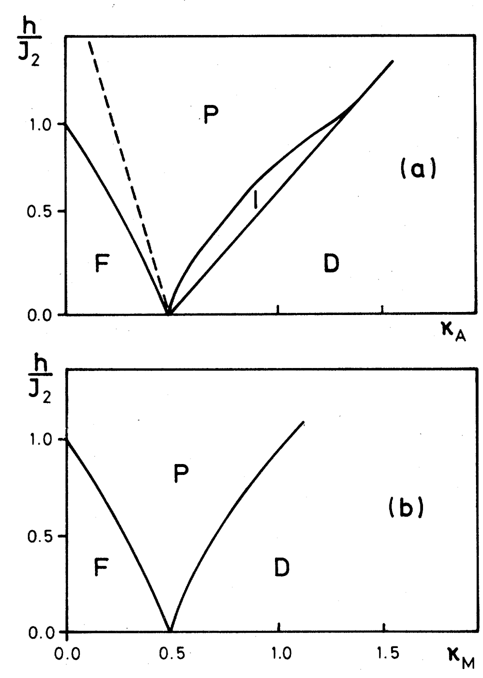

FIG. 1. Phase diagrams of the two models under study. The phases are identified as P (paramagnetic), F (ferromagnetic), and D (degenerate, fourfold and eightfold). Part (a) shows the ANNNI case (Ref. 5), with the incommensurate region sketched (I), part (b), the $2+4$ spin case (Ref. 9). The disorder line (Ref. 4) is indicated (dotted).

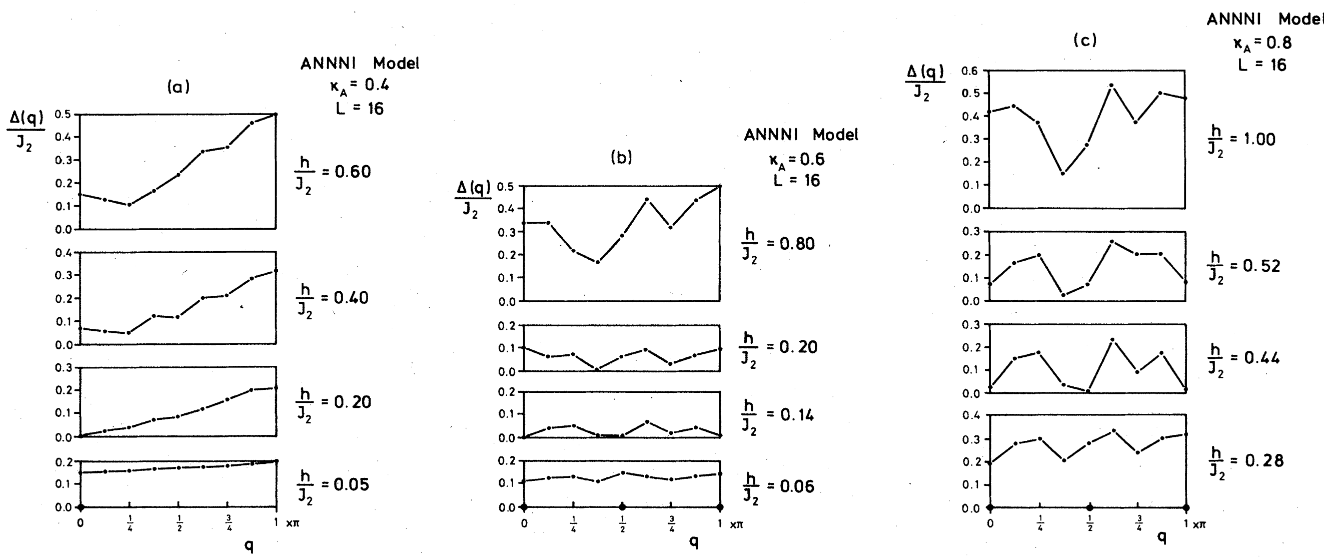

FIG. 2. Low-lying, wave-vector-dependent energy spectrum $\Delta(q)$ of the ANNNI model. Part (a) shows the paramagnetic to ferromagnetic transition as h is lowered ($\kappa=0.4$). Parts (b) ($\kappa=0.6$) and (c) ($\kappa=0.8$) show the paramagnetic to degenerate transition. There is a clear minimum for a wave vector $q_c \neq \pi/2$ that shifts to $q=\pi/2$ as h is lowered. In the spectra for the lowest h/J for each κ the degenerate ground-state energies have been shown by large dots.

ANNNI model in two dimensions, evidence for such a phase has been found using different approaches.[3] So far the $2+4$ spin model has not been investigated for this phase.

The incommensurate phase is expected to manifest itself in the low-temperature spectrum, where the energy gap is expected to vanish at a finite value $q_c = q_c(h,\kappa)$, which varies with h and κ. The energy gaps for a given wave vector q are defined by the energy difference between the lowest-lying states in this subspace and the ground state.

The interpretation of finite-chain results is complicated by the following fact:[5] for a chain of length L only the values $q=(2\pi/L)n$, n integer, are accessible and the gaps vanish only as $L\to\infty$. Furthermore, already in the paramagnetic phase, the spectrum may have a minimum at a value $q=q(h,\kappa)\neq m(\pi/2)$, m integer. Nevertheless, the spectra presented here show a clear difference for the two models under consideration. We compare, in Fig. 2, for the

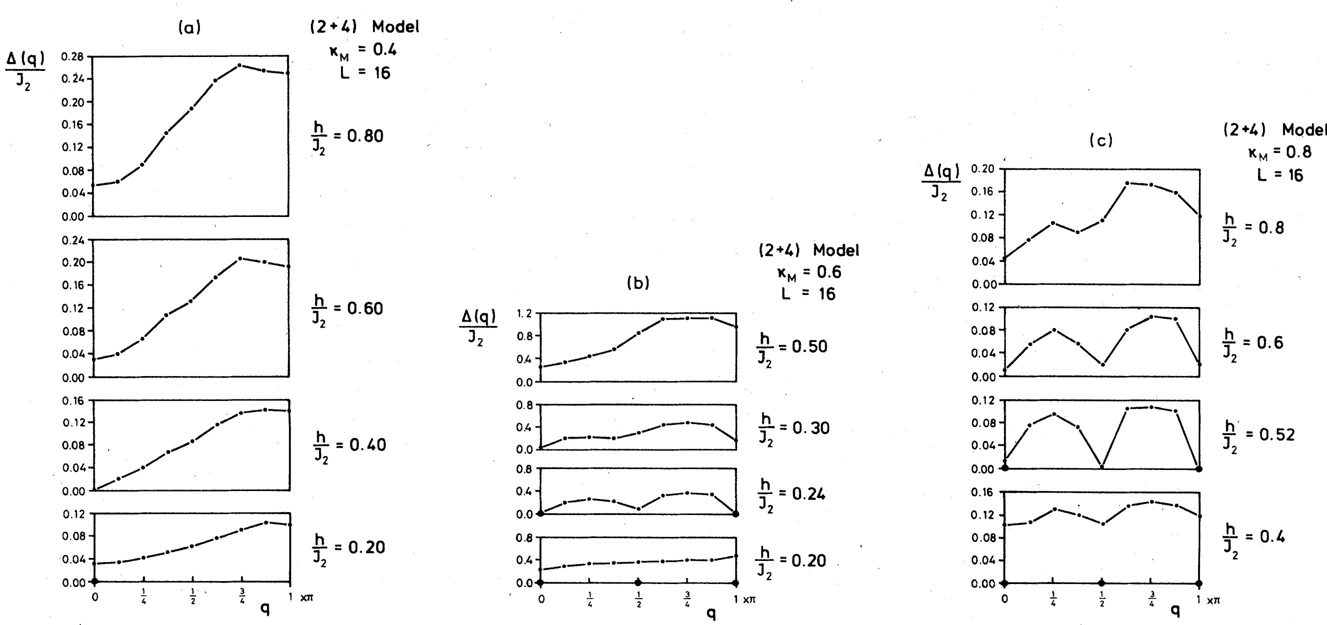

FIG. 3. Same wave-vector-dependent energy spectrum $\Delta(q)$ as for the ANNNI model in Fig. 2, here for the $2+4$ spin model. Part (a) shows the paramagnetic/ferromagnetic ($\kappa=0.4$), parts (b) ($\kappa=0.6$) and (c) ($\kappa=0.8$) the paramagnetic/degenerate transitions. In contrast to Fig. 2, there is no minimum developing for a $q_c \neq \pi/2$. Here, also, the large dots indicate the degenerate ground states.

ANNNI model and, in Fig. 3, for the $2+4$ spin model the spectra for the largest size that we can handle numerically, $L=16$. Parts (a) of Figs. 2 and 3 show the paramagnetic-ferromagnetic transition ($\kappa < 0.5$), which for both models is of the same type as the simple transverse Ising model. In the ANNNI case, a signal of the $q \neq \pi/2$ phase is visible.[4] Parts (b) and (c) of Figs. 2 and 3 show the region where the competition between the interactions is relevant ($\kappa > 0.5$). While the limited number of q values do not allow us to accurately determine at what value h_c (for $\kappa = \text{const}$) and for what wave vector q_c the gap vanishes,[5] it is clear that the two models display an entirely different spectrum. The ANNNI model develops a minimum for fixed κ and decreasing h at a value $q \neq \pi/2$ which gradually shifts towards $q = \pi/2$ as $h \rightarrow 0$. On the other hand, the $2+4$ spin model develops its minimum at $q=\pi/2$, and it remains at that value as h is lowered. Smaller L curves support this picture.

In conclusion, the difference between the ANNNI and the $2+4$ spin models expected on the basis of qualitative arguments is supported by finite-size calculations. For the ANNNI case there are indications of an incommensurate phase, whereas no such indications can be seen in the $2+4$ model.

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